

Preface

This short monograph is the first one to deal exclusively with the study of almost convergence and statistical convergence of double sequences. In almost every branch of science, engineering, medical science, life science, social science, and business, we often come across some sort of sequences. So the study of sequences or sequence spaces in broader sense becomes a very important topic to deal with such situations. To get or check the limit of a sequence, there are several ways and methods. Sometimes, the limit of a sequence may not exist, but its generalized limit or weak limit may exist to serve our purpose. The notion of almost convergence is one of the most useful notions to obtain a weak limit of a bounded nonconvergent sequence. There is another notion of convergence, known as the statistical convergence, which was introduced by H. Fast, who attributed this concept to Hugo Steinhaus. Fast introduced an extension of the usual concept of sequential limit, which he called statistical convergence. In 1953, this concept arose as an example of “convergence in density,” which was also studied as a summability method. Even unbounded sequences can be dealt with this method.

In Chap. 1, we give an overview of almost convergence, statistical convergence, and some related methods for ordinary (single) sequences. In Chap. 2, we discuss the notion of almost convergence and almost Cauchy for double sequences. Some more related spaces for double sequences, associated sublinear functionals, and various inclusion relations are also studied. The theory of sequence spaces gave rise to another important topic, matrix transformations, where we study the methods to transform one sequence space into the same or another sequence space. Such a study of matrix transformations generalizes some special methods of summability. These methods are studied in Chap. 3, in which we characterize the classes of four-dimensional almost conservative, almost regular, strongly regular, and almost strongly regular matrices. In Chap. 4, we study the concept of absolute almost convergence for double sequences and use this notion to characterize the absolute almost conservative and absolute almost regular matrices. In Chap. 5, we define different cores of double sequences and establish various core theorems analogous to the well-known Knopp’s core theorem. In Chap. 6, we give some applications of almost convergence of double sequences to prove Korovkin-type approximation

theorems for functions of two variables through different sets of test functions and also show that these results are more applicable. Chapter 7 is devoted to the study of statistical convergence of double sequences, and in Chap. 8, we apply this to statistical approximation of positive linear operators. The last chapter is devoted to the study of convergence of double series and describe various convergence tests. We give many interesting examples. For the convenience of the reader, all chapters of this book are written in a self-contained style.

This brief monograph relies mostly on both authors' recent research work as well as other eminent authors' work. Advanced courses can be taught out of this short book. All necessary background and motivation are given per chapter. Besides applications in approximation theory, the presented results are expected to find applications in many other areas of pure and applied mathematics, such as mathematical analysis, probability, fixed point theory, statistics, etc. As such, this brief monograph is suitable for researchers, graduate students, and seminars on the above subjects.

This book was basically started when the first author visited King Abdulaziz University, Jeddah, Tabuk University, Tabuk, Yildiz Technical University, Istanbul, and University Putra Malaysia during 2011 and 2012, to whom he is greatly thankful for providing him kind hospitalities during the stay at these institutions. Both the authors would like to thank their respective families for their moral support during the writing of this monograph.

Aligarh, India
Jeddah, Saudi Arabia

M. Mursaleen
S.A. Mohiuddine

Convergence Methods for Double Sequences and
Applications

Mursaleen, M.; Mohiuddine, S.A.

2014, IX, 171 p., Hardcover

ISBN: 978-81-322-1610-0