

Basic Driving Dynamics of Cyclists

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Abstract. In this work we introduce the Necessary-Deceleration-Model (NDM) which is a car-following-model developed to investigate driving behavior of bicycles. For this purpose the derivation of the mathematical description of the NDM is investigated. For the sake of calibration and validation of the model, several experiments are performed. The results of the experiments are presented and examined. Finally, the limits and possibilities of the NDM are discussed.

Keywords: Driving dynamics · Bicycles · Car-following-model · Ordinary differential equations

1 Introduction

Recently people's awareness of the environment changed due to the global warming and lack of resources. Many people use their bicycle not only as a leisure activity but also in order to reach their work site [8]. Therefore it is not surprising that the popularity of cycling has grown in the last years and is still increasing. In Central Europe more and more tracks for cyclists are built or are still in planning. Furthermore, the use of e-bikes and pedelecs increased, such that special routes for fast bicycles are in consideration.

In some situations, where special routes for bicycles are missing, cyclists must use together with cars the same route. However, neither traffic systems in which only cyclists are involved nor heterogeneous traffic systems with cars, motorcycles, bicycles, e-bikes and pedestrians are well investigated [7, 11, 12].

A long-range goal is to find and determine characteristics which are necessary for planning and designing roadway facilities and for controlling and regulating traffic flow [10].

In order to achieve this goal we investigate basic driving dynamics with help of the Necessary-Deceleration-Model scrutinized in the following sections.

2 The Necessary-Deceleration-Model

2.1 Miscellaneous

The Necessary-Deceleration-Model (NDM) is a uni-dimensional car-following-model continuous in space especially designed to simulate the driving behavior of bicycles.

Like car-following-models in general [1, 2], the NDM describes the driving behavior of a vehicle from its own perspective while it is in a traffic system. That means the state variables of each vehicle are updated at each time step and determined by mathematical regularities (see Fig. 1).

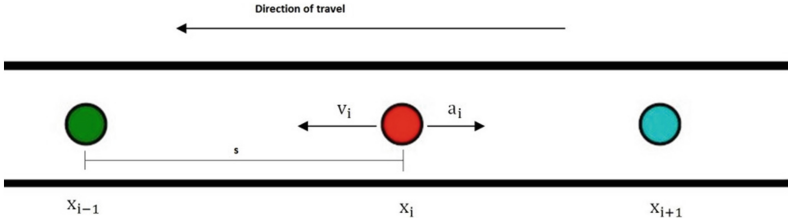


Fig. 1. Spatial coordinate, velocity and acceleration of a virtual vehicle in a uni-dimensional car-following-model while it is in a breaking progress.

A driver, regardless of the type of vehicle he uses, has a desired speed which he tries to reach if there is no slower vehicle or obstacle in front of him.

In case there is a slower moving predecessor, the driver will decrease his speed by decelerating until his speed is aligned to the speed of the predecessor. The amount of the deceleration depends on the current headway s [1–3] and the current difference of the velocities $\Delta v (= v_{i-1} - v_i)$ between the considered driver and his predecessor [1, 3, 4, 9]. Considering s and Δv in every time step the deceleration has to be high enough to avoid collisions [1, 4, 9].

As all vehicles can only decrease their speed with a certain maximum deceleration, the driver has to maintain a safety distance $d(v)$ depending on his own speed [1, 3].

2.2 Derivation of the NDM

The NDM consists of three components acc , dec_1 and dec_2 representing the acceleration and the deceleration of a vehicle. As the NDM is discrete in time, the velocity and the spatial coordinate of a driver in the next time step are numerically calculated by means of an adequate numerical solver and considering the superposition of the acceleration and deceleration terms as expressed in the following equations:

$$x(t + \Delta t) = x(t) + v(t) \cdot \Delta t, \quad (1)$$

$$v(t + \Delta t) = v(t) + (acc + \min(dec_1 + dec_2, b_{max})) \cdot \Delta t, \quad (2)$$

with: Δt = a time constant.

Acceleration. The first term acc is representing the free acceleration of a vehicle until it reaches its desired speed v_0 . For this purpose the following expression [5] is used:

$$acc = \frac{v_0 - v}{\tau}. \quad (3)$$

The relaxation time τ regulates how fast a vehicle can accelerate to its desired speed.

If a driver falls below the safety distance $d(v)$ it is not necessary and furthermore not useful for him to continue accelerating. That means the term acc is only supposed to be effective if the vehicle's current distance s is bigger than the safety distance $d(v)$:

$$acc = \begin{cases} 0, & s \leq d(v) \\ \frac{v_0 - v}{\tau}, & s > d(v). \end{cases} \quad (4)$$

For simplicity the safety distance $d(v)$ is assumed to be linearly velocity-dependent:

$$d(v) = s_0 + l + T \cdot v, \quad (5)$$

with:

- s_0 = distance between two standing vehicles (see Fig. 2),
- T = constant of proportionality,
- l = length of the considered vehicle.

Deceleration. As mentioned earlier a driver has to decrease his speed with a deceleration that is high enough to avoid a collision with an obstacle in front of him. For this purpose we introduce the fundamental physical equation

$$s_{nec} = \frac{(\Delta v)^2}{2b}. \quad (6)$$

In this case s_{nec} is representing the necessary braking distance of a vehicle to avoid a collision. It depends on the square of the relative speed if the vehicle decreases its speed with a certain deceleration b . If we rearrange Eq. (6) to

$$b_{nec} = \frac{(\Delta v)^2}{2s} \quad (7)$$

we obtain the necessary deceleration b_{nec} to avoid a collision depending on the current difference of the speeds of the considered vehicle and his predecessor and the current headway s .

However, s describes the distance between the centers of two vehicles. As vehicles of all type have a physical length, we need to increase the braking distance, so that the foremost point of a vehicle does not touch the tail of the front vehicle. Furthermore it is common to preserve some security distance between the vehicles even if all of them

had come to a standstill. Taking these considerations into account we obtain from Eq. (7)

$$b_{nec} = \frac{(\Delta v)^2}{2\left(s - 2 \cdot \frac{l}{2} - s_0\right)} \quad (8)$$

as the necessary deceleration for a vehicle to avoid a collision and to keep a certain distance to the front vehicle after the braking process (see Fig. 2). Hereby we assume that all vehicles have the same length l .

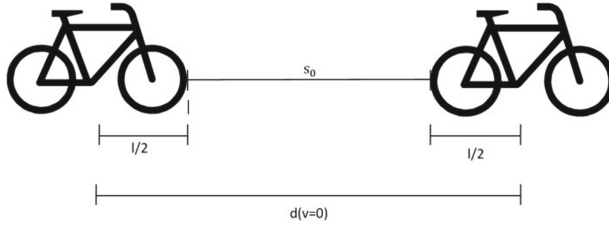


Fig. 2. Aspired distance $d(v)$ between two standing vehicles.

As the necessary deceleration b_{nec} is expected to be high enough to avoid headways below $s_0 + l$, there is for now no need to consider the case $s - s_0 - l \leq 0$.

Since no type of vehicle is able to slow down with a deceleration higher than a certain maximum, we have to consider the limitation of the necessary deceleration b_{nec} to a maximum b_{max} . We obtain:

$$b_{nec} = \min\left(\frac{(\Delta v)^2}{2(s - l - s_0)}, b_{max}\right). \quad (9)$$

Because of the limitation of b_{nec} to the maximum b_{max} the minimal distance $s_0 + l$ can be undercut in dangerous braking situations. However, using realistic values for the model parameters these undercuts are negligibly small. Hence, they can be tolerated.

To distinguish b_{nec} from further, later introduced, deceleration terms we define:

$$dec_1 := b_{nec}. \quad (10)$$

The NDM is already up to now able to simulate plausible driving behaviour of vehicles in the three fundamental traffic situations (free accelerating, moving in a group, approaching an obstacle) of the longitudinal dynamic [1], except for the following situation:

As a driver approaches his predecessor, he decreases his speed until it is aligned to the front vehicle's speed. Assuming the considered driver has to decelerate with a high deceleration, so that he undercuts the safety distance, there is no deceleration which let him fall back to maintain the safety distance again. Because $dec_1 = 0$ if the velocities

of two considered vehicles are aligned, an additional deceleration term must be considered, namely:

$$dec_2 = \frac{b_{max}}{(l - d(v))^2} \cdot (s - d(v))^2. \quad (11)$$

The second deceleration term dec_2 is only effective if $s \leq d(v)$.

The fact that dec_2 vanishes if $s = d(v)$ guarantees a continuous deceleration in every situation.

Since the distance between two drivers quickly increases if the front one is vastly faster, it is only necessary for the second deceleration term to be effective if the difference between the velocities is lower than a constant parameter ϵ . We obtain:

$$dec_2 = \begin{cases} \frac{b_{max}}{(l - d(v))^2} \cdot (s - d(v))^2, & s \leq d(v); \Delta v \leq \epsilon \\ 0, & \text{otherwise.} \end{cases} \quad (12)$$

As dec_1 and dec_2 can be effective simultaneously in certain situations we have to limit their summation to the maximum possible deceleration b_{max} due to the previously mentioned fact, that a vehicle can only decrease its speed with a deceleration not more than a certain maximum.

3 Calibration and Validation of the NDM

3.1 Miscellaneous

For calibration and validation of the NDM we evaluate the bicycle experiments dated on May, 6th 2012 in Wuppertal, Germany performed by the University of Wuppertal in cooperation with Jülich Supercomputing Center. About 30 participants aged 7–66 years were involved. Two of the participants used an e-bike.

Using the videos filmed by two cameras that overlooked the whole area of the car park (see Fig. 3) we were able to extract the trajectories of the cyclists. As part of the test runs two different types of experiments were carried out.

The first experimental run called *Single Experiments* was performed to investigate the individual behaviour of cyclists while accelerating. For this purpose participants have to increase their speed from zero until the desired speed is reached.

We carried out the second experimental run called *Group Experiments* to investigate the collective driving behaviour of cyclists while moving in a group. There for a settled number of cyclists are supposed to drive through an oval track simultaneously without being allowed to overtake (see Fig. 3). The participants were told to drive normally without haste.

3.2 Data Analysis Methods

The measuring range was located at one of the straight lines of the oval track. Its length Δx has been set to 20 m.



Fig. 3. Experimental set-up (*Group Experiment*) with 33 participants.

The *Single Experiments* or rather the individual acceleration progress of a cyclist took place in the measuring area without exception. For every recorded time step j of an acceleration phase we calculate the current speed of the regarded vehicle i (see Eq. 13).

$$v_{i,j} = \frac{x_j - x_{j-1}}{t_j - t_{j-1}}. \quad (13)$$

For the sake of measuring the density and velocity of the system while performing the Group Experiments we make use of the following data analysis methods:

For every pass of a bicycle through the measuring area ($t \in [t_{in}, t_{out}]$) its mean speed is determined by using Eq. (14).

$$v_i = \frac{\Delta x}{t_{out} - t_{in}} [6]. \quad (14)$$

Additionally at every time step a bicycle is located in the measuring area the amount of bicycles N in the area is counted. We use Eq. (15) to obtain the mean density assigned to each participant passing through the measuring range. (In Comparison with [6] Eq. (15) is changed for our purpose to analysis discrete one-dimensional data.)

$$\rho_i = \frac{1}{(t_{out} - t_{in}) \cdot \Delta x} \cdot \sum_{t_i=t_{in}}^{t_{out}} N_{i,t_i} \quad [6]. \quad (15)$$

Taking into account the mean speed and mean density assigned to a pass through of a vehicle as a tuple we are able to set up the velocity-density and flow-density relation. The flow J_i is calculated by Eq. (16).

$$J_i = v_i \cdot \rho_i. \quad (16)$$

For the sake of investigating the velocity-headway relation (see Fig. 6) we calculate the velocity with the aid of Eq. (14) and make use of the following headway definition:

$$d_i = \frac{1}{\rho_i}. \quad (17)$$

3.3 Driving Behaviour While Accelerating

Experimental Results. The average desired speed of the participants is about 4.3 m/s (corresponds 15.5 km/h) (see Fig. 4). The standard deviation of the desired velocity

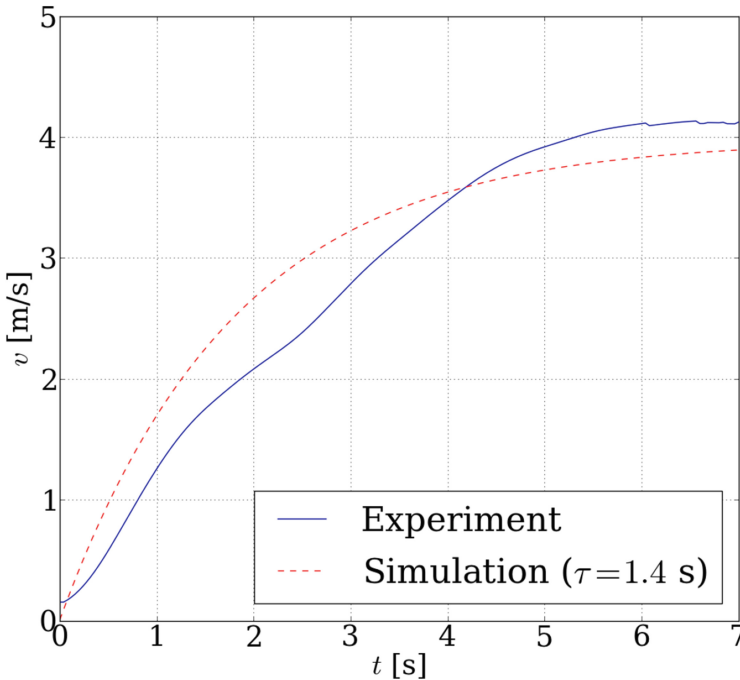


Fig. 4. Free flow acceleration of real cyclists and cyclists simulated using the NDM.

resulted in 0.57 m/s (corresponds 2.05 km/h). On average it takes 20–25 m to accelerate to the desired speed and the duration of the acceleration phase is about 7 s.

Calibration. As shown in Fig. 4 the exponential acceleration that emerges from Eq. (3) is in good agreement with the empirical acceleration process of the participants. Therefore, the model parameter v_0 is assumed as Gaussian-distributed with a mean value of 4.3 m/s and a standard deviation of 0.57 m/s. The parameter τ is set to 1.4 s.

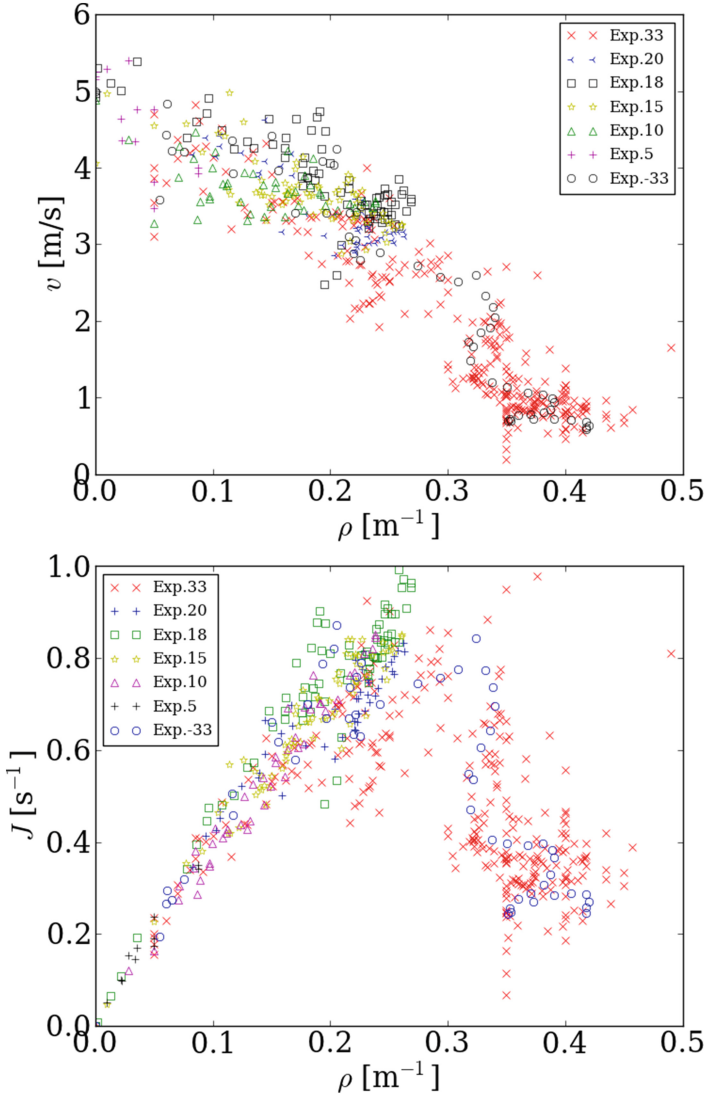


Fig. 5. Experimental flow/density-velocity relation (Exp. -33 = Exp. 33 (anti-clockwise)).

3.4 Driving Behaviour While Moving in a Group

Experimental Results. By analyzing the results of several experimental runs of the *Group Experiment* with various numbers of participants the fundamental diagrams can be set up. For this purpose experimental runs with 5, 7, 10, 15, 18, 20 and 33 participants have been performed. In the run with 33 vehicles the drivers had to move clock- and anti-clockwise. The length of the circuit was set to 86 m.

The following characteristics of driving behaviour can be extracted from Fig. 5.

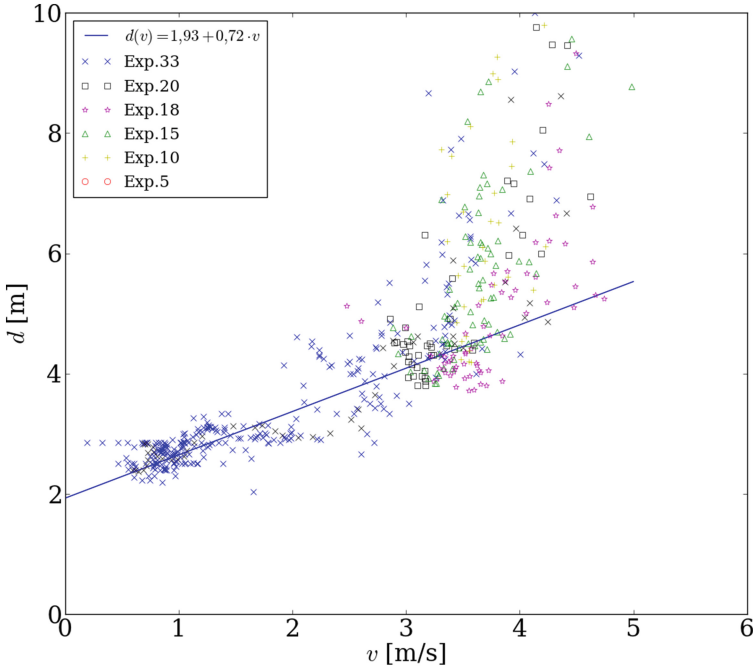


Fig. 6. The headway of cyclists with respect to the speed.

Table 1. Adapted model parameters of the NDM calibrated by using the measurements of the bicycle experiments.

Model parameter	Value
v_0	Gauss-distributed (mean-value: 15.5 km/h; standard deviation: 2 km/h)
τ	1.4 s
T	0.72 s
s_0	0.2 m
l	1.73 m
b_{max}	-5.5 m/s
ϵ	1.8 km/h

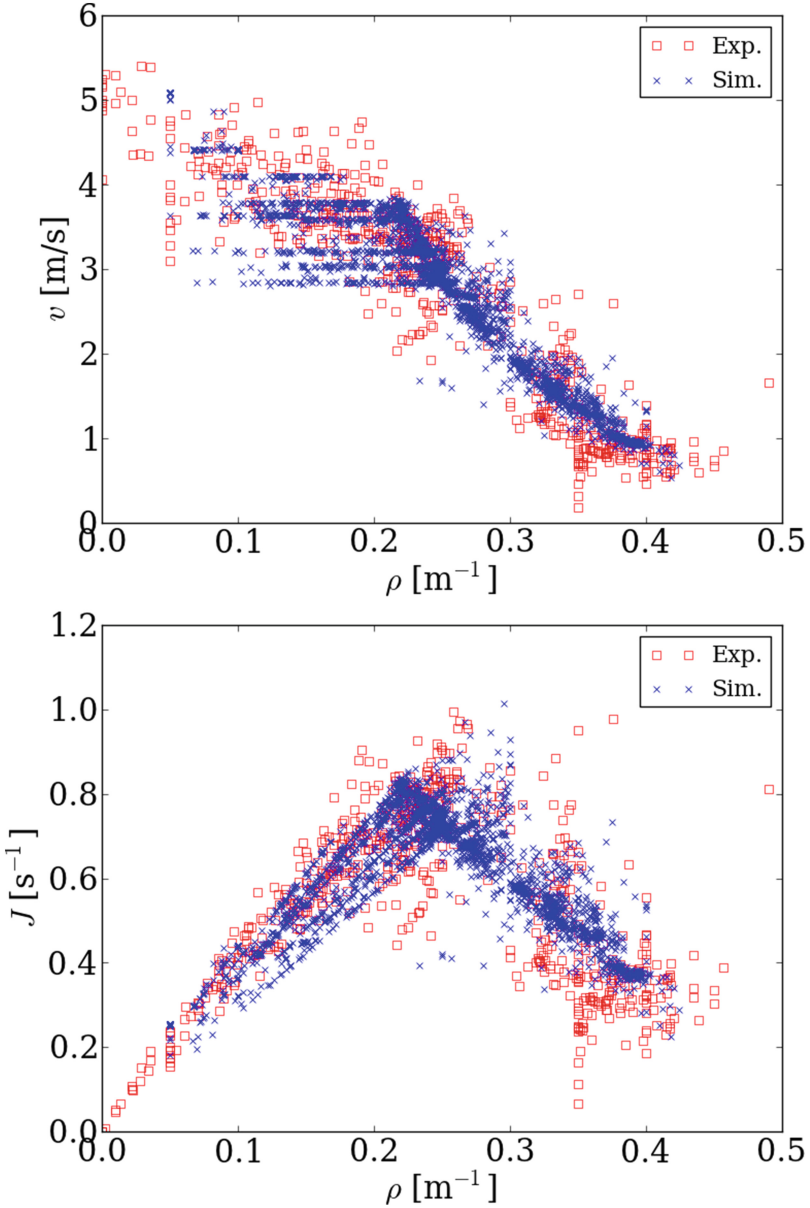


Fig. 7. Comparison of the fundamental diagrams of the experimental runs and the simulation runs ($N = 1, 2, 3, 4, 5, 8, 10, 15, 18, 20, 22, 25, 28, 30, 33$) performed by the NDM using the model parameter listed in Table 1 and the numerical integration interval $\Delta t = 0.01\text{s}$.

The maximum flow of ca. 0.8 s^{-1} is located at the density of $\sim 0.25 \text{ m}^{-1}$. By density regions below this density, the mean speed of drivers is mainly determined by the speed of the slowest one, since he is the only one who can freely accelerate,

respectively, moving with his desired speed. Above the density 0.25 m^{-1} the slowest driver is hindered by his predecessor as well, that is, the speed of the system is no more determined by the desired speed of the slowest vehicle.

Calibration. We already could determine the real values of the desired speed and the relaxation time of the acceleration progress to calibrate the NDM by investigating the *Single Experiments*. Furthermore we obtained the mean length of a bicycle (1.73 m) by surveying all bicycles before starting the experiments.

Adapting the results of experiments according to the maximum deceleration of bicycles performed by [13] we set the parameter b_{\max} to 5.5 m/s^2 .

We make use of the results of the experimental runs to calibrate the remaining model parameters. To adjust the parameters s_0 and T we build up the headway-velocity relation of the experiment (see Fig. 6).

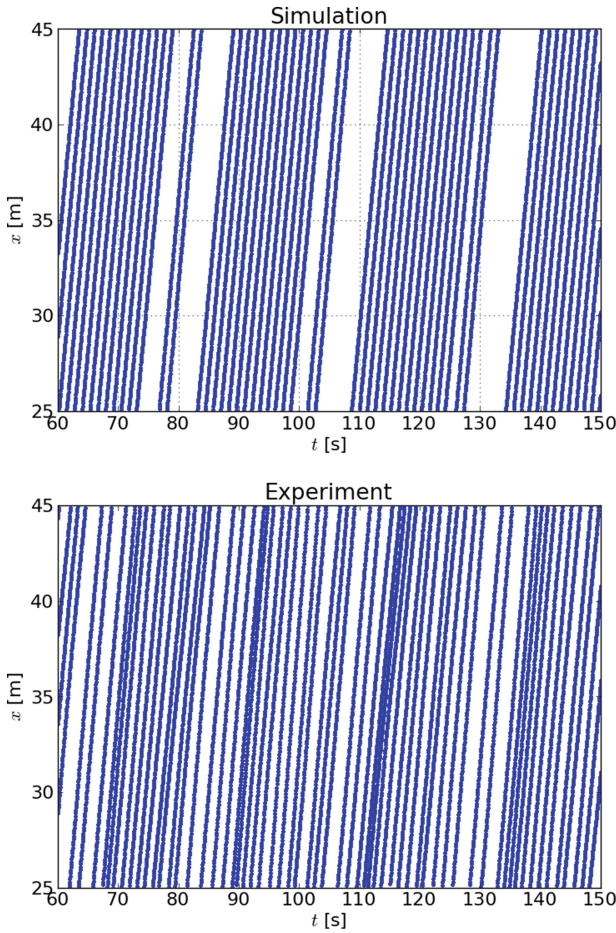


Fig. 8. Comparison of the trajectories ($N = 15$) of the experimental and the simulation runs performed by the NDM using the model parameter listed in Table 1 and the numerical integration interval $\Delta t = 0.01s$.

We assume that only the velocity-headway relation located lower than the velocity of the slowest driver, namely ~ 3.35 m/s, is relevant to calibrate the parameters of the safety distance. That means we solely regard the systems in which the density is high enough so that every driver is hindered to drive freely by his predecessor. Hence, the distance between two vehicles can be understood as the safety distance in this case.

Using two tuples of the headway-velocity relation taken from the area below the velocity of ~ 3.35 m/s we fit a linear function which represents the safety distance (see Fig. 6). Note that the linear function can only approximately describe the safety distance. However, for the sake of calibration our model parameters this procedure is sufficient.

The linear relationship of the safety distance and the speed is found to be as:

$$d(v) = 1.93 + 0.72 \cdot v \quad (18)$$

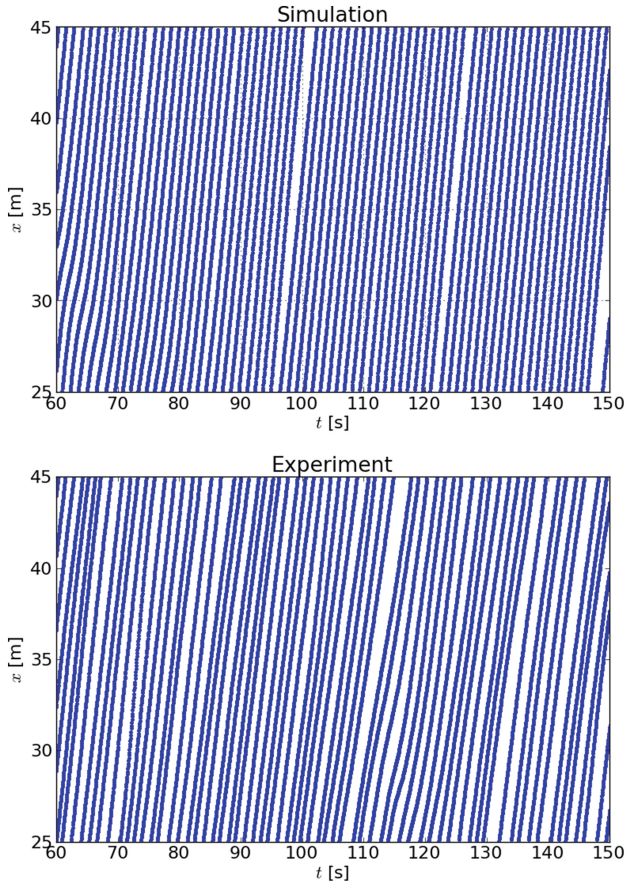


Fig. 9. Comparison of the trajectories $N = 20$ (Adjustment of the parameters: see Caption Fig. 8).

Accordingly we can set the parameter T to 0.72 s. As the safety distance is described by $s_0 + l$ and we already have measured the length of a vehicle l as 1.73 m (see above) we notice 0.2 m to calibrate the parameter s_0 .

We have calibrated every single model parameter by using experimental data except for the parameter ϵ . We set ϵ to 1.8 km/h as an preliminary estimate.

The adapted model parameters are summarized in Table 1.

Validation. Considering the calibrated model parameters of the NDM (Table 1) the relation of velocity, density and flow can be realistically reproduced by the NDM (see Fig. 7).

The qualitative behaviour of cyclists is investigated by comparing the experimental and simulated trajectories of selected systems. In Fig. 8 we show the trajectories of a

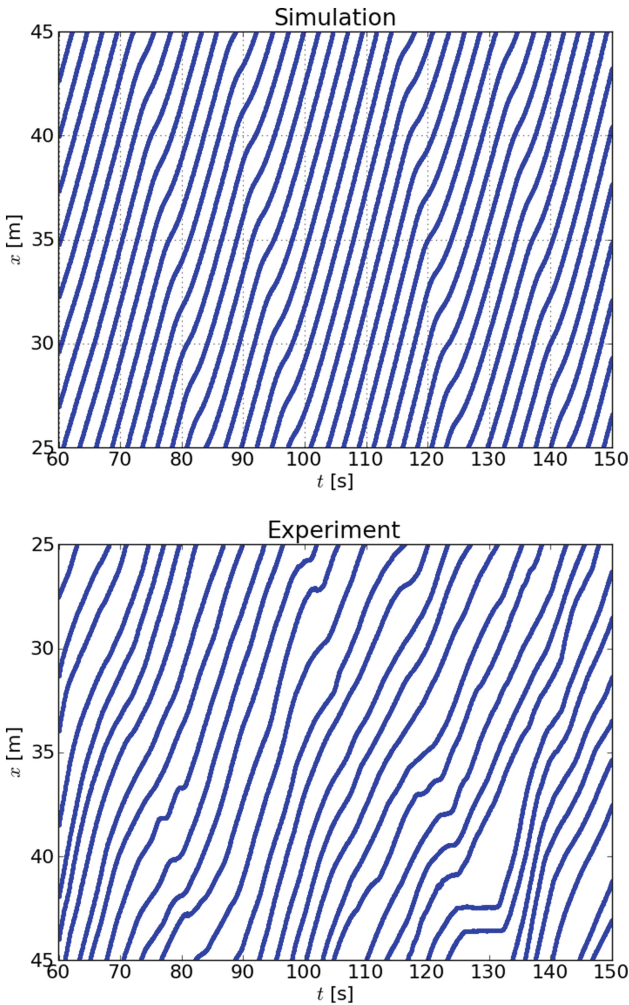


Fig. 10. Comparison of the trajectories $N = 33$ (Adjustment of the parameters: see Caption Fig. 8).

run with 15 participants. In this system the group building or rather queue building of the cyclists is predominating. There is at least one driver who can drive freely.

We see this driving behaviour as well in the simulation as in the experiment.

By comparing the subfigures of Fig. 9 we notice that the cyclists of the experimental and the simulated run with 20 drivers show similar patterns again. The system is predominated by an equal distribution of the participants. The drivers still drive with a constant speed.

In Fig. 10 the trajectories of the system with 33 participants are shown. By investigating the figure we notice the occurrence of congestions. That is, although the drivers do not come to a standstill most of the times they have to vary their speed partly noticeably. Again, the simulated and the real cyclists show these patterns.

4 Conclusion, Limits and Possibilities of the NDM

Since the NDM is developed by using fundamental physical relationships, the model parameters reflect physical characteristics of the driver or the vehicle. The NDM can model plausible driving behaviour in the three traffic situations of the longitudinal dynamic (free accelerating, moving in a group, approaching an obstacle).

By using the calibrated parameters, collisions do not occur. The driving dynamics responds only slightly to changes in the model parameters. Figures 4, 5, 6, 7, 8, 9, and 10 show that the model provides reasonable quantitative and qualitative results when plausible values for the model parameters are used. Especially the comparison of the fundamental diagrams (see Fig. 7) shows that the NDM can model realistic driving behaviour of cyclists moving in a group. Furthermore, as seen in Fig. 4 the NDM is able to replicate the free acceleration progress of cyclists.

By comparing the trajectories of simulated and real drivers (see Figs. 8, 9, and 10) we notice similar behavioural features in various closed traffic systems with different densities.

Although originally the NDM has been developed to model the dynamics of cyclists in a traffic system, it is technically possible to simulate the dynamics of cars or shared-used routes as well. However, a thoroughly investigation of the NDM to investigate the exact dynamics of the mentioned systems is scheduled for future works.

Acknowledgements. We would like to thank Wolfgang Mehner and Maik Boltes working at Jülich Research Center for filming and extracting the trajectories of the bicycle experiments dated on 6th May 2012. Furthermore we would like to thank all the participants and helpers who were involved at the bicycle experiments and remain disciplined, so that excellent results of the experimental runs could be achieved.

References

1. Treiber, M., Kesting, A.: Verkehrsdynamik und -simulation. Springer Lehrbuch, Heidelberg (2010)
2. Bando, M., Hasebe, K., Nakayama, A., Shibata, A., Sugiyama, Y.: Dynamical model of traffic congestion and numerical simulation. *Phys. Rev. E* **51**, 1035–1042 (1995)

3. Gipps, P.G.: A behavioural car-following model for computer simulation. *Transp. Res. B Methodol.* **15**, 105–111 (1981)
4. Treiber, M., Hennecke, A., Helbing, D.: Congested traffic states in empirical observations and microscopic simulations. *Phys. Rev. E* **62**, 1805–1824 (2000)
5. Pipes, L.A.: An operational analysis of traffic dynamics. *J. Appl. Phys.* **24**, 274 (1953)
6. Zhang, J., Klingsch, W., Schadschneider, A., Seyfried, A.: Transitions in pedestrian fundamental diagrams of straight corridors and T-junctions. *J. Stat. Mech. Theor. Exp.* **2011**(06), P06004 (2011)
7. Faghri, A., Egyhaziova, E.: Development of a computer simulation model of mixed motor vehicle and bicycle traffic on an urban road network. *Transp. Res. Rec.: Safety and Human Performance*, **1674**(1999), 86–93 (1999)
8. Gould, G., Karner, A.: Modeling bicycle facility operation: a cellular automaton approach. *Transp. Res. Rec.: J. Trans. Res. Board*, **2140**, 157–164 (2009)
9. Jiang, R., Wu, Q., Zhu, Z.: Full velocity difference model for a car-following theory. *Phys. Rev. E* **64**, 017101 (2001)
10. Navin, F.P.D.: Bicycle traffic flow characteristics: experimental results and comparisons. *ITE J.* **64**, 31–36 (1994)
11. Arasan, V.T., Koshy, R.Z.: Methodology for modeling highly heterogeneous traffic flow. *J. Transp. Eng.* **131**, 544–551 (2005)
12. Minh, C.C.: Kazushi Sano and Shoji Matsumoto (2005) The speed flow and headway analyses of motorcycle traffic. *J. East. Asia Soc. Transp. Stud.* **6**, 1496–1508 (2005)
13. Bäumler, H.: Bremsverzögerung von modernen Fahrrädern 2.3.4., *Verkehrsunfall und Fahrzeugtechnik, Fachblatt für Kraftfahrzeug* 47.11:347 (2009)

Simulation of Urban Mobility

First International Conference, SUMO 2013, Berlin,
Germany, May 15-17, 2013. Revised Selected Papers

Behrisch, M.; Krajzewicz, D.; Weber, M. (Eds.)

2014, VIII, 175 p. 98 illus., Softcover

ISBN: 978-3-662-45078-9