Rough Dynamic Response Prediction for Simple Railway Bridges Subjected to High-Speed Trains

Christoph Adam and Patrick Salcher

Abstract This paper describes an efficient approach for prediction of the dynamic peak response of shear deformable railway bridges subjected to high-speed trains, which is based on response spectra. In the proposed response spectra the modal peak response is presented as a function of a non-dimensional modal speed parameter and the bridge span to wagon length ratio. A rough estimate of the dynamic peak bridge response is found by modal combination of the modal peak responses identified from readily available response spectra considering the actual train and bridge parameters.

1 Introduction

If a high-speed train passes a railway bridge with a critical speed, resonance effects may have a severe impact on the train-bridge interaction system. In such a situation the serviceability may be impaired, and critical stresses exceeded, and thus a quasistatic computation of the bridge response is insufficient. Higher modes may contribute significantly in particular to the bridge acceleration, however they cannot be captured with a simplified quasistatic analysis. A large bridge acceleration response leads to instability of ballast, and passengers discomfort. Depending on the fundamental bridge frequency and bridge geometry, Eurocode 1 [1] allows for single-span bridges a quasistatic analysis, if the maximum travel speed is smaller than or equal to 200 km/h. However, comparative analyses of the authors have shown that a quasistatic computation may underestimate the actual dynamic bridge response even in the admitted parameter range of Eurocode 1 [1].

Various mechanical models of different degrees of sophistication have been developed to predict the dynamic response of railway bridges subjected to

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high-speed trains, see e.g. [2–4]. For example, Cojocaru et al. [5] model the train as an additional elastic beam, which crosses the bridge. Some of the numerical studies have also been validated by experiments [6]. A comprehensive state-of-the-art of the analysis of high-speed train-bridge interaction is provided in the textbook of Yang et al. [7].

Most generally, the assessment of the dynamic bridge response is based on complex numerical models leading to time-consuming time history analysis. In an effort to reduce this effort Hauser and Adam [8] have translated the response spectrum methodology from earthquake engineering into bridge dynamics. This methodology permits for simple bridges a rough, however quick and easy to apply assessment of the peak response induced by high-speed trains, which is particular useful in the initial design phase. Simultaneously, Fink and Mähr [9] have developed independently a similar response prediction concept. Salcher [10] and Adam and Salcher [11] derived for a large number of different characteristic train sets response spectra for both single-span and continuous two-span bridges modeled as Bernoulli-Euler beam. Later, also Spengler [12] has seized this idea studying the effect of high-speed trains on the response of railway bridges. In the present study, a modified response spectrum concept is introduced to include the effect of shear deformations of simply supported bridges for the considered train-bridge interaction problem, compare also with [13].

2 Mechanical Model of the Bridge-Train Interaction System

The bridge and the passing train vehicle represent a rather complex interactive system with time-dependent mechanical properties. Depending on the response quantity to be predicted and on the required accuracy mechanical modeling of this system may be performed with different degree of sophistication [7]. In a detailed model a spring-mass system describes the dynamic behavior of each train car consisting of body, bogies, and viscoelastic connection elements [4]. For example, the contact problem between rails and wheels, and the non-linear behavior of the ballast should be specified appropriately. The numerical solution of the resulting mechanical model is in general computationally expensive, and might come along with numerical stability problems. Thus, detailed system modeling is not efficient in the process of initial bridge design.

Since in the design process of a bridge the properties of all passing trains to be developed during the bridge life cycle cannot be foreseen, analysis should not be performed considering particular characteristics of the vehicle [3]. Consequently, in the simplest approach the passage of a bridge by a high-speed train is considered as a sequence of moving concentrated forces of constant speed v. Each concentrated load represents the static reaction force of a train axle. This model, which is adopted for the present study, disregards the inertia effect of the train, and thus, it leads in general to slightly conservative bridge response predictions [3].

In this paper simply supported single span bridges with a single track are analyzed. In contrast to previous studies [11, 12] the effect of shear deformation is taken into account, which may play a significant role for truss and/or short span bridges. Consequently, it is assumed that lateral bridge vibrations $w(x,t)$ induced by the N axle loads F_i, $i = 1, 2, \ldots, N$ of the considered train, are described sufficiently accurate by means of the partial equations of motion of a shear beam with constant structural parameters (mass per unit length ρA, bending stiffness EI, shear stiffness GA_S) [14]

$$\rho A \ddot{w} + EI w_{,xxxx} - \rho A \frac{EI}{GA_S} \ddot{w}_{,xx}$$
$$= \sum_{i=1}^{N} F_i \left[\delta(x - \xi_i) - \frac{EI}{GA_S} \delta_{,xx}(x - \xi_i) \right] \left[H(t - t_i^0) - H(t - t_i^E) \right] \quad (1)$$

The Dirac delta function $\delta(x - \xi_i)$ describes mathematically the action of the ith axle load with amplitude F_i, which is located at time t at length coordinate $\xi_i = vt - s_i$ (of the bridge). The unit step functions H specify the arrival and departure of F_i at time instants $t_i^0 = s_i/v$ and $t_i^E = (s_i + L)/v$, respectively. v is the constant train speed, s_i denotes the initial location of F_i, and L is the span of the bridge [11].

Modal decomposition of the lateral displacement $w(x,t)$ into the mode shapes ϕ_n, $n = 1, \ldots, \infty$, of the actual boundary value problem, $w(x,t) = \sum_{n=1}^{\infty} q_n(t)\phi_n(x)$, leads to an infinite set of ordinary oscillator equations of motions for the modal coordinates q_n, viscous damping is modally added,

$$\ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \omega_n^2 q_n = \frac{1}{m_n} \sum_{i=1}^{N} F_i \phi_n(\xi_i) \left[H(t - t_i^0) - H(t - t_i^E) \right], n = 1, \ldots, \infty \quad (2)$$

The solution of this equation can be found by standard methods of structural dynamics such as Duhamel's integral [15].

The nth mode shape ϕ_n, the corresponding natural circular frequency ω_n, and modal mass m_n of a simply supported shear beam are derived as [14]

$$\omega_n = \frac{n^2\pi^2}{L^2} \left(\frac{EI}{\rho A} \right)^{1/2} \left(1 + \frac{n^2\pi^2}{L^2} \frac{EI}{GA_S} \right)^{-1/2},$$
$$\phi_n = \sin \frac{n\pi x}{L}, \quad m_n = \frac{\rho A L}{2} \quad (3)$$

Structural bridge damping is a fundamental bridge parameter, in particular for excitation at resonance. In Eurocode 1 [1] lower limit values for ζ_n used in this study are defined. They depend on the bridge structure and on span L.

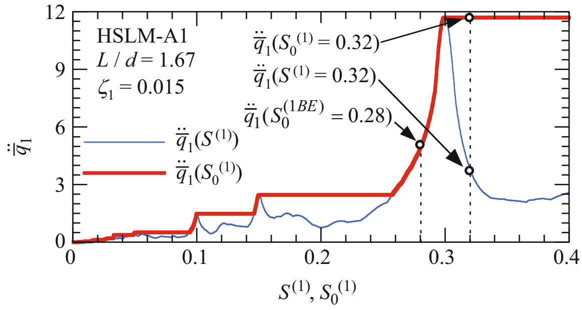

Fig. 1 Comparison of modal peak accelerations $\ddot{\bar{q}}_1(S_0^{(1)})$ and $\ddot{\bar{q}}_1(S^{(1)})$ of the first mode

3 Proposed Response Spectra

In a response spectrum the dynamic peak response of a single-degree-of-freedom oscillator is presented as a function of characteristic excitation and structural parameters. A readily available response spectrum provides the design engineer with a tool to predict the dynamic peak response without performing computational expensive time-history analyses. The characteristic excitation parameters of the considered problem are the ratio of bridge length L to wagon length d, L/d, and the modal speed parameters $S^{(n)}, n = 1, 2, \ldots$. For given length ratio L/d and nth speed parameter $S^{(n)}$ the nth modal peak bridge response is presented in non-dimensional form,

$$\bar{q}_n = \max|q_n(t)|\,\frac{\rho AL}{F_{\max}}\left(\frac{\omega_n}{2\pi}\right)^2, \quad \ddot{\bar{q}}_n = \max|\ddot{q}_n(t)|\,\frac{\rho AL}{F_{\max}}, \quad S^{(n)} = \frac{\pi v}{\omega_n L} \tag{4}$$

F_{max} is the maximum single force of the analyzed train model. Since in general the peak response does not occur at the maximum admissible speed $S_0^{(n)}$ but at a lower speed $S^{(n)} < S_0^{(n)}$, coefficients $\bar{q}_n(S_0^{(n)})$ and $\ddot{\bar{q}}_n(S_0^{(n)})$ denote the nth modal peak displacement and acceleration, respectively, in the range $0 \leq S^{(n)} \leq S_0^{(n)}$. As an example, Fig. 1 shows for the specific train load model HSLM-A1 according to [1], a length ratio of $L/d = 1.4$, and viscous damping of $\zeta_1 = 0.015$ the peak acceleration of the first mode, plotted against the corresponding modal speed parameter. The bold line shows the modal peak acceleration as a function of the maximum admissible speed parameter $S_0^{(1)}$ (which enters the response spectrum), while the thin line corresponds to the actual modal peak response at the specific speed parameter $S^{(1)}$.

Based on the mechanical model of the bridge-train interaction system presented before, the authors have derived modal response spectra for simply supported bridges subjected to high-speed trains by series of time history analyses for the HSLM-A load models of Eurocode 1 [1] and for real European high-speed trains. For each train load model and viscous damping specified according to Eurocode 1 [1] these readily available spectra are presented as a function of $S_0^{(n)}$ and L/d.

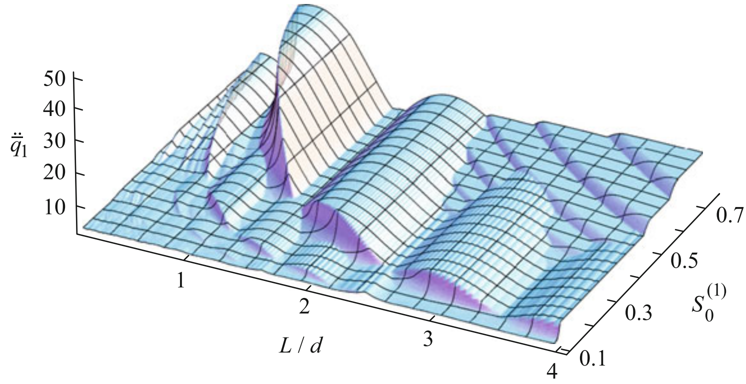

Fig. 2 Acceleration response spectrum of the first mode. HSLM-A1 load model. Damping coefficient $\zeta_1 = 0.015$

Figure 2 shows as a showcase the acceleration response spectrum of the first mode for the HSLM-A1 train set and bridge damping of $\zeta_1 = 0.015$. It can be seen that for certain values of $S_0^{(1)}$ and L/d the gradient of the response surface is very steep, which makes the peak response vulnerable to small parameter variations.

4 Application

Based on the parameters $S_0^{(n)}$ and L/d of the actual considered bridge problem the modal peak responses $\bar{q}_n(S_0^{(n)})$, $\ddot{\bar{q}}_n(S_0^{(n)})$ are identified from the corresponding response spectra. The number of included modes k ($n = 1, \ldots, k$) depends on the considered response quantity. In general, the fundamental mode approximates sufficiently accurate the peak deflection $\max|w(x,t)|$, i.e. $k = 1$. When estimating the maximum acceleration $\max|\ddot{w}(x,t)|$, three modes should be taken into account ($k = 3$). In all cases, the selection of k should be based on a convergence test taking into account the symmetry and antisymmetry of the mode shapes. Note that the peak response does not occur necessarily at mid-span because of higher mode effects.

The modal peak responses must be superposed to obtain an estimate of the actual maximum bridge response. Since in a response spectrum representation information of the phase shift between the individual modal peak responses is not available, modal combination rules such as the ABSUM method [15]

$$\max|w|(x) \approx \frac{4\pi^2 F_{\max}}{\rho A L} \sum_{n=1}^{k} \left| \frac{\bar{q}_n}{\omega_n^2} \phi_n(x) \right|,$$

$$\max|\ddot{w}|(x) \approx \frac{F_{\max}}{\rho A L} \sum_{n=1}^{k} \left| \ddot{\bar{q}}_n \phi_n(x) \right| \qquad (5)$$

and the SRSS method [15]

$$\max |w|(x) \approx \frac{4\pi^2 F_{\max}}{\rho A L} \sqrt{\sum_{n=1}^{k} \left(\frac{\bar{q}_n}{\omega_n^2} \phi_n(x) \right)^2},$$

,

$$\max |\ddot{w}|(x) \approx \frac{F_{\max}}{\rho A L} \sqrt{\sum_{n=1}^{k} \left(\ddot{\bar{q}}_n \phi_n(x) \right)^2} \tag{6}$$

well known from applications in earthquake engineering, are utilized. The SRSS rule provides in general more accurate results than the ABSUM method, but may underestimate the peak response. The ABSUM rule gives always an upper bound of the peak acceleration in the context of the underlying mechanical model.

5 Example

In an example problem the peak displacement and peak acceleration at mid-span of a composite steel-reinforced concrete bridge subjected to the train model HSLM-A1 is assessed. The train and bridge parameters are specified as: $L = 30\,\text{m}$, $\rho A = 18{,}000\,\text{kg/m}$, $EI = 1.40 \cdot 10^{11}\ \text{N/m}^2$, $GA_s = 5.5 \cdot 10^9\ \text{N}$, $\zeta_n = 0.015$, $v_{max} = 300\,\text{km/h}$ ($= 83.3\,\text{m/s}$), $d = 18\,\text{m}$, $F_{max} = 170\,\text{kN}$.

Based on these parameters the first three natural circular frequencies of the shear deformable bridge (Eq. 3) are evaluated: $\omega_1 = 27.1\,\text{rad/s}$, $\omega_2 = 84.5\,\text{rad/s}$, $\omega_3 = 149\,\text{rad/s}$. At mid-span the amplitudes of the corresponding mode shapes are: $\phi_1(x = 0.5L) = 1$, $\phi_2(x = 0.5L) = 0$, $\phi_3(x = 0.5L) = -1$. Thus, in a three mode approximation only the first and third mode contribute to the peak response at mid-span. The non-dimensional parameters of this train-bridge interaction problem required for application of response spectra are: $L/d = 1.67$, $S_0^{(1)} \approx 0.32$, $S_0^{(3)} \approx 0.06$. From the two-dimensional representation of the corresponding response spectra shown in Figs. 3 and 4 the following modal peak response quantities for the first and third mode are identified: $\bar{q}_1 = 0.48$, $\bar{q}_3 = 0.23$, $\ddot{\bar{q}}_1 = 12.0$, $\ddot{\bar{q}}_3 = 1.66$. The SRSS combination yields a maximum mid-span peak deflection of $\max w(x = 0.5L) = 0.0081\,\text{m}$, which is identical to the exact solution (from a complete time history analysis) of the considered beam problem. Note that only the first mode contributes significantly to peak mid-span deflection. Thus, the ABSUM combination rule, $\max w(x = 0.5L) = 0.0083\,\text{m}$, overestimates slightly the peak deflection, when both the first and the third modal displacements are considered.

Evaluation of the peak acceleration leads to the following outcomes. SRSS: $\max \ddot{w}(x = 0.5L) = 3.80\,\text{m/s}^2$, ABSUM: $\max \ddot{w}(x = 0.5L) = 4.29\,\text{m/s}^2$, exact: $\max \ddot{w}(x = 0.5L) = 3.83\,\text{m/s}^2$, one mode approximation: $\max \ddot{w}(x = 0.5L) = 3.76\,\text{m/s}^2$. The results show that for this example the ABSUM rule

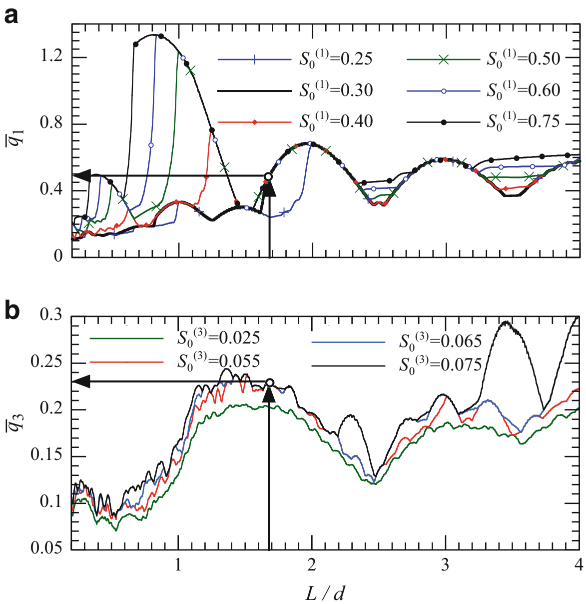

Fig. 3 Response spectra of the modal deflection for (**a**) the first mode, and (**b**) the third mode. HSLM-A1 load model. Damping coefficients $\zeta_1 = \zeta_3 = 0.015$

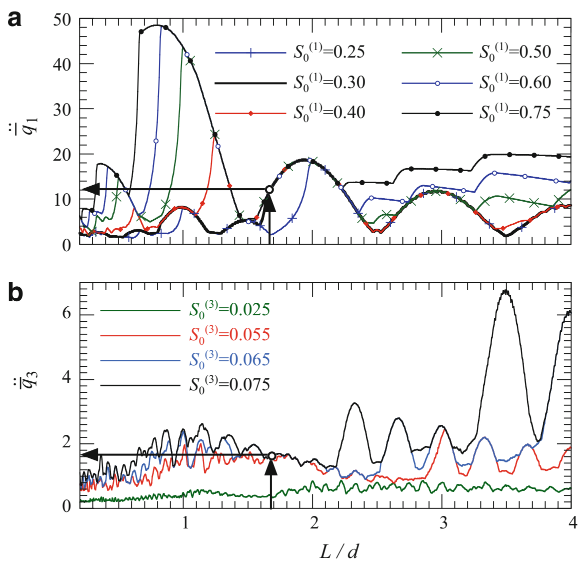

Fig. 4 Response spectra of the modal acceleration for (**a**) the first mode, and (**b**) the third mode. HSLM-A1 load model. Damping coefficients $\zeta_1 = \zeta_3 = 0.015$

overestimates the exact peak acceleration by 12 %, however, the result from the SRSS rule gives a very accurate estimate. The difference between the exact peak acceleration and its one mode approximation is 2 %.

Subsequently, the effect of shear deformation is assessed comparing the derived peak bridge responses with outcomes based on the corresponding Bernoulli-Euler beam (which is rigid in shear). The first natural circular frequencies of the Bernoulli-Euler beam, $\omega_{1(BE)} = 30.6$ rad/s, $\omega_{2(BE)} = 122$ rad/s, $\omega_{2(BE)} = 275$ Hz, show that these quantities are significantly affected by shear. In Fig. 1 the effect of the frequency shift is visualized. Since $\omega_{1(BE)}$ is larger than ω_1, the corresponding maximum admissible speed drops from $S_0^{(1)} \approx 0.32$ to $S_0^{(1BE)} \approx 0.28$. According to Fig. 1 at $S_0^{(1BE)}$ the modal peak acceleration is much smaller than at $S_0^{(1)}$. Thus, the exact peak displacement and peak acceleration based on the Bernoulli-Euler theory, $\max w_{(BE)}(x = 0.5L) = 0.0064$ m and $\max \ddot{w}_{(BE)}(x = 0.5L) = 1.90\,\mathrm{m/s}^2$, respectively, underestimate considerable the response based on the more accurate shear beam theory. It can be concluded that for certain bridges response spectra considering the effect of shear deformation must be utilized for a reliable peak response prediction.

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