

Chapter 2

Models and Frameworks

Abstract I discuss theoretical and empirical models used in environmental economics in a basic manner. I discuss constrained optimization models with an example of a non-renewable resource that is depleted over two periods. I then briefly look at models that use game theory and differential equations. In the discussion of regression models, I use the idea of a causal graph to help distinguish between association and causality. I then briefly set out the institutions and development framework of Elinor Ostrom. I conclude with a framework for thinking about diverse models—both theoretical and statistical.

Keywords Models • Optimization • Game theory • Differential equation • Causality • Statistical

Modern economics uses theoretical and empirical models; in several economics journals, it is difficult to see research articles without a model. Theoretical models are typically mathematical while the empirical models are statistical. Environmental economists use constrained optimization models to capture the essence of choices made by firms or households; these choices have environmental implications. Often, game theory is used to see how actors interact. Theoretical models are often dynamic, using differential equations, which link today's actions to the world tomorrow. Empirical work in environmental economics uses econometrics—the adaptation of statistics by economists—extensively. Increasingly, empirical work aims to establish causality, which is vital to policy, but difficult to establish.

In this chapter I discuss these theoretical and empirical models in a basic manner, briefly, and separately. In the research published in environmental economics journals the models are far more complex, and theoretical and empirical models are often used together. While environmental economics seems to use models universally, it is possible to use alternative frameworks—Nobel Laureate Elinor Ostrom (2009) developed the Institutions and Development Framework, which I discuss briefly at the end of the chapter. I conclude this chapter with a framework to think about models.

2.1 Constrained Optimization Models

Economists frequently use constrained optimization models. According to Screpanti and Zamagni (2005, p. 381), ‘Samuelson’s argument, that all the problems faced by economics (in the neoclassical approach) can be reduced to problems of constrained maximization, was very important.’ An objective function is maximized subject to one or more constraints.

In dynamic optimization we consider problems where the choices we make today affect our choices tomorrow.

Consider the extraction of a stock of oil. This will benefit those who consume oil, but will be costly to extract.

Let the net benefits (i.e. benefits minus costs) of extracting Q units of oil be $aQ - b/2Q^2 - cQ$ and the total oil reserves $= R$.

We first examine the problem of maximizing net benefits from extracting oil without worrying about the total oil reserves; this gives us the static optimum. We then examine the extraction of oil over two periods, given that total extraction is limited by finite reserves.

If oil is extracted to maximize net benefits in a particular period, the optimal static quantity ($Q^{\text{STATIC}*}$) is given by taking the derivative of the net benefits (called marginal net benefits), and equating to zero

$$a - bQ^{\text{STATIC}*} - c = 0$$

$$\text{So } Q^{\text{STATIC}*} = (a - c)/b.$$

$$\text{If } a = 10, c = 2 \text{ and } b = 0.5, Q^{\text{STATIC}*} = 16.$$

We now consider the dynamic problem. Consider only two periods, now or later, indexed by 0 or 1. Let Q_0 and Q_1 denote extraction in periods 0 and 1. Then $Q_0 + Q_1$ cannot be more than R . If R is more than twice the static optimum extraction, i.e. $Q^{\text{STATIC}*}$, then choices now do not affect choices later. If $R < Q^{\text{STATIC}*}$, we have a dynamic optimization problem.

We use dynamic optimization to solve this problem. We have a discrete time optimization problem, and will use the Lagrange approach.

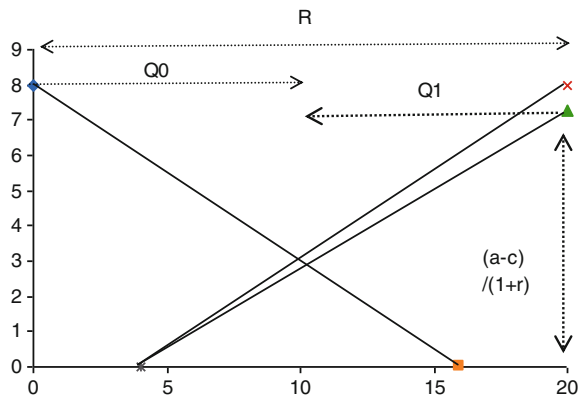
In the Lagrange approach, we use a methodological construct—we form an expression called the Lagrangian:

Lagrangian = Objective + Multiplier $\times$ Constraint, and then maximize the Lagrangian. This has the added benefit of yielding the multiplier that has an economic interpretation of a ‘shadow price’.

In our situation the objective = net benefits in period 0 + net benefits in period 1. The constraint is that $Q_0 + Q_1 = R$. Therefore, the Lagrangian, L , is given by:

$$\begin{aligned} L &= \{aQ_0 - \frac{1}{2} b Q_0^2 - c Q_0\} + \{[aQ_1 - \frac{1}{2} b Q_1^2 - c Q_1]/(1+r)\}, \\ &\quad - \lambda [Q_0 + Q_1 - R], \\ &= \left\{ \sum [aQ_t - \frac{1}{2} b Q_t^2 - c Q_t]/(1+r)^t \right\} - \lambda [Q_0 + Q_1 - R] \end{aligned}$$

Fig. 2.1 Extraction of oil over two periods. Total resource available ($R = 20$) is represented on the horizontal axis, Q_0 (extraction in period zero) goes from left to right, Q_1 (extraction in period one) goes from right to left; marginal net benefits on the y axis



where λ is the multiplier.

The partial derivative with respect to $Q_0 = a - bQ_0 - c - \lambda = 0$

The partial derivative with respect to $Q_1 = \{[a - bQ_1 - c]/(1 + r)\} - \lambda = 0$

$$\text{So } a - bQ_0 - c = \{[a - bQ_1 - c]/(1 + r)\} \quad (2.1)$$

The marginal net benefit (in present value terms) across the two periods is equated.

This combined with the constraint, gives us the answer. About 10.3 units are used in the first time period and about 9.7 units in the second. Thus more than half the resource should be extracted in the first time period.

Figure 2.1 provides an intuitive feel for the problem. On the x axis we have quantity of oil. Total reserves are 20; moving rightwards from zero we have Q_0 and moving leftwards from 20 we have Q_1 . On the y-axis we plot the marginal net benefits; with the discounted (i.e. divided by $1 + r$) net benefit curve for the second period being lower than the undiscounted curve. Where the two net benefit curves for the two periods intersect we have the solution to Eq. 2.1. The higher is r , the more is Q_0^* relative to Q_1^* . In other words, we extract more now and leave less for later—a recurrent theme in resource economics. We shall encounter a dynamic optimization model in the Appendix of Chap. 6.

2.2 Game Theory Models

Game theory models help us model interdependent decisions for more than one person. If we consider two persons, each person's decision depends on the other's. While seeming like a purely technical extension of single person optimization, it often leads to quite different insights. Notably, game theory can show the tension between individual and social rationality and its resolution through collective action.

Table 2.1 Payoff matrix for a two player game

Player A	Player B	
	Left	Right
Up	(9, 27)	(3, 24)
Down	(0, 0)	(6, 3)

A game consists of: a set of players; a set of strategies for each player; and the payoffs to each player for every possible list of strategy choices by the players.

We will only study games in which there are two players, each of whom can choose between two strategies. Here, the players are called A and B. Player A has two strategies, called “Up” and “Down”. Player B has two strategies, called “Left” and “Right”. The table showing the payoffs to both players for each of the four possible strategy combinations is the game’s payoff matrix (Table 2.1).

If A plays Up and B plays left then A’s payoff is 9 and B’s payoff is 27 (Table 2.1). A play of the game is a pair such as (U, L) where the first element is the strategy chosen by A and the second is the strategy chosen by B.

We examine which play is likely given the game’s payoff matrix, by examining each cell in Table 2.1 in turn. Here, If B plays Right then A’s best reply is Down rather than Up since $6 > 3$. So (U, R) is not a likely play.

If B plays Right then A’s best reply is Down. If A plays Down then B’s best reply is Right. So (D, R) is a likely play.

A play of the game where each strategy is a best reply to the other is a Nash equilibrium. Our example in Table 2.1 has two Nash equilibria: (U, L) and (D, R). A Nash equilibrium represents individual rationality—it may not be the best outcome for the players collectively; both may be better off with some other play. In Chaps. 3 and 4, we will see examples of the Prisoner’s Dilemma where individual rationality is at odds with collective rationality.

2.3 Differential Equation Models

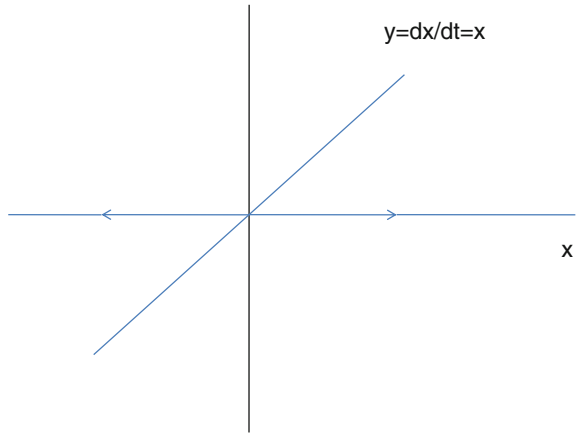
Differential equations are used commonly in the physical and natural sciences, and extensively in mathematical ecology.

In a differential equation, a variable is changing continuously with time, and the change in the variable is described by the equation.

An example is $y = dx/dt = x$; here the change in x is itself a variable and equal to x . Visualizing this gives us an insight into the evolution of x over time (Fig. 2.2).

If $x = 0$ then $dx/dt = 0$, i.e. x will not change. We say that $x = 0$ is an equilibrium. If $x = 1$ then $dx/dt = 1$, so x will increase. If $x = -1$, then $dx/dt = -1$, so x will decrease. Because a slight push to x from $x = 0$ will result in x moving further away, we say that $x = 0$ is an unstable equilibrium (Fig. 2.2).

Fig. 2.2 Graph of $y = dx/dt$ ($= x$) versus x . There is an unstable equilibrium at $x = 0$



If $y = dx/dt = -x$, then by similar reasoning, $y = 0$ will be a stable equilibrium (Fig. 2.3).

Stewart (2010) explains how mathematical models can help us probe beyond the surface. Non-linear dynamics can be complex. A graph of carbon (C) and temperature (T) over time (t) gives the impression that because temperature leads carbon, carbon dioxide emissions are not causing changes in temperature. We need to first model the natural process and Stewart uses non-linear differential equations to model the mutual dependence of carbon and temperature (Fig. 2.4), the equations are such that temperature leads carbon in a graph of their time series. The set of equations (Stewart 2010, p. 168) is: (1) $dT/dt = \sin t - 0.25 C - 0.01 T^2$, and (2) $dC/dt = 0.1 T - 0.01 C^2$.

To this natural process we can add the shock to the system because of high emissions of carbon dioxide by human beings (Fig. 2.5). In the resulting output

Fig. 2.3 Graph of y or dx/dt ($= -x$) versus x . There is a stable equilibrium at $x = 0$

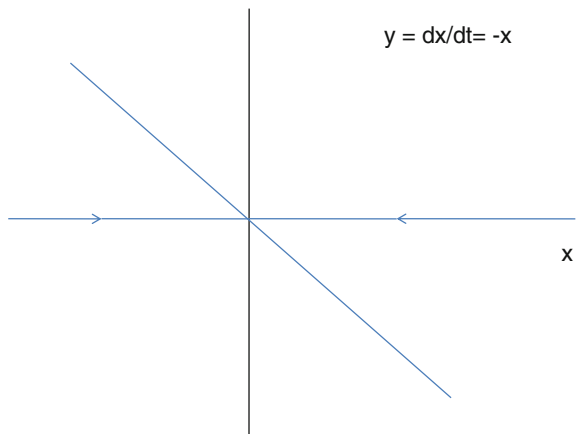


Fig. 2.4 Natural process (in which C and T cause each other) modeled by Ian Stewart; the resulting graph of time series of output of natural process shows T leading C

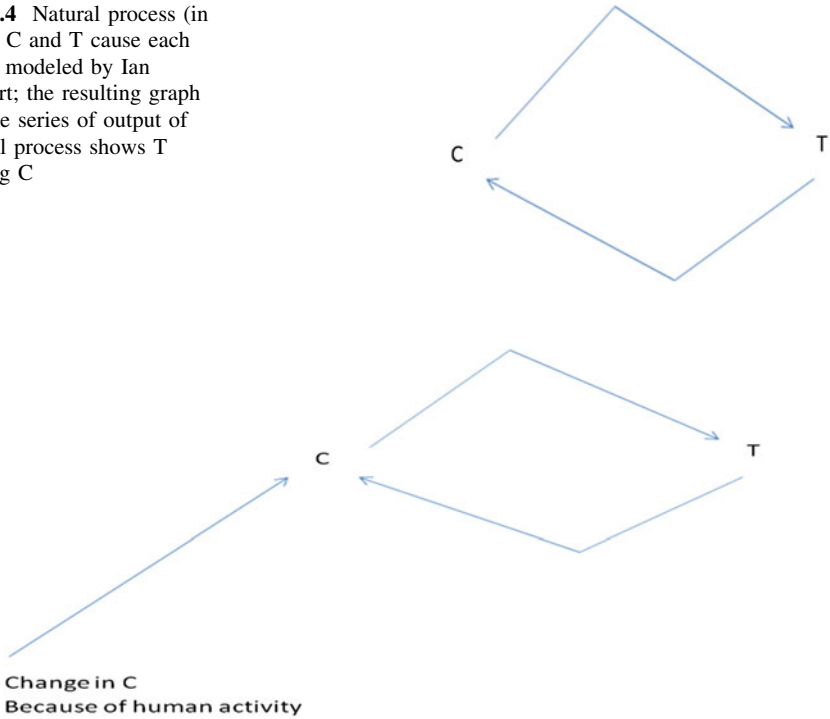


Fig. 2.5 Human intervention leading to increases in C, graph of time series still shows T leading C

from the model, although we have increased carbon dioxide from outside the system, temperature continues to lead carbon dioxide. However, if a change in temperature is plotted along with the change in carbon dioxide over time, we see that the temperature increases at the same time as the increase in carbon dioxide.

In other words, in the model Stewart (2010) created with non-linear differential equations, although carbon dioxide causes temperature, the appearance of the system as captured in time series graphs would lead to a superficial, and wrong, impression that temperature causes carbon dioxide emissions. [Chapter 5](#) on complex ecology uses simple differential equations.

2.4 Regression Models and Causality

In a regression model we have observations on a dependent variable, say y , and one or more independent variables, say x . y is likely to be related to x , or we think it is, but not exactly, so we model this as

$$y = \beta_1 + \beta_2 x + \varepsilon$$

where ε is a disturbance term.

Econometricians often use the notion of a data generating process—we observe data that has been generated in the world by some process and we hope to use regression to recover that. One of the key assumptions of the classical linear regression model, the specification assumption, is that we can estimate the dependent variable as a linear function of some independent variables plus a disturbance term (Kennedy 2003). For example, if the data generating process for example is $y = 0.2 + 20x$, then a regression of y on x helps us recover the equation—we use a formula to estimate β_2 so that we come as close to 20 (the true value) as possible.

Regressions may indicate correlation or association rather than causality.

Causality is important because it is implicit in assertions on, or advocacy of, policies. We can distinguish between regression that is predictive or descriptive (conditional expectation) and regression that is causal.

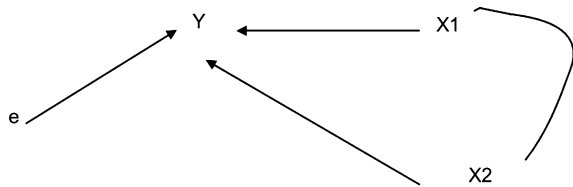
The difficulty in dealing with causal issues is related to the nature of the research questions and the complexity of the systems. For example, we can contrast the following two questions: (1) which of the following interventions has a greater effect on educational outcomes, school bag or blackboard, versus (2) what are the causes of deforestation in different countries? The causal issue is easier in the case of the effect of interventions on educational outcomes than in the case of the causes of deforestation. Pearl (1999) emphasizes the distinction between causal thinking and probability theory:

The word cause is not in the vocabulary of standard probability theory. It is an embarrassing yet inescapable fact that probability theory, the official language of many empirical sciences, does not permit us to express sentences such as ‘Mud does not cause rain’; all we can say is that the two events are mutually correlated, or dependent—meaning that if we find one, we can expect to encounter the other. Scientists seeking causal explanations for complex phenomena or rationales for policy decisions must therefore supplement the language of probability with a vocabulary for causality, one in which the symbolic representation for the causal relationship ‘Mud does not cause rain’ is distinct from the symbolic representation for ‘Mud is independent of rain’ (p. 1)

To summarize, we use probability for empirical work, but causal thinking requires us to put in a directional arrow: rain $\rightarrow$ mud. A regression $Y = a + bX_1 + cX_2 + e$ can be interpreted predictively or causally. Predictively, we say the regression gives us the conditional expectation of Y . If we interpret it causally, we have in mind an implicit causal graph (as in Fig. 2.6); the correctness of this causal interpretation depends on the correctness of the implicit causal graph for the situation that we are studying. If the causal pathways do not correspond with what appears in Fig. 2.6, then the causal interpretation is problematic.

If we want to explore the effect of X on Y , following Morgan and Winship (2007), we can list the key ingredients of causal graphs as follows:

Fig. 2.6 Implicit causal graph for regression
 $Y = a + b X_1 + c X_2 + e$



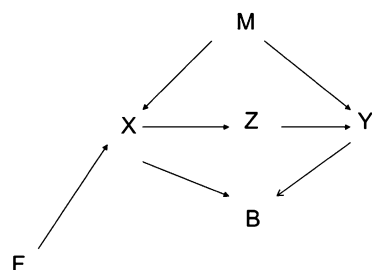
- Common cause (controlling for M, the common cause, will block path): $X \leftarrow M \rightarrow Y$. Here M causes X as well as Y.
- Mediator (controlling for Z, will block path): $X \rightarrow Z \rightarrow Y$. Here X causes Z and Z causes Y.
- Collider (controlling for B, opens path): $X \rightarrow B \rightarrow Y$. Here X and Y are both causes of B.

Whether we should control for a variable depends on whether it is a common cause, a mediator, or a collider. However, we need to see the causal graph as a whole. Figure 2.7 illustrates three specification options for studying the effect of X on Y, given the causal graph.

The intuition for excluding the collider is best given with an example. Assume a situation where a car fails to start in cold weather when there is petrol in the tank. While petrol in the tank of a car and very cold weather are uncorrelated, they can be common causes of a car failing to start in cold climates. The car not starting is the collider. Whether there is petrol in the car or not tells me nothing about cold weather. But if my car fails to start, and there is petrol in the tank, I would guess that cold weather is the cause.

If we wish to study the causal effect of X on Y, we can also experimentally vary X and see how the outcome of the experiment, Y, varies between the ‘treatment’ and the ‘control’ group. In this case we free X from the influence of M if our experiment is carefully designed. In the next chapter, we will look at John Snow’s famous use of a ‘natural experiment’ to study cholera. In Chap. 4, we see how

Fig. 2.7 Causal graph and regression options



Options, regress:

1. Y on X and M
 2. Y on X, with F as an instrumental variable
 3. Regress Z on X * regress Y on Z
- In 1, 2, and 3, do not include B

Somanathan and co-authors used regression and matching to establish the effects of management regime on forest stock. In [Chap. 6](#), we discuss the causal issues that arise while studying carbon storage and livelihoods. In [Chap. 7](#), we see two applications that use the method of instrumental variables to overcome simultaneous causation bias.

2.5 The Institutions and Development Framework

Vatn (2005) distinguishes between three different kinds of institutions: conventions, norms and formally sanctioned rules. But Vatn (2005) sees a tension between the individualist and social constructivist positions in the social sciences with regard to the approach to institutions—a tension that can be plotted in terms of the direction of causality between individuals and institutions. As Hodgson (2000) puts it: “In the writings of Veblen and Commons there is both upward and downward causation; individuals create and change institutions, just as institutions mold and constrain individuals” (p. 326). Ostrom (2005) points out that institutional analysis requires “digging deeper than markets and hierarchies” (p. 819), the analyst has to recognize institutions that are often intangible, and not easily measurable. Alston (1996) also draws attention to the difficulty of observing institutions quantitatively:

Institutional changes usually have some unique features limiting the data points and thus generally preventing conventional statistical analysis ... Frequently, quantitative measures of the causes or consequences of institutional change are simply not available; even when they are available, better evidence may come from the qualitative historical record (pp. 29–30).

In this brief I focus on models. But we can think of frameworks—which are more general than models—and include more elements of reality. A framework may include more than one model. The Nobel Laureate Elinor Ostrom developed the Institutions and Development framework. In this framework, the context influences the action arena and interactions, which lead to outcomes (see [Fig. 2.8](#)). Outcomes can feedback into the context and the action arena and interactions. In the case of some action arenas and interactions, these may be represented by a game theory model. Among the contextual or exogenous variables are rules or institutional arrangements. Thus, we have a more inclusive view—institutions can help resolve social dilemmas represented by game theory models. Rules and



Fig. 2.8 The institutions and development framework

Table 2.2 Four-fold classification of goods

	Excludable	Non-excludable
Rival	Private goods, e.g. coffee cup	Common-pool resources, e.g. forests, fisheries
Non-rival	Club goods, e.g. cable TV	Public goods, e.g. reducing greenhouse gases

institutional arrangements can be examined at multiple levels—operational and collective choice (rules for making rules) levels.

Although Ostrom (2009) worked with experiments to study game theory models in the laboratory, she also documented the immense variety of rules that are used by people in the world. The framework was also used when she and her co-workers conducted a meta-analysis—to systematically learn from diverse studies of the commons.

In her Nobel lecture, Ostrom (2009) said that she and her colleagues found that the focus on two organizational forms—the state and the market—did not do justice to the wide variety of institutional arrangements that humans design. They found that the assumption of rational individuals worked well in certain market situations, but was inadequate in diverse social dilemmas. They rejected Samuelson’s two-fold classification of goods—private and public. James Buchanan had developed the concept of club goods, and Ostrom and her colleagues added a fourth type of good, common pool resources (Table 2.2). Common pool resources like forests and fisheries, which are non-excludable and rival, affect millions of people in developing countries. Reducing greenhouse gases is a global public good—it is non-excludable and non-rival (Table 2.2).

2.6 Conclusions and Pointers: A Framework for Thinking about Diverse Models—Both Theoretical and Statistical

Dasgupta (2002), responding to critics of modern economics, said that most economists go ahead and do economic analysis. I think there is an element of truth in the observation that modern economists aim to master sophisticated techniques in their training and then use them, and don’t spend their time on philosophical reflection. However, Bromley (2006, p. 87), who could be regarded as a critic of standard economics, reflects on how economists fix belief: ‘The standard approach to economics is embedded in the hypothetico-deductive method. On this approach, primitive axioms (covering laws) inform the search for particular assumptions and applicability postulates that will then suggest hypotheses to be tested against data from the “real” world. The axioms entail postulates of rationality, self-interest, stable preferences, and the alleged desire to maximize utility. Indeed, the core axioms of economic theory are rarely subjected to tests of their veracity.’

Bromley's discussion is deep and sophisticated, requiring repeated study. He does not explicitly discuss statistical models. He does, however, distinguish between mechanical cause and reason—if a man switches on a light, the switch is the mechanical cause of the light coming on, but not the reason.

King (1998) attempted to unify the methodology of political science, but he does so under a general theory of statistical inference. King et al. (1994) aimed to include even qualitative inquiry under such a theory. The purpose here is not to unify like the hedgehog, but to consider plural extensions of core economic models, like the fox. In this framework I identify elements of diverse models.

In this brief, while discussing statistical models, I pay attention to issues of causality, and I use informal causal graphs, thus drawing on Pearl's pioneering formal work on causal graphs. I view a causal graph as implicit in a statistical model, with a theoretical model helping us think about a causal graph. Here, I think of environment and development economics as resulting from an interplay between theoretical and statistical models.

We make use of models (M) in environment and development economics; both theoretical models (M^T) and statistical models (M^S).

2.6.1 Theoretical Models

A theoretical model (M^T) can be thought of as having the following list of attributes, listed below:

M^T : $Tech^T$, A^T , V^T

where $Tech^T$ represents a technique (for example, game theory); A^T denotes assumptions and axioms; and V^T denotes variables. The technique used could be mathematical or verbal. Assumptions and axioms are to some extent tied to the technique—constrained optimization techniques in neoclassical microeconomics use the axiom of rationality (Bromley 2006). Over a long period, the average economist invests in such techniques, so such axioms may become part of the economist's mental reflexes. The set of variables V^T varies from problem to problem. In neoclassical economics V^T is usually prices and quantities.

2.6.2 Statistical Models

A statistical model, M^S , can be thought of as having the following list of attributes: a set of variables V^S , a causal graph CG , a set of parameters that are to be estimated θ , a technique T^S , and statistical assumptions A^S . We would have data on the variables V^S , which may be seen to be connected with an implicit, default causal graph or an explicit causal graph (see Figs. 2.6 and 2.7). When we use regressions, the parameters we estimate would include the coefficients and their

standard errors. Often attention is centred on the ‘significance’ (often equated with statistical significance) of θ , A^S , and not so much on CG.

A particular economic analysis addresses a specific problem, by harnessing elements of a theoretical model (M^T) and/or elements of a statistical model (M^S). A problem may be framed in a new way by mapping a model used for a different purpose to the phenomenon being studied—we may use models of investment not just for the stock of machinery but also investment in human capital (education), or nature (forest preservation). We may go from the output of a regression of sulphur dioxide on per capita income to the metaphor of the Environmental Kuznets Curve, and develop a theoretical model—i.e. go from M^S to M^T . Or we may go from a theoretical model of monetary valuation of air pollution to a statistical model—i.e. go from M^T to M^S .

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