

Contents

1	Nevanlinna Theory of Meromorphic Functions	1
1.1	The First Main Theorem	1
1.2	The Second Main Theorem	12
1.3	Examples of Functions of Finite Order	19
2	The First Main Theorem	25
2.1	Plurisubharmonic Functions	25
2.1.1	One Variable	25
2.1.2	Several Variables	33
2.2	Poincaré–Lelong Formula	42
2.3	The First Main Theorem	50
2.3.1	Meromorphic Mappings, Divisors and Line Bundles	50
2.3.2	Differentiable Functions on Complex Spaces	54
2.3.3	Metrics and Curvature Forms of Line Bundles	58
2.4	The First Main Theorem for Coherent Ideal Sheaves	66
2.4.1	Proximity Functions for Coherent Ideal Sheaves	66
2.4.2	The Case of $m = 1$	71
2.5	Order Functions	73
2.5.1	Metrics	73
2.5.2	Cartan’s Order Function	77
2.5.3	A Family of Rational Functions	79
2.5.4	Characterization of Rationality	83
2.6	Nevanlinna’s Inequality	86
2.7	Ramified Covers over \mathbf{C}^m	88
3	Differentiably Non-degenerate Meromorphic Maps	91
3.1	Lemma on Logarithmic Derivatives	91
3.2	The Second Main Theorem	93
3.3	Applications and Generalizations	102
3.3.1	Applications	102
3.3.2	Non-Kähler Counter-Example	105
3.3.3	Generalizations	110

4	Entire Curves in Algebraic Varieties	113
4.1	Nochka Weights	113
4.2	The Cartan–Nochka Theorem	123
4.3	Entire Curves Omitting Hyperplanes	131
4.4	Generalizations and Applications	133
4.4.1	Derived Curves	133
4.4.2	Generalization to Higher Dimensional Domains	134
4.4.3	Finite Ramified Covering Spaces	134
4.4.4	The Eremenko–Sodin Second Main Theorem	135
4.4.5	The Second Main Theorem of Corvaja–Zannier, Evertse–Ferretti and Ru	136
4.4.6	Krutin’s Theorem	136
4.4.7	Moving Targets	136
4.4.8	Yamanoi’s Second Main Theorem	137
4.4.9	Applications	137
4.5	Logarithmic Forms	138
4.6	Logarithmic Jet Bundles	144
4.6.1	Jet Bundles in General	144
4.6.2	Jet Spaces	146
4.6.3	Logarithmic Jet Bundles and Logarithmic Jet Spaces	146
4.7	Lemma on Logarithmic Forms	148
4.8	Inequality of the Second Main Theorem Type	150
4.9	Entire Curves Omitting Hypersurfaces	157
4.10	The Fundamental Conjecture of Entire Curves	159
5	Semi-abelian Varieties	161
5.1	Semi-tori	161
5.1.1	Definition	161
5.1.2	Characteristic Subgroups of Complex Semi-tori	164
5.1.3	Holomorphic Functions	166
5.1.4	Semi-abelian Varieties	167
5.1.5	Presentations	169
5.1.6	Presentations of Semi-abelian Varieties	170
5.1.7	Inequivalent Algebraic Structures	171
5.1.8	Choice of Presentation	171
5.1.9	Construction of Semi-tori via Presentations	172
5.1.10	Morphisms and GAGA	173
5.2	Reductive Group Actions	176
5.3	Semi-toric Varieties	180
5.3.1	Toric Varieties	180
5.3.2	Semi-toric Varieties	181
5.3.3	Key Properties of Semi-toric Varieties	182
5.3.4	Quasi-algebraic Subgroups	185
5.3.5	Compactifiable Groups and Kähler Condition	187
5.3.6	Examples of Non-semi-toric Varieties	190
5.4	Jet Bundles over Semi-toric Varieties	191

5.5	Line Bundles on Toric Varieties	192
5.5.1	Ample Line Bundles	192
5.5.2	Leray Spectral Sequence	195
5.5.3	Decomposition of Line Bundles	196
5.5.4	Global Span and Very Ampleness	198
5.5.5	Stabilizer and Bigness	201
5.6	Good Position and Stabilizer	203
5.6.1	Good Position	203
5.6.2	Good Position and Choice of Compactification	204
5.6.3	Regular Subgroups	209
5.6.4	More Facts on Semi-tori	210
6	Entire Curves in Semi-abelian Varieties	215
6.1	Order Functions	215
6.2	Structure of Jet Images	220
6.2.1	Image of f (Case $k = 0$)	220
6.2.2	Jet Projection Method	220
6.2.3	A Counter-Example	224
6.3	Compact Complex Tori	225
6.3.1	Entire Curves	225
6.3.2	Applications to Differentiably Non-degenerate Maps	233
6.4	Semi-tori: Truncation Level k_0	235
6.5	Semi-abelian Varieties: Truncation Level 1	248
6.5.1	Truncation Level 1	248
6.5.2	The Second Main Theorem for Jet Lifts	249
6.5.3	Higher Codimensional Subvarieties of $X_k(f)$	254
6.5.4	Proof of Theorem 6.5.1	268
6.6	Applications	270
6.6.1	Algebraic Degeneracy of Entire Curves	270
6.6.2	Kobayashi Hyperbolicity	278
6.6.3	Complements of Divisors in Projective Space	279
6.6.4	Strong Green–Griffiths Conjecture	281
6.6.5	Lang’s Questions on Theta Divisors	283
6.6.6	Algebraic Differential Equations	285
7	Kobayashi Hyperbolicity	289
7.1	Kobayashi Pseudodistance	289
7.2	Brody’s Theorem	293
7.2.1	Brody’s Reparametrization	293
7.2.2	Hyperbolicity as an Open Property	300
7.3	Kobayashi Hyperbolic Manifolds	301
7.4	Kobayashi Hyperbolic Projective Hypersurfaces	309
7.5	Hyperbolic Embedding into Complex Projective Space	315
7.6	Brody Curves and Yosida Functions	321
7.6.1	Growth Conditions and Yosida Functions	322
7.6.2	Characterizing Brody Maps into Tori	331

7.6.3	Brody Curves with Prescribed Points in the Image	332
7.6.4	Ahlfors' Currents	333
8	Nevanlinna Theory over Function Fields	341
8.1	Lang's Conjecture	341
8.2	Nevanlinna–Cartan Theory over Function Fields	345
8.3	Borel's Identity and Unit Equations	350
8.4	Generalized Borel's Theorem and Applications	355
9	Diophantine Approximation	361
9.1	Valuations	361
9.1.1	Definition and the Basic Properties	361
9.1.2	Extensions of Valuations	364
9.1.3	Normalized Valuations	364
9.2	Heights	368
9.3	Theorems of Roth and Schmidt	377
9.4	Unit Equations	383
9.5	The <i>abc</i> -Conjecture and the Fundamental Conjecture	385
9.6	The Faltings–Vojta Theorem	388
9.7	Distribution of Rational Points	389
	References	393
	Index	411
	Symbols	415

Nevanlinna Theory in Several Complex Variables and
Diophantine Approximation

Noguchi, J.; Winkelmann, J.

2014, XIV, 416 p. 6 illus., Hardcover

ISBN: 978-4-431-54570-5