

Chapter 2

High-Energy Heavy Ion Collisions

2.1 Overview

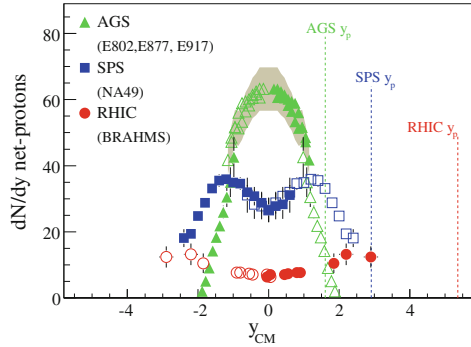
Nuclear collisions have been playing an important role in high-energy nuclear physics as they provide quite unique opportunities to experimentally approach the quark matter under extreme conditions. Prior to the nuclear collisions, observation of the cosmic ray events are the main means of exploring high-energy physics. It is noteworthy that the theoretical studies for the analyses of multi-particle production had built part of the foundation of collider physics today [1]. The heavy ion collisions have the advantage of being able to study the hot medium with larger volume in detail under controlled environments, as beam energy and colliding nuclei can be changed to explore wider regions in the phase diagram of QCD. The collider experiments have been providing great chances and challenges for developing realistic theories of the many-body systems of quarks and gluons including the quark-gluon plasma.

2.1.1 Past Achievements

One of the earliest experiments of heavy ion collisions dates back to Bevalac in Lawrence Berkeley National Laboratory. The heavy ion collisions with more higher energies were carried out in the Alternating Gradient Synchrotron (AGS) at the Brookhaven National Laboratory (BNL) for Au nuclei at $\sqrt{s_{NN}} = 5$ GeV and the Super Proton Synchrotron (SPS) at the European Organization for Nuclear Research (CERN) for Pb nuclei at $\sqrt{s_{NN}} = 17$ GeV. Those accelerators were fixed-target experiments. While the energy would not have been sufficient to fully produce the QGP, their results indicated the existence of collective behavior in heavy nuclear collisions. The observation of the J/Ψ suppression, which is possibly due to the color screening, at SPS also suggests the emergence of primordial medium effects.

One of the biggest achievements of the heavy ion experiments is the discovery of the QGP in the Relativistic Heavy Ion Collider (RHIC) [2–5] at BNL which started

Fig. 2.1 The net proton distribution in the beam direction in momentum space at AGS (E802, E877, E917), SPS (NA49) and RHIC (BRAHMS) experiments for the most central 0–5 % events. (Figure from Ref. [6], American Physical Society.)



in 2000. Designed for collider experiments, the facility has performed Cu–Cu and Au–Au collisions at $\sqrt{s_{NN}} = 130 \text{ GeV}$ and $\sqrt{s_{NN}} = 200 \text{ GeV}$. It is noteworthy that AGS is used as an early-stage accelerator and injector for RHIC. The existence of the hot and dense matter is strongly indicated because sizable medium effects are observed in comparison to the results of proton–proton collisions. One example is jet quenching, where a small bunch of high momentum hadrons called mini-jet gets suppressed when it travels through the bulk medium due to the strong interaction with the medium. The hot medium is expected to be the first human-made QGP since the temperature of the quark-gluon system at RHIC estimated from the photon radiation is above the crossover temperature suggested from the lattice QCD calculations. Estimated energy density of the hot medium is $\sim 5 \text{ GeV/fm}^3$ at RHIC.

The net baryon distribution, which is defined as the distribution of baryons minus that of antibaryons, at AGS, SPS and RHIC is shown in Fig. 2.1 [6] to illustrate how the remnant of shattered colliding nuclei evolves in the heavy ion collisions. The average rapidity of the net baryon number tends to be smaller than the beam rapidity because part of the energy is converted to the production of the hot medium. This phenomenon is known as *baryon stopping*. It is notable that the collisions become increasingly transparent for the nuclear collisions with higher center-of-mass energies. Since the momentum distribution respects the coordinate-space one, this would indicate the creation of a hot medium with finite volume.

Amongst the great achievements at RHIC, it was very surprising that the QGP was found as a nearly-perfect relativistic fluid in the vicinity of the quark-hadron crossover since many had speculated the QGP to be a gas because of the asymptotic freedom of QCD. This is shown by comparing the anisotropy in momentum space [7] against that in coordinate space. The QGP typically has an elliptic shape in the transverse plane for non-central collisions as explained in detail in Sect. 2.2. If the interaction among the constituent quasi-particles is weak, the system is gas-like and the momentum anisotropy would not be produced; on the other hand, if the interaction is strong, i.e. the system is liquid-like, the momentum distribution would reflect the spatial azimuthal anisotropy. The experimental data [8, 9] show that the existence of sizable anisotropy in momentum space and it reaches the hydrodynamic limit

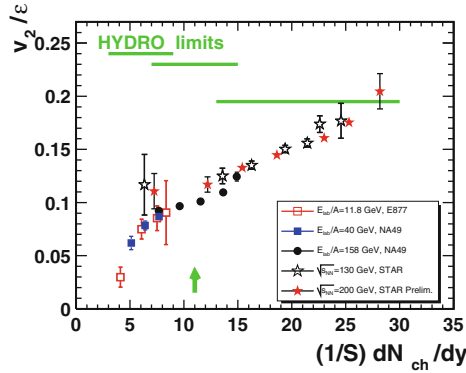


Fig. 2.2 The ratio of azimuthal ellipticity in momentum space v_2 to that in coordinate space ϵ over the charged particle yield per transverse unit area, which increases as the collision energy does, for AGS (E877), SPS (NA49) and RHIC (STAR) experiments compared with the hydrodynamic expectations. (Figure from Ref. [10], American Physical Society.)

at $\sqrt{s_{NN}} = 200$ GeV collisions (Fig. 2.2 [10]). $(1/S)dN_{ch}/dy$ in the figure means the yield of charged hadrons per rapidity over the overlapping area of the colliding nuclei, which tends to increase as the energy increases.

The RHIC experiments also allow one to peer into various high-energy phenomena such as gluon saturation in the cold nuclear matter, i.e., a matter at high energies without the effects of hot medium. The evidence of the gluon saturation in the colliding nuclei is considered to be found because the color glass condensate picture (Sect. 2.4.1) gives a good description of both d -Au and Au–Au collisions at RHIC. Note that the hot medium would be produced only in the latter environment.

2.1.2 Current Status and Future Prospects

With the beginning of the heavy ion program at the Large Hadron Collider (LHC) in CERN in 2010, heavy ion physics has entered a new energy regime. LHC has run Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, and its first data started to provide valuable information for the quark matter on the high temperature frontier. The currently-planned goal for the collision energy is $\sqrt{s_{NN}} = 5.5$ TeV. It is believed that the properties of the hot medium does not fundamentally change from RHIC to LHC [11], though several intriguing anomalies are reported in particle production [12]. The analyses of azimuthal anisotropy show that the medium still behaves as a fluid with small viscosity, which is important information since it has been naïvely expected that the QGP becomes slightly more weakly-coupled with increasing energy due to the QCD asymptotic freedom. On the other hand, since it is the first experiment to explore the energy dependence of the QGP, it had been possible that the system became too strongly-coupled even for ideal hydrodynamic description, in which case

one would need a different mechanism for explaining the data. The energy density at LHC is estimated to be $\sim 15 \text{ GeV/fm}^3$, which is about three times larger than that at RHIC.

RHIC has started to explore the system dependence of the hot matter. The facility recently performed the U–U collision at $\sqrt{s_{NN}} = 193 \text{ MeV}$ and the Au–Cu collision at $\sqrt{s_{NN}} = 200 \text{ MeV}$ [13]. The former is the collision of nuclei with prolate deformation, and thus tip-to-tip collisions and body-to-body collisions are expected to show different properties. The uranium tip-to-tip collisions produce more dense matter than what is produced in the previous Au–Au collisions by $\sim 30\%$ in the energy density. The latter is the first asymmetric collision at high energies. It provides intrinsic triangular and higher order transverse anisotropies in momentum space that does not originate from fluctuation. At the most central events, Cu nucleus is going to be completely buried in Au nucleus, producing the QGP in a corona of nucleons.

The search for the QCD critical point on the phase diagram is also a very important topic. One can access a wider region on the T – μ_B plane by changing the beam energy, because the net baryon chemical potential becomes higher as the temperature becomes lower and vice versa. The beam energy scans (BES) are performed at RHIC at $\sqrt{s_{NN}} = 62.4, 39, 27, 19.6, 11.5, 7.7$ and 5.5 GeV [14]. Although the critical point is yet to be found, the momentum anisotropy is found to persist until around 19.6 MeV [15]. BES experiments are also performed at SPS for light nuclei at $\sqrt{s_{NN}} = 4.9 - 17.3 \text{ MeV}$, and planned at Nuclotron-based Ion Collider fAcility (NICA) in Joint Institute for Nuclear Research with Au nuclei at $\sqrt{s_{NN}} = 3.9 - 11 \text{ MeV}$ and at Facility for Antiprotons and Ion Research (FAIR) in GSI Helmholtzzentrum für Schwerionenforschung with Au nuclei for $\sqrt{s_{NN}} = 4.9 - 17.3 \text{ MeV}$. Japan Proton Accelerator Research Complex (J-PARC) also has a plan for low energy collisions.

2.2 Geometrical Setup

The two nuclei collide nearly at the speed of light in high-energy heavy ion collisions. Thus they are squeezed in the direction of beam axis due to Lorentz contraction in the laboratory frame. At RHIC energy $\sqrt{s_{NN}} = 200 \text{ GeV}$, Lorentz dilation factor is $\gamma \sim 100$ for a projectile nuclei, which means the nucleus of diameter $\sim 14 \text{ fm}$ is reduced to $\sim 0.1 \text{ fm}$. Likewise at LHC energy $\sqrt{s_{NN}} = 2.76 \text{ TeV}$, $\gamma \sim 1500$ and the nucleus is squeezed to $\sim 0.01 \text{ fm}$. The hot medium would be produced in the overlapping area between the two passing nuclei. The collision axis is conventionally chosen as z -axis, and often referred to as the longitudinal direction as opposed to the transverse plane, which is perpendicular to the collision axis. Collisions can be non-central, and the resulting geometry of a hot medium is almond-shaped in the transverse directions. The nucleons which collide are called participants, and those which do not are called spectators. The schematic pictures of the collision geometry of symmetric nuclei are shown in Fig. 2.3.

It is more convenient to introduce the relativistic τ – η_s coordinate system to describe the heavy ion systems, where

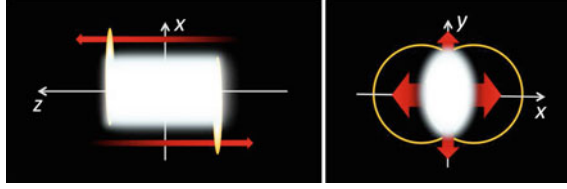


Fig. 2.3 Schematic picture of the geometry of non-central high-energy heavy ion collisions for (left) the longitudinal relativistic expansion and (right) the transverse expansion

$$\tau = \sqrt{t^2 - z^2}, \quad (2.1)$$

$$\eta_s = \frac{1}{2} \ln \frac{t+z}{t-z}, \quad (2.2)$$

are the proper time and the space-time rapidity. The space-time rapidity is a dimensionless variable that can be interpreted as a hyperbolic angle. They satisfy the relations $t = \tau \cosh \eta_s$ and $z = \tau \sinh \eta_s$. $\eta_s = 0$ corresponds to the t axis and $\eta_s = \pm\infty$ the light cone. Similarly, one defines the transverse mass m_T and the rapidity y in momentum space as

$$m_T = \sqrt{E^2 - p_z^2}, \quad (2.3)$$

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}. \quad (2.4)$$

In collider physics, the transverse momentum $p_T = \sqrt{m_T^2 - m^2}$ and the pseudo-rapidity $\eta_p = \frac{1}{2} \ln [(|p| + p_z)(|p| - p_z)]$ are useful variables because they are independent of mass and thus of particle species. At relativistic energies, they are also fairly close to the transverse mass and the rapidity, respectively, and become identical in relativistic massless limit.

The polar coordinate system is often employed in analyses of the transverse dynamics. The angle in the configuration space is denoted as ϕ and that in the momentum space as ϕ_p . They are related to the variables in Cartesian coordinates as $(x, y) = (r \cos \phi, r \sin \phi)$ and $(p_x, p_y) = (p_T \cos \phi_p, p_T \sin \phi_p)$.

The non-centrality of collisions is characterized by the impact parameter b , which is defined as the distance between the centers of the two colliding nuclei on the transverse plane. *Centrality* is defined by groups of events per the number of participants because the collision would have more participant nucleons for more central collisions. The groups are ordered from the most central events, e.g., 0–20% centrality means that the most central collisions are selected up to 20% of the total events, 20–40% centrality the next 20% events and likewise. The variable is preferred in collider experiments as the impact parameter is not a direct observable. The number of participants still reflects the centrality in asymmetric collisions where the smaller nuclei is buried in the larger one because of the effects of fluctuating geometry.

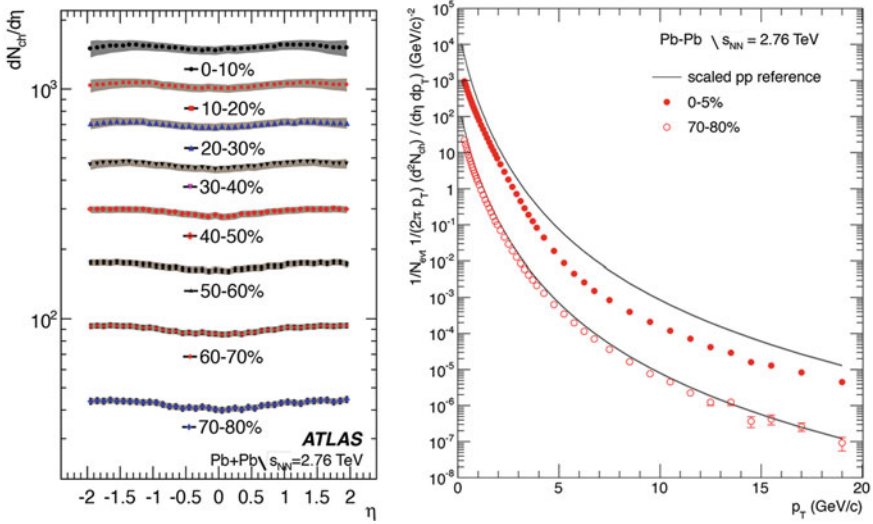


Fig. 2.4 (Left) The pseudo-rapidity distributions of the charged particles for different centralities in Pb–Pb collisions at 2.76 TeV by ATLAS Collaboration. (Figure from Ref. [16], Elsevier.) (Right) The p_T spectra of the charged particles for central and peripheral collisions in the same collisions by ALICE Collaboration. (Figure from Ref. [21], Elsevier.)

2.3 Experimental Observables

2.3.1 Hadronic Particle Spectra

Hadronic particle yield is an essential observable to study high-energy heavy ion collisions, since hadrons constitute the bulk part of the produced medium, i.e., most of the initial energy is carried by hadrons. Various insights can be obtained by analyzing its dependences on transverse momentum, rapidity, centrality, beam energy and particle species. Due to the strong interaction in the medium, the particle spectra is considered to contain the information at the latest stage of heavy ion collisions. Thus detailed profiles of QGP time evolution is integrated out from the observable.

The rapidity distribution dN/dy , or the pseudo-rapidity distribution $dN/d\eta_p$, is a basic observable to quantify particle production in the system (Fig. 2.4, left [16]). The charged particle multiplicities at mid-rapidity are $dN_{ch}/d\eta_p \sim 650$ at RHIC ($\sqrt{s_{NN}} = 200$ GeV) and $dN_{ch}/d\eta_p \sim 1600$ at LHC ($\sqrt{s_{NN}} = 2.76$ GeV) in the most central events [11, 16–20]. It is note-worthy that the multiplicity of charged particles at LHC was larger than most of the theoretical predictions made prior to the experiment. A possible solution to the discrepancy will be presented in Chap. 4. The rapidity distribution of the net baryon number originates only from the colliding nuclei and thus is sensitive to kinetic energy loss of the nucleons which becomes available for the production of the medium as seen in Sect. 2.1.1. This would be revisited later in Chap. 5.

The transverse momentum distribution is very informative (Fig. 2.4, right [21]). The Fourier harmonics in the series expansion of p_T spectra is an essential observable since the hot medium is almond-shaped in non-central collisions [7, 22]. The particle spectrum is expressed as

$$\frac{dN}{d\phi_p p_T dp_T dy} = \frac{1}{2\pi} \frac{dN}{p_T dp_T dy} \left[1 + 2 \sum_n v_n(p_T, y) \cos(n\phi_p - n\Psi) \right], \quad (2.5)$$

with Fourier harmonics $v_n(p_T, y)$ and the reaction plane Ψ . This leads to

$$v_n(p_T, y) = \int d\phi_p \cos(n\phi_p - n\Psi) \frac{dN}{d\phi_p p_T dp_T dy} \bigg/ \int d\phi_p \frac{dN}{d\phi_p p_T dp_T dy}. \quad (2.6)$$

The space-time azimuthal anisotropy in the hot medium suggests that the second order harmonics v_2 , *elliptic flow*, would be larger than other harmonics for non-central collisions in heavy ion collisions. If one takes the reaction plane as the minor axis of the ellipsoid based on the average over many events, the odd-order harmonics vanishes because of the symmetry around the collision axis.¹ The large v_2 observed in the high-energy heavy ion collisions is well quantified by hydrodynamic models, supporting the fact that the QGP is a strongly-coupled medium. It is known to be roughly proportional to the spatial anisotropy ε_2 at RHIC and LHC. The elliptic flow is known to be very sensitive to viscosity in the hydrodynamic phase, as deviation from equilibrium would lead to a less strongly-coupled medium. The experimental data typically shows agreement with the hydrodynamic calculations up to $p_T \sim 1$ GeV if there is no viscosity, and $p_T \sim 3$ GeV if shear viscosity is introduced. The flow harmonics can be used to experimentally constrain the transport coefficients in the medium. It should be noted that the applicability of hydrodynamics naturally breaks down for high- p_T regions as quasi-particles do not reach local equilibrium and v_2 decreases as the deviation from the fluid picture becomes larger.

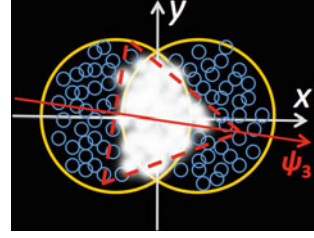
Higher flow harmonics should remain finite in event-by-event analyses since initial conditions have geometrical fluctuations that break the symmetry [23, 24]. This provides new sets of observables such as triangular flow v_3 for the analyses of a hot medium (Fig. 2.5). Here the variable would be defined as

$$v_n^{\text{ebe}}(p_T, y) = \int d\phi_p \cos(n\phi_p - n\Psi_n) \frac{dN}{d\phi_p p_T dp_T dy} \bigg/ \int d\phi_p \frac{dN}{d\phi_p p_T dp_T dy}, \quad (2.7)$$

where Ψ_n is chosen for each event and harmonics. The analyses of the observed higher flow harmonics [25, 26] are important in precision physics.

¹ Directed flow v_1 is the exception because in non-central collisions, rapidity-dependent flow asymmetry is created as the expansion gets tilted from the collision axis. It is an odd function of rapidity, and should be distinguished from the one from transverse geometrical fluctuation, which would be an even function of rapidity.

Fig. 2.5 A schematic picture of the triangular anisotropy and the event plane ψ_3 originating from the fluctuation in an initial condition



The particle spectrum in a high- p_T region ($p_T \geq 5$ GeV) shows non-hydrodynamic behavior. At peripheral collisions the spectrum exhibits simple power law behavior which is expected from perturbative QCD in Fig. 2.4 (right). At central collisions, it is clearly less than the scaled reference because the medium effects become more prominent. The deviation is quantified by the nuclear modification factor R_{AA} as

$$R_{AA}(p_T, y) = \frac{dN_{AA}}{p_T dp_T dy} \bigg/ \langle N_{\text{coll}} \rangle \frac{dN_{pp}}{p_T dp_T dy}, \quad (2.8)$$

where $\langle N_{\text{coll}} \rangle$ is the average number of binary collisions. AA denotes a nucleus–nucleus collision and pp a proton–proton reference. R_{AA} is mostly smaller than 1 for hadrons in central collisions. At low energy collisions, it can become larger than unity due to the multiple parton scattering called Cronin effect. Experimental data shows that the Cronin effect is not large enough to counter the suppression in high-energy nucleus–nucleus collisions.

2.3.2 Jets and Quarkonia

Jet quenching is a very strong evidence of the presence of a hot and dense medium, the concept of which is originally proposed in the context of hadron–hadron collisions [27]. In nucleus–nucleus collisions, a pair of streaks of partons at high momentum called (mini)-jets are produced. They are not thermalized and considered to be dominant in hadronic particle spectra at high- p_T . The triggered event is called the near-side jet, and the other side the away-side jet. For proton–proton binary collisions and deuteron–gold collisions, the two streaks are clearly observed [28]. On the other hand, the away-side jet is absorbed, if not completely, in nucleus–nucleus collisions [29]. The results are summarized in Fig. 2.6 (left) [28]. This phenomenon is called jet quenching, and is considered to indicate the creation of a chromodynamically interacting hot matter only in the Au–Au collisions. The magnitude of parton energy loss is expected to be sensitive to the density properties of a medium. The away-side jet is reduced to low momentum partons spread in wide angle and the rest of energy is expected to go back to the medium itself.

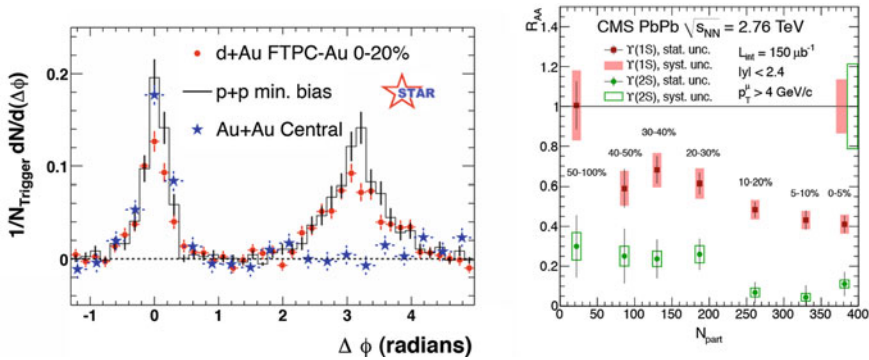


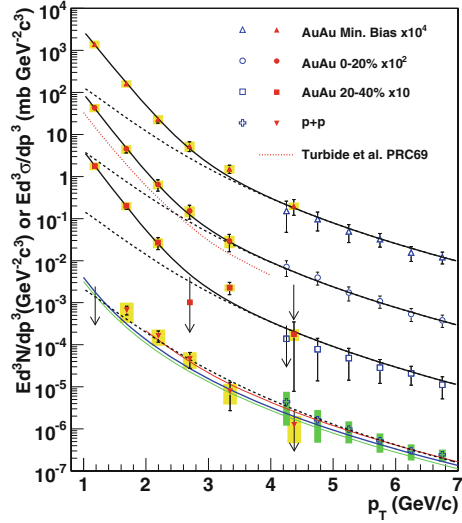
Fig. 2.6 (Left) The correlation of the near-side and the away-side jets ($4 < p_T < 6$ GeV) are clearly visible for p - p and d -Au collisions, whereas it is lost for Au–Au collisions at $\sqrt{s_{NN}} = 200$ GeV. (Figure from Ref. [28], American Physical Society.) (Right) R_{AA} for $\Upsilon(1S)$ and $\Upsilon(2S)$ as a function of the number of participants at Pb–Pb collisions from CMS Collaboration at LHC. (Figure from Ref. [31], American Physical Society.)

Quarkonia, bound states of a heavy quark and its antiquark, are expected to be able to survive in the QGP as the lattice QCD calculations indicate that the binding energy for the ground state of charmonium $J/\Psi(1S)$ is ~ 0.6 GeV and that of bottomonium $\Upsilon(1S)$ is ~ 1.2 GeV, which are higher than the QCD scale $\Lambda_{\text{QCD}} \sim 0.2$ GeV. On the other hand, the quarkonia melt if the QGP temperature is sufficiently high because of the color screening effect. The higher excited states are less tightly bound and thus easier to melt as the temperature becomes higher. The phenomenon is called sequential melting. This suggests that quarkonia can be used as a thermometer of the produced medium. The suppression of the charmonium production in medium is called J/Ψ suppression [30]. The experimental data show a clear evidence of the suppression of J/Ψ at SPS, RHIC and LHC. It is note-worthy that J/Ψ regeneration process in later stages may be important for quantitative analyses in higher energy collisions. The LHC also observes Υ suppression where the excited state is more suppressed than the ground state (Fig. 2.6, right) [31]. It is interesting to note that the J/Ψ suppression in the Au–Cu collisions [13] shows stronger suppression for the Cu-going side whereas it remains the same for the Au-going side compared with that in the Au–Au collisions.

2.3.3 Electroweak Probes

The experimental data indicate that the quark-gluon plasma is a very opaque medium with respect to the color charge. On the other hand, it is suggested to be reasonably transparent in terms of electromagnetic interactions. This motivates one to use electromagnetically charged particles with no color as probes for the hot matter since

Fig. 2.7 Transverse momentum dependence of direct photon particle spectra of Au–Au collisions at RHIC. Minimum bias, 0–20% centrality and 20–40% centrality events are shown together with p - p results. The *dashed lines* are the scaled p - p data. The *three lines* on the p - p results are of next-to-leading order pQCD calculations. The *dotted line* on the Au–Au results is also a theoretical calculation. (Figure from Ref. [32], American Physical Society.)



they should retain the information of the medium at the time of their creation unlike the colored observables.

Photons play an important role in estimating the temperature of the hot medium. There are multiple sources of photons in heavy ion collisions. The photons created at the time of a collision are called *prompt photons*. This is a heavy-ion analogy to the cosmic microwave background in the early universe. The photons emitted from the QGP as an analogy to the black-body radiation is called *thermal photons*, which is relevant to the estimation of the medium temperature. Finally, photons are also produced when hadrons decay in later stages; this is called *decay photons*. Decay photons are the majority ($\sim 90\%$) of the inclusive photons for $1 < p_T < 3 \text{ GeV}$ where the thermal photons become important. The prompt and thermal photons together are called direct photons. Direct photon spectra for Au–Au and p - p collisions are shown along with next-to-leading order pQCD calculations in Fig. 2.7 [32]. One can find clear enhancement of photon yields for the Au–Au data. The comparison with the fit by an exponential function of the slope parameter—or the effective temperature—yields $221 \pm 38 \text{ MeV}$ for the medium that emitted thermal photons in the central 0–20% collisions. Since this is the average over time evolution, the initial temperature T_{init} should be larger. Hydrodynamic model calculations suggest $T_{\text{init}} \sim 300\text{--}600 \text{ MeV}$ for the thermalization time $\tau_0 \sim 0.6\text{--}0.15 \text{ fm}/c$. This is well beyond the crossover temperature $T_c \sim 170 \text{ MeV}$, which strongly suggests the hot medium is the quark-gluon plasma rather than a hadronic matter at RHIC. The enhancement is not observed for d -Au collisions. Latest experiments at LHC indicate a slightly higher slope parameter 304 MeV [33] for the thermal photon production on time average.

One can also consider the elliptic flow of photons in analogy to the hadronic case. The elliptic flow v_2 of direct photons reflects the thermalization time of the QGP

because prompt photons do not have v_2 since the momentum anisotropy has not developed yet at time zero. Theoretical estimation suggests that if the thermalization is early, v_2 of thermal photons become small. This seems rather counterintuitive but could be understood that the energy density decreases faster in hydrodynamic evolution and the elliptic flow cannot fully develop for too early thermalization. v_2 of direct photons can be as large as that of hadrons for late thermalization times. The latest experimental data [34] show that there is a serious underestimation by the theoretical estimations of hydrodynamics, making it one of the important clues to uncover the yet-unknown features of the hot medium.

The production of W and Z bosons in Pb–Pb collisions at LHC, on the other hand, follows that of p - p collisions scaled by the number of participants very well [35, 36]. This suggests that they are created at the time of the collision and do not interact with the QCD medium. The same trend can be found for large momentum photons at RHIC and LHC.

2.4 Theoretical Modeling

The QCD system in high-energy heavy ion collisions goes through various stages from nuclei to the quark-gluon plasma to hadrons. Since the system is suggested to be strongly coupled, one would not be able to naïvely employ perturbative QCD for estimating the evolution of the bulk medium. The dynamical first-principle simulation of a non-perturbative QCD medium at finite temperature and chemical potential is currently beyond the capacity of any theoretical method developed so far. The fact that collective behavior exists in the hot medium, on the other hand, allows one to build an effective model of the hot matter. It is sometimes referred to as a hydrodynamic model, but hydrodynamic picture is applicable for the intermediate stage in heavy ion collisions around $\tau \sim \mathcal{O}(1) - \mathcal{O}(10)$ fm/c. A “standard model” of high-energy nucleus–nucleus collisions consists of the color glass condensate, glasma, relativistic hydrodynamics and hadronic gas (Fig. 2.8) which will be explained in order in the following sections.

2.4.1 Color Glass Condensate

The colliding nuclei is accelerated to near-light speed in high-energy nuclear collisions. At those energies, the life-time of gluons emitted from the valence quarks or other gluons become long enough to allow additional emissions of soft gluons from themselves. The cascading process keeps increasing the number density of gluons until gluons become saturated as recombination of gluons becomes non-negligible. This state of matter is called color glass condensate (CGC) [37–49]. The nonlinear behavior is described in Balitsky–Kovchegov equation [43, 44]. The gluon occupation number is typically of order $\sim 1/\alpha_s$. The gluon emission can be characterized

Fig. 2.8 A “standard model” of the space-time evolution of a hot medium in high-energy heavy ion collisions. The hyperbolic geometry is due to the Lorentz dilation of relativistic longitudinal expansion

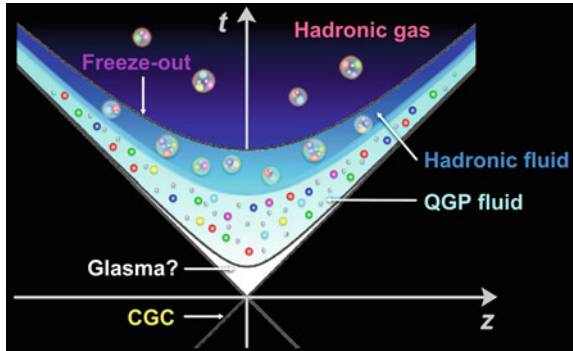
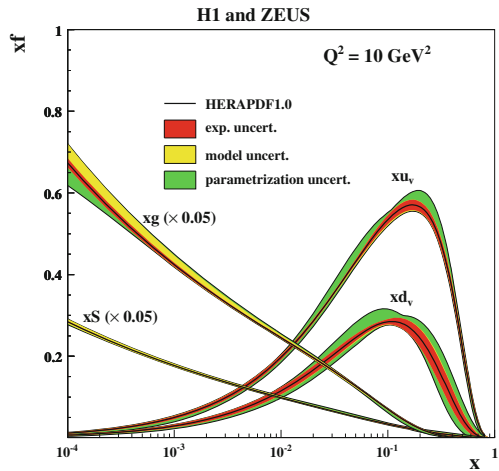


Fig. 2.9 The distributions of valence quarks $xu_v(x)$ and $xd_v(x)$, sea quarks $xS(x)$ and gluons $xg(x)$ as functions of x for the proton from H1 and ZEUS Collaborations at HERA. (Figure from Ref. [50], Springer Science+Business Media.)



by momentum fraction x of incident particles. It gets smaller as more partons are produced at higher energies. Figure 2.9 shows the distribution functions of valence up and down quarks, sea quarks and gluons observed at Hadron Electron Ring Accelerator (HERA) experiments for protons [50]. The valence quarks are important for large x and their distributions peak around $x \sim 1/3$. On the other hand, the gluons become dominant against the valence quarks for small x . The gluon saturation is suggested to be realized in high-energy heavy ion collisions since $x \sim 10^{-2}$ at RHIC and $x \sim 5 \times 10^{-4}$ at LHC. Saturation scale $Q_s(x)$ is the typical momentum of a gluon when the gluons starts to overlap in the transverse plane. It is parametrized as $Q_s^2(x, A) \sim A^{1/3}x^{-\lambda}$ where A is the atomic number of the nucleus. λ is a parameter that is experimentally suggested to be in the range 0.2–0.3 [51, 52].

The color glass picture has been successful in reproducing the centrality dependence of charged particle yields for the deuteron/proton–nucleus and nucleus–nucleus collisions assuming the number of hadrons much that of gluons at RHIC, and various studies have been made for LHC as well [53–64]. Its relation to the

hydrodynamic space-time evolution for Au–Au collisions at RHIC and Pb–Pb collisions at LHC will be investigated later in Chap. 4.

2.4.2 Early Thermalization

It is implied from the experiments that the system locally thermalizes in a surprisingly short time ($\sim 0.6\text{--}1\text{ fm}/c$) after the collisions. The precise mechanism of this early thermalization is not fully known. If one solves Yang–Mills equation [65] with the CGC initial condition, color tube structures appear between the collided nuclei, which is known as *glasma* [66]. A typical transverse size of the tube is $\sim 1/Q_s$. The word is a mixture of the color *glass* condensate and the quark-gluon *plasma*. Although it is expected to provide a bridging picture between the two models towards thermalization, the glasma currently has the problem that the longitudinal pressure stays negative during its time evolution and never reaches the hydrodynamic limit where the pressure is almost isotropic in all directions. One possible solution to the problem is instability in the glasma [67–71]. The non-boost invariant picture may play an important role in the longitudinal thermalization [72]. It is also reported that isotropization may be encouraged by the Bose–Einstein condensate that might be realized in the medium [73].

2.4.3 Hydrodynamic Stage

The correlation between the azimuthal anisotropy in coordinate space and that in momentum space at high energies suggests that the quark-gluon plasma would need to be described by relativistic hydrodynamic picture for a certain period of its lifetime. It is note-worthy that the description of relativistic hydrodynamics apply not only to the quark-gluon plasma but also to the hadronic matter near the pseudo-critical temperature as well, making it a simple and effective method to treat the time-evolution in the crossover regime. Historically speaking, the relativistic hydrodynamic study of hadron collisions started with the Landau model [74] where the hot medium created in the middle of the two shattered nuclei is speculated have no collective velocity before it starts to expand into vacuum. It should be noted the picture was originally proposed for hadron–hadron collisions. The full nuclear stopping picture later turned out to be rather unrealistic in the early heavy ion experiments since the stopping power is finite, and the hot matter was found to be described better by the scaling expansion in the longitudinal direction. This is called the Bjorken model [75], and to the present day the boost-invariant ansatz can be found in many hydrodynamic and non-hydrodynamic models of relativistic heavy ion collisions.

The fluidity of a hot medium was first observed in analyses of the Au–Au collisions at RHIC. The first relativistic ideal hydrodynamic analyses were performed by Kolb et al. [76–78] (Fig. 2.10, left), followed by Teaney et al. [79, 80]. They

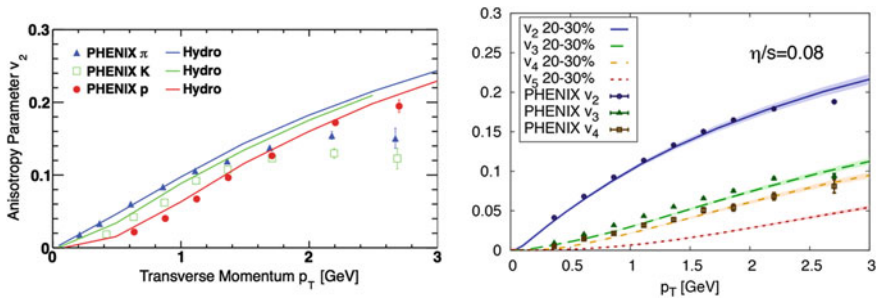


Fig. 2.10 (Left) The first ideal hydrodynamic calculations for v_2 of p_T with particle identification (π , K , p) by Huovinen et al. (Figure from Ref. [77], Elsevier.) (Right) State-of-art viscous hydrodynamic calculations with fluctuation for v_2 , v_3 , v_4 and v_5 of p_T for 20–30% centrality with the shear viscosity to entropy ratio $\eta/s = 0.08$ by B. Schenke et al. (Figure from Ref. [134], American Physical Society.)

have shown that the elliptic flow is quantitatively well-explained by hydrodynamic models, which indicates the existence of the strongly-coupled QGP in high-energy heavy ion collisions. An ideal hydrodynamic model that includes non-boost invariant longitudinal expansion in addition to the transverse one was introduced by Hirano [81, 82]. The equation of state is assumed to be of first order phase transition for those calculations. Various hybrid approaches of ideal hydrodynamic and hadronic transport models are also investigated [79, 80, 83–89] along with improvements in initial conditions and the equation of state by introduction of first principle-based results.

As mentioned in Sect. 2.3.1, the experimentally observed elliptic flow v_2 is described well by ideal hydrodynamic models up to $p_T \sim 1$ GeV, and the picture breaks down for mid-high p_T because those components would not be fully-thermalized. The agreement with the experimental data becomes better by introducing viscosity, which would embody deviation from equilibrium in hydrodynamic systems. Viscous flow analyses have been performed in boost-invariant geometry with shear viscosity [90–112] and also with shear and bulk viscosities [113–118]. Elliptic flow results with viscous corrections typically show good agreements with the data up to $p_T \sim 3$ GeV. Systematic studies on parameter dependences are also performed [119, 120]. The higher-order harmonics are explained in event-by-event hydrodynamic models well, further strengthening the understanding that the QGP is a relativistic fluid [121–125]. Hadronic transport picture for the later stages of the collision is taken into account in Refs. [126–128].

All the viscous hydrodynamic calculations mentioned above assumes boost-invariant expansion and vanishing baryon chemical potential possibly due to the difficulty in viscous formalism of relativistic expansion. The importance of the longitudinal expansion is being recognized at RHIC and LHC with shear viscosity [129], with shear and bulk viscosities with a CGC initial condition [130] and with viscosities and baryon diffusion at finite density [131]. Figure 2.10 (right) shows state-of-art

(3+1)-dimensional viscous hydrodynamic simulation results by Schenke et al. [132–134], which include shear viscosity. Their work is also followed by shear and bulk viscous calculations by Bozek [135–137] and by yet-another shear viscous simulation by Vredevogd and Pratt [138].

Recent trends in hydrodynamic studies include analyses with temperature dependent transport coefficients [139–141], which would be important to go beyond the conventional conjecture that the shear viscosity is proportional to the entropy density. Highly-anisotropic hydrodynamic models are venturing frameworks that assume only transverse thermalization [142–145]. This is motivated by the fact that the glasma picture yields the negative longitudinal pressure. The importance of hydrodynamic fluctuation coming from fluctuation–dissipation theorem was also investigated [146]. The detailed analyses on flow harmonics [147, 148] and the event plane correlations [149] have been performed as well.

There are a couple of choices for the hydrodynamic initial condition due to the fact that ambiguities still remain in the early thermalization stage. Monte–Carlo Glauber model is a phenomenological approach, which is constructed by geometrically placing wounded nucleons according to the nuclear density function. Monte–Carlo Kharzeev–Levin–Nardi (MC-KLN) model is based on CGC with k_T -factorization formula [150, 151]. The gluon energy or entropy density distribution is matched with that at the beginning of hydrodynamic evolution. Monte–Carlo running coupling Balitsky–Kovchegov (MCrcBK) is an advanced CGC based method which takes into account the effect of QCD running coupling [62]. IP-Glasma is the combination of impact parameter dependent saturation model (IP-Sat) and the glasma picture [152, 153].

2.4.4 Freeze-Out and Hadronic Transport

The quark-gluon system cools down with time evolution to the point where hydrodynamic description is no longer applicable due to the break-down of the strongly coupled picture. Freeze-out is a concept that is conventionally employed for describing the transition from the liquid picture to the gas one. The boundary between the two pictures form a 3-dimensional hypersurface Σ in Minkowski space-time. The total number of type- i particles N_i on the freeze-out hypersurface in relativistic kinetic theory is given as

$$N_i = \int_{\Sigma} d\sigma_{\mu} N_i^{\mu} = \int_{\Sigma} d\sigma_{\mu} \int \frac{g_i dp_i^3}{(2\pi)^3 E_i} p_i^{\mu} f_i, \quad (2.9)$$

where g_i is the degeneracy of the particle and $\sigma_{\mu}(x)$ is the local freeze-out hypersurface element which is perpendicular to Σ . The difference from a simple Gauss's law is that particles can be dynamically created and annihilated in the medium. The above equation gives the particle number that is frozen out at the hypersurface. Then its differential form gives the Cooper–Frye formula [154],

$$E_i \frac{dN_i}{d^3p} = \frac{dN_i}{d^2p_T dy} = \frac{g_i}{(2\pi)^3} \int_{\Sigma} p_i^{\mu} d\sigma_{\mu} f_i, \quad (2.10)$$

which gives particle spectra in phase space. The freeze-out hypersurface is usually defined by the freeze-out temperature $T_f \sim 0.13\text{--}0.16\text{ GeV}$ but one may choose some other criteria such as the magnitude of flow derivatives [112] which would be sensitive to hydrodynamic applicability. It is note-worthy that when the system is completely in local equilibrium, input from a hydrodynamic model is the flow, the temperature and the chemical potential that appears in the distribution f_i , and the energy density, the pressure and the net charge density are not used. The distribution is subject to viscous corrections, which can add extra modifications to particle spectra at freeze-out [112, 117, 155–157]. Estimations of effects of shear and bulk viscous corrections on the distribution function will be discussed in Appendix A.

The hadronic matter after freeze-out is considered to further evolve in a weakly-coupled picture. Hadronic transport models are originally used for describing a hot matter at the low-energy nuclear collisions at AGS and SPS which does not enter a fully-developed hydrodynamic phase. The heavy baryons and mesons created at freeze-out decay during the hadronic evolution, which is called hadronic cascade. Notable transport models include, but are not limited to, Jet AA Microscopic transport model (JAM) [158] and Ultra-relativistic Quantum Molecular Dynamics (UrQMD) [159, 160]. It is note worthy UrQMD also serves initial conditions for hydrodynamic simulations, which exhibit agreement with experimental data [87–89]. The hadronic transport is considered to be essential in understanding identified particle spectra from the collider experiments quantitatively.

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