Mathematical Models of Visual Information Processing in the Human Brain and Applications to Visual Illusions and Image Processing

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Abstract This chapter is a survey of a series of joint works of Shinobu Arai and me. The main purpose of our study is to construct mathematical models of visual information processing in the brain, and to give applications to study of visual illusions and image processing. In this chapter I will describe a part of our simulations of some visual illusions, creations of the so-called "fuyuu illusions", and an application to image processing.

Keywords Framelet · Pinwheel framelet · Image processing · Visual illusion

1 Introduction

On the past few decades, several studies have been made on mathematical models of visual information processing in the human brain. Our new models are constructed by using simple pinwheel framelets ([5]) and pinwheel framelets ([7]), which are new classes of the so-called framelets. The general scheme of framelets was established by Daubechies et al. [9]. Our simple pinwheel framelets and pinwheel framelets are new framelets appropriate to study of vision. Before going to the main body of this chapter, I will review (simple) pinwheel framelets.

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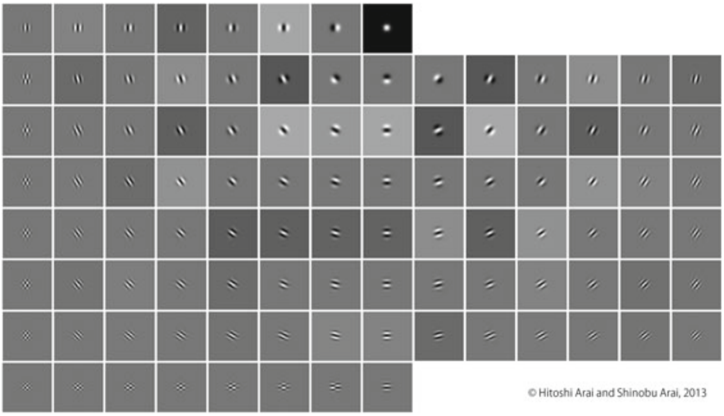

Fig. 1 Maximal overlap version of the pinwheel framelet of degree 7 (level 2)

2 Pinwheel Framelets

Some filters have been well known as models of simple cells in V1. For example, Gabor filters, DOG filters and ODOG filters. Recently R. Young pointed out that Gaussian derivatives are more appropriate than these filters (see Young [13]). Adopting his theory, Escalante-Ramírez and Silván-Cárdenas [10] proposed a mutliresolution model which consists of Gaussian derivatives. However Gaussian derivatives are not compactly supported: in other words, filters have infinite length in nature unless the filters are cut artificially. On the other hand, our simple pinwheel framelets [5] and pinwheel framelets [7] are produced by finite length filters (or compactly supported functions). The Fig. 1 is the maximal overlap version of our pinwheel framelets of degree 7 at the second level.

3 Simulations of Visual Illusions

Based on the maximal overlap version of our pinwheel framelet we constructed a nonlinear model of visual information processing in the striate cortex. This model produces computer simulations of several lightness illusions in a unified way: This method covers the Hermann grid illusion, Mach band illusion, the Chevreul illusion etc. Here I show as an example simulations of the Hermann grid illusion and some variations.

The Hermann grid is shown in Fig. 2. In this image, small gray spots are perceived at the intersections of white bands, however there are no such spots. (Note that the illusion disappear in foveal vision.) Our simulation of the illusion is Fig. 3.

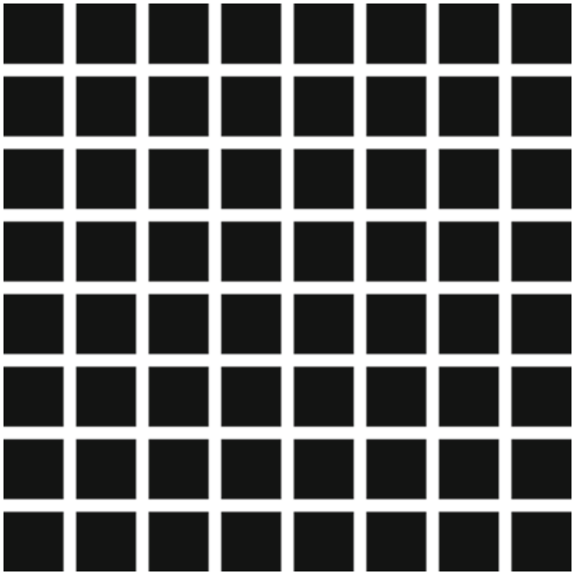

Fig. 2 Hermann grid

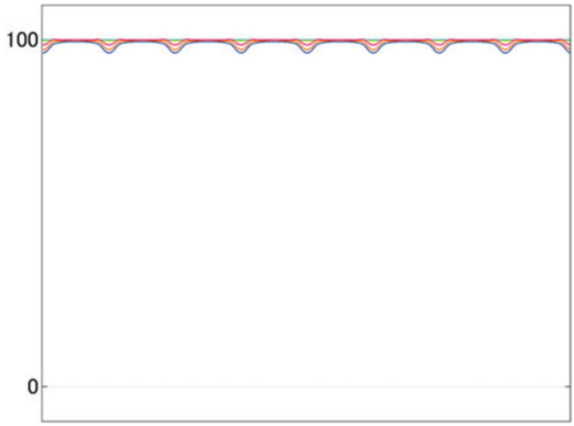

Fig. 3 Simulation of the Hermann grid illusion by using our nonlinear model with the pinwheel framelet of degree 5

So far it was thought that the Hermann grid illusion is caused by lateral inhibition in the retina ([8]). However as reported in Spillmann [11] and Wolfe [12], some evidences of a postretinal contribution to the illusion were found: the illusion is weakened when the grid is presented diagonally, and the strength of the illusion is effected by the number of intersections (see Fig. 4). Our mathematical model can simulate these phenomena. For example I show the simulation of the former phenomenon in Fig. 5.

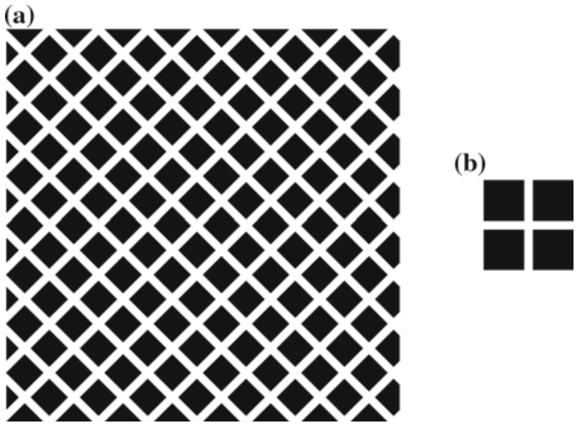

Fig. 4 **a** Diagonally presented Hermann grid, **b** Hermann grid with one intersection of white roads

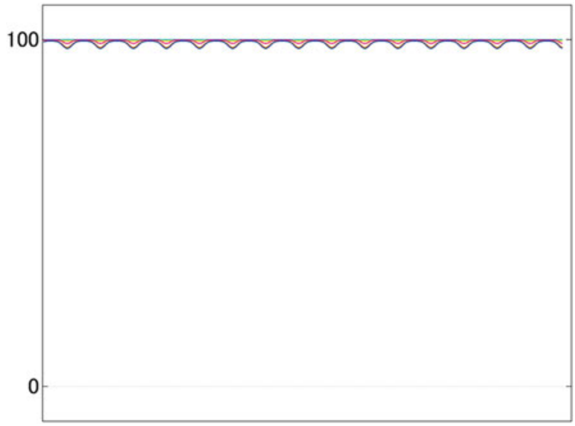

Fig. 5 Simulation of the diagonally presented Hermann grid illusion by using our nonlinear model with the pinwheel framelet of degree 5

4 Applications to Image Processing

Our nonlinear models of visual information processing in the brain have many applications not only to visual illusions, but also to image processing. A nonlinear model which has given us simulations of several lightness illusions and some illusions related to color perception provides a new method of sharpening of natural images. Our method can sharpene the parts a person generally wants to see without losing the overall image. For an example of our image processing, see [4]. This technique is patent pending (H. Arai and S. Arai, JST).

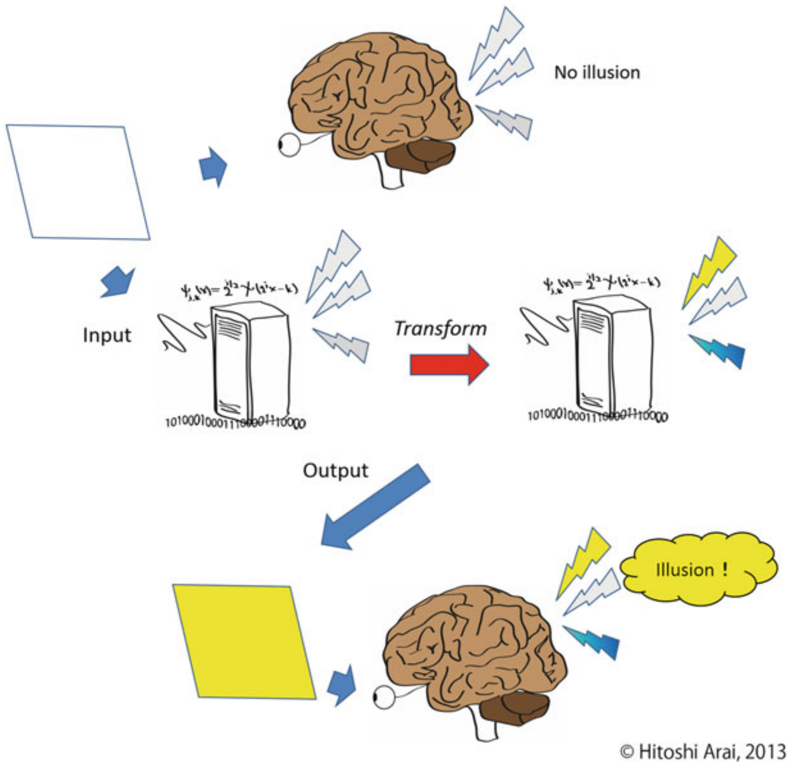

Fig. 6 Idea of transforming to fuyuu illusions

5 Creation of Visual Illusions

By one of our mathematical models of visual information processing, we can transform usual images or designs to certain illusory images: still images and designs seem to be moving, although they are at rest in the images (patent, H. Arai and S. Arai, JST). We call illusions produced by our method "fuyuu illusions". For examples of our fuyuu illusions, see [4] or the following site:

Arai, H and Arai, S.: The Birth of "New Age Op Art",—new op art born from mathematical science of optical illusions (2011).
http://www4.ocn.ne.jp/~arai/Exhibition/VisualIllusions.html

Why can our mathematical model create such illusions? A main idea is indicated in Fig. 6.

6 Other Results

We have other results related to mathematical models of visual information and applications to image processing. For example, mathematical algorithm of creating letter-row tilt illusions, noise reduction, edge detection, etc (patents, Arai and Arai, JST).

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