Chapter 2
Transformer, Transmission Line, and Load

2.1 Transformer

2.1.1 Single-Phase Transformer

The single-phase transformer consists of a core and two or more windings. Figure 2.1 shows a transformer with two windings.

Let the transformer be ideal: the windings have zero resistance and the core has infinite permeability [1]. Infinite permeability means that there is no flux outside the core.

N_1 and N_2 are the number of turns in the windings. If the flux in the core is ϕ, the induced emfs in the windings are

$$e_1 = N_1 \frac{\mathrm{d}\phi}{\mathrm{d}t} \tag{2.1}$$

$$e_2 = N_2 \frac{\mathrm{d}\phi}{\mathrm{d}t} \tag{2.2}$$

From (2.1) and (2.2),

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} \tag{2.3}$$

The relation between i_1 and i_2 is obtained from Ampere's law. Due to infinite permeability, the magnetic field intensity in the core is zero. Application of Ampere's law to the closed path in the core, shown in Fig. 2.1, gives

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} \tag{2.4}$$

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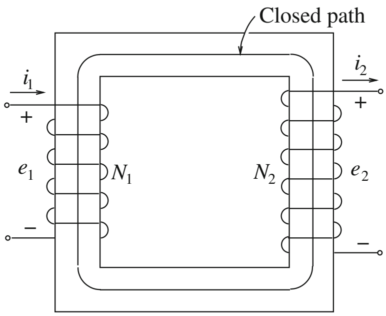


Fig. 2.1 Single-phase transformer with two windings

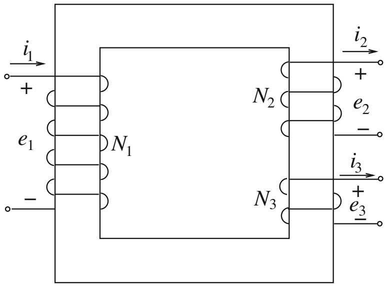


Fig. 2.2 Single-phase transformer with three windings

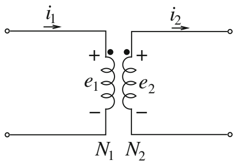


Fig. 2.3 Representation of ideal transformer

For the three-winding transformer shown in Fig. 2.2, where the number of turns in the windings are N_1, N_2, and N_3,

$$\frac{e_1}{e_2} = \frac{N_1}{N_2}, \frac{e_1}{e_3} = \frac{N_1}{N_3} \tag{2.5}$$

$$N_1 i_1 = N_2 i_2 + N_3 i_3 \tag{2.6}$$

The two-winding ideal transformer can be represented by the equivalent circuit shown in Fig. 2.3. The dots shown at a terminal of each winding indicate the winding terminals which simultaneously have the same polarity due to the emfs induced.

There are applications where the ideal transformer cannot be used. Then the equivalent circuit of the transformer is given by Fig. 2.4. R_1 and R_2 are the resistances of the two windings. Though the permeability of the core is high, it is not infinite, and hence there is flux outside the core which links some or all turns of only one winding and induces an emf. This flux is called leakage flux and its effect is modelled by leakage inductances L_1 and L_2. e_1 and e_2 are related by (2.3). Due to finite permeability of the core, (2.4) is not exact but is used as an approximation.

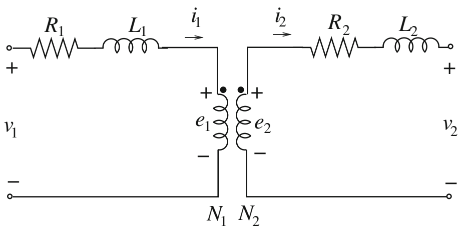


Fig. 2.4 Equivalent circuit of transformer

2.1.2 Three-Phase Transformer

A three-phase transformer can be obtained from three identical single-phase transformers. Figure 2.5 shows the equivalent circuit of the wye-wye-connected transformer. The two windings of a single-phase transformer are shown parallel to each other.

Let v_{1a}, v_{1b}, and v_{1c} be the potentials of the terminals $1a$, $1b$, and $1c$, respectively, with respect to the neutral. Let v_{2a}, v_{2b}, and v_{2c} be the potentials of the terminals $2a$, $2b$, and $2c$, respectively, with respect to the neutral. The equations for the wye-wye-connected transformer are

$$v_{1a} = i_{1a}R_1 + L_1\frac{\mathrm{d}i_{1a}}{\mathrm{d}t} + e_{1a} \tag{2.7}$$

$$v_{1b} = i_{1b}R_1 + L_1\frac{\mathrm{d}i_{1b}}{\mathrm{d}t} + e_{1b} \tag{2.8}$$

$$v_{1c} = i_{1c}R_1 + L_1\frac{\mathrm{d}i_{1c}}{\mathrm{d}t} + e_{1c} \tag{2.9}$$

$$v_{2a} = \frac{N_2}{N_1}e_{1a} - \frac{N_1}{N_2}i_{1a}R_2 - \frac{N_1}{N_2}L_2\frac{\mathrm{d}i_{1a}}{\mathrm{d}t} \tag{2.10}$$

$$v_{2b} = \frac{N_2}{N_1}e_{1b} - \frac{N_1}{N_2}i_{1b}R_2 - \frac{N_1}{N_2}L_2\frac{\mathrm{d}i_{1b}}{\mathrm{d}t} \tag{2.11}$$

$$v_{2c} = \frac{N_2}{N_1}e_{1c} - \frac{N_1}{N_2}i_{1c}R_2 - \frac{N_1}{N_2}L_2\frac{\mathrm{d}i_{1c}}{\mathrm{d}t} \tag{2.12}$$

Elimination of induced emfs from (2.7) to (2.12) gives

$$v_{2a} = \frac{N_2}{N_1}v_{1a} - \left(\frac{N_2}{N_1}R_1 + \frac{N_1}{N_2}R_2\right)i_{1a} - \left(\frac{N_2}{N_1}L_1 + \frac{N_1}{N_2}L_2\right)\frac{\mathrm{d}i_{1a}}{\mathrm{d}t} \tag{2.13}$$

$$v_{2b} = \frac{N_2}{N_1}v_{1b} - \left(\frac{N_2}{N_1}R_1 + \frac{N_1}{N_2}R_2\right)i_{1b} - \left(\frac{N_2}{N_1}L_1 + \frac{N_1}{N_2}L_2\right)\frac{\mathrm{d}i_{1b}}{\mathrm{d}t} \tag{2.14}$$

$$v_{2c} = \frac{N_2}{N_1}v_{1c} - \left(\frac{N_2}{N_1}R_1 + \frac{N_1}{N_2}R_2\right)i_{1c} - \left(\frac{N_2}{N_1}L_1 + \frac{N_1}{N_2}L_2\right)\frac{\mathrm{d}i_{1c}}{\mathrm{d}t} \tag{2.15}$$

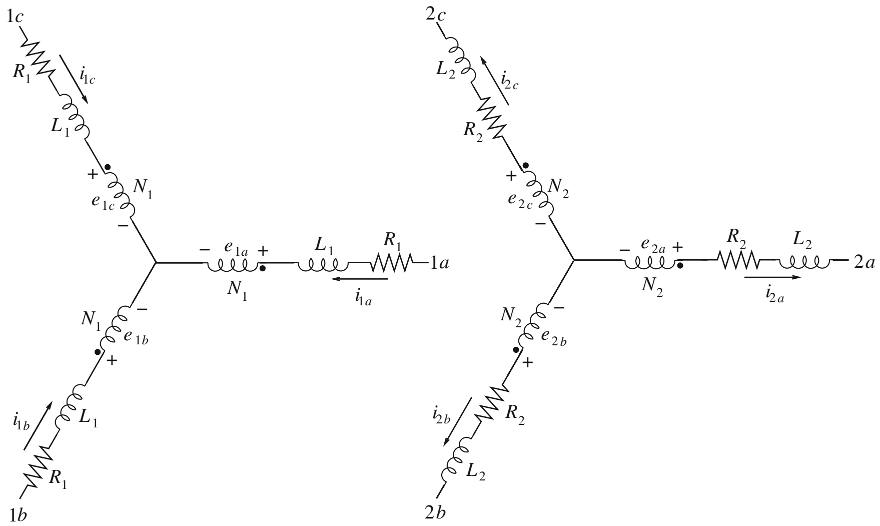


Fig. 2.5 Wye-wye-connected transformer

In order to obtain the equations in per unit quantities, the equations are divided by the base voltage. If V_{1B} is the base voltage on the transformer side with N_1 turns, the base voltage on the other side of the transformer is

$$V_{2B} \triangleq \frac{N_2}{N_1} V_{1B} \tag{2.16}$$

The base values of other quantities are obtained as follows.

$$I_{1B} = \frac{S_B}{V_{1B}},\ I_{2B} = \frac{S_B}{V_{2B}},\ Z_{1B} = \frac{V_{1B}}{I_{1B}},\ Z_{2B} = \frac{V_{2B}}{I_{2B}} \tag{2.17}$$

Dividing (2.13)–(2.15) by V_{2B} gives

$$\overline{v}_{2a} = \overline{v}_{1a} - \left(\overline{R}_1 + \overline{R}_2\right)\overline{i}_a - \frac{1}{\omega_B}\left(\overline{X}_1 + \overline{X}_2\right)\frac{\mathrm{d}\overline{i}_a}{\mathrm{d}t} \tag{2.18}$$

$$\overline{v}_{2b} = \overline{v}_{1b} - \left(\overline{R}_1 + \overline{R}_2\right)\overline{i}_b - \frac{1}{\omega_B}\left(\overline{X}_1 + \overline{X}_2\right)\frac{\mathrm{d}\overline{i}_b}{\mathrm{d}t} \tag{2.19}$$

$$\overline{v}_{2c} = \overline{v}_{1c} - \left(\overline{R}_1 + \overline{R}_2\right)\overline{i}_c - \frac{1}{\omega_B}\left(\overline{X}_1 + \overline{X}_2\right)\frac{\mathrm{d}\overline{i}_c}{\mathrm{d}t} \tag{2.20}$$

where $X_1 \triangleq \omega_B L_1$ and $X_2 \triangleq \omega_B L_2$. The subscripts 1 and 2 are not necessary in the notation for currents since $\overline{i}_{1a} = \overline{i}_{2a}$, $\overline{i}_{1b} = \overline{i}_{2b}$, and $\overline{i}_{1c} = \overline{i}_{2c}$.

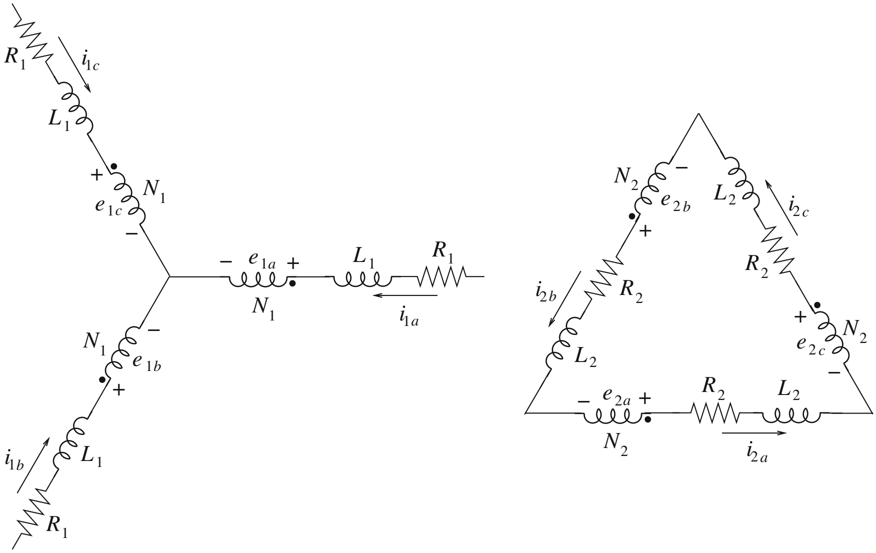


Fig. 2.6 Wye-delta-connected transformer

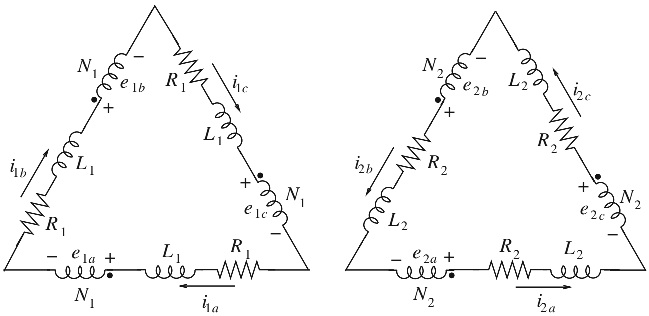


Fig. 2.7 Delta-delta-connected transformer

The equivalent circuit of the wye-delta-connected transformer is shown in Fig. 2.6. For the wye-delta-connected transformer, if V_{1B} is the base voltage on the transformer side with N_1 turns, the base voltage on the other side of the transformer is

$$V_{2B} \triangleq \frac{N_2}{\sqrt{3}N_1} V_{1B} \tag{2.21}$$

For balanced sinusoidal operation, if resistance and leakage inductance are neglected, the phase shift between terminal voltages on the two sides of the transformer is 30°.

The equivalent circuit of the delta-delta-connected transformer is shown in Fig. 2.7. For the delta-delta-connected transformer, if V_{1B} is the base voltage on the transformer side with N_1 turns, the base voltage on the other side of the transformer is

$$V_{2B} \triangleq \frac{N_2}{N_1} V_{1B} \tag{2.22}$$

2.2 Transmission Line

A transmission line has four parameters: series resistance, series inductance, shunt conductance, and shunt capacitance. These parameters are distributed uniformly throughout the length of the transmission line. The series resistance in each phase is denoted by R. For an overhead transmission line, the shunt conductance represents the effects of leakage current over the surface of the insulator and corona. The shunt conductance in each phase is denoted by G.

The expression for inductance and capacitance are derived for overhead transmission lines. The derivations assume that the conductors are straight.

2.2.1 Inductance

2.2.1.1 Transmission Line with Three Conductors

Let the transmission line consist of three conductors, one for each phase, of radius r as shown in Fig. 2.8. Let the current in these conductors be i_a, i_b, and i_c with uniform current density. The expression for inductance is derived assuming that

$$i_a + i_b + i_c = 0 \tag{2.23}$$

It is assumed that the three conductors are transposed if not spaced symmetrically, in order to have a symmetrical system; the transmission line is divided into three sections of equal lengths and each conductor occupies each of the three positions 1, 2, and 3 for one third of the transmission line length. Let the conductors a, b, c occupy positions 1, 2, 3, respectively, in the first section, positions 2, 3, 1, respectively, in the second section, and positions 3, 1, 2, respectively, in the third section.

Consider a tube of radius $x < r$ and thickness dx in phase a conductor in section 1 as shown in Fig. 2.8; the tube is coaxial with the conductor. Consider a filament in this tube with cross-sectional area $x\mathrm{d}x\mathrm{d}\theta$; $\mathrm{d}\theta$ is the angle subtended at the axis of the conductor by the filament [2]. Consider the closed path consisting of this filament and an arbitrarily located (at P) straight line parallel to the conductors. Let ψ_{fa}, ψ_{fb}, and ψ_{fc} be the flux linkage of this closed path in section 1, due to i_a, i_b, and i_c, respectively. The power delivered to this closed path, due to i_a, is equal to

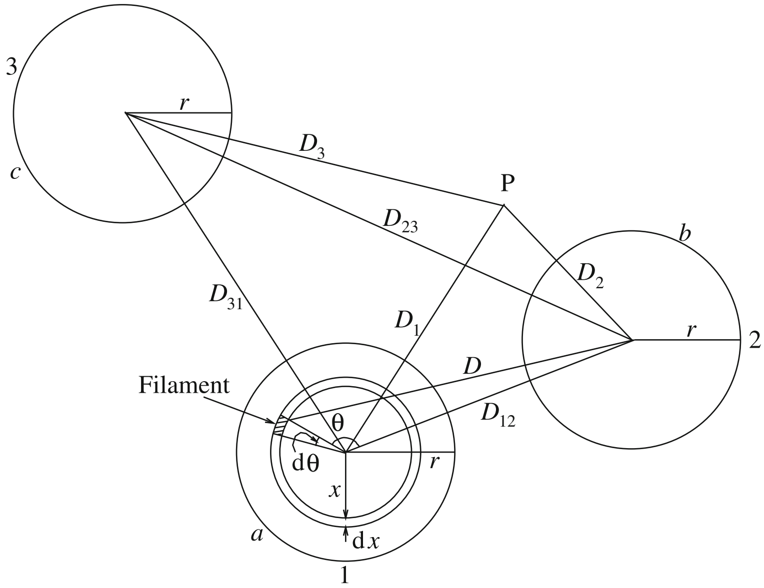


Fig. 2.8 Cross section of transmission line conductors

$$\frac{\mathrm{d}\psi_{fa}}{\mathrm{d}t} i_a \frac{x\mathrm{d}x\mathrm{d}\theta}{\pi r^2} = \frac{1}{\pi r^2} \psi_{fa} \frac{\mathrm{d}i_a}{\mathrm{d}t} x\mathrm{d}x\mathrm{d}\theta \tag{2.24}$$

The power delivered to phase a, due to i_a, is obtained by integrating this expression over the cross-sectional area of the conductor as follows.

$$p = \frac{1}{\pi r^2} \frac{\mathrm{d}i_a}{\mathrm{d}t} \int_{\theta=0}^{2\pi} \int_{x=0}^{r} \psi_{fa} x\mathrm{d}x\mathrm{d}\theta \tag{2.25}$$

Let ψ_{aa}, ψ_{ab}, and ψ_{ac} be the flux linkages of phase a in section 1 due to i_a, i_b, and i_c, respectively. The expression for p can also be written in terms of flux linkage of phase a due to i_a, as

$$p = \frac{\mathrm{d}\psi_{aa}}{\mathrm{d}t} i_a = \psi_{aa} \frac{\mathrm{d}i_a}{\mathrm{d}t} \tag{2.26}$$

From (2.25) and (2.26),

$$\psi_{aa} = \frac{1}{\pi r^2} \int_{\theta=0}^{2\pi} \int_{x=0}^{r} \psi_{fa} x\mathrm{d}x\mathrm{d}\theta \tag{2.27}$$

Similarly, it can be shown that

$$\psi_{ab} = \frac{1}{\pi r^2} \int_{\theta=0}^{2\pi} \int_{x=0}^{r} \psi_{fb} x \mathrm{d}x \mathrm{d}\theta \tag{2.28}$$

$$\psi_{ac} = \frac{1}{\pi r^2} \int_{\theta=0}^{2\pi} \int_{x=0}^{r} \psi_{fc} x \mathrm{d}x \mathrm{d}\theta \tag{2.29}$$

Hence, the flux linkage of a phase is the average of the flux linkages of all the filamentary closed paths in that phase.

The flux linkage of a phase can be determined from the flux densities due to currents in the three conductors. The flux density B_a due to i_a can be obtained from Ampere's law.

$$B_a = \begin{cases} \dfrac{\mu_0 i_a x'}{2\pi r^2} & \text{if } x' \le r \\ \dfrac{\mu_0 i_a}{2\pi x'} & \text{if } x' \ge r \end{cases} \tag{2.30}$$

where x' is the distance from the axis of conductor a and μ_0 is permeability of free space; permeability of air and conductor are almost equal to that of free space. Then,

$$\psi_{fa} = \frac{l}{3} \int_{x}^{D_1} B_a \mathrm{d}x' = \frac{\mu_0 i_a l}{6\pi} \left(\frac{1}{2} - \frac{x^2}{2r^2} + \ln \frac{D_1}{r} \right) \tag{2.31}$$

where l is the length of the transmission line. From (2.27) and (2.31),

$$\psi_{aa} = \frac{1}{\pi r^2} \int_{\theta=0}^{2\pi} \int_{x=0}^{r} \frac{\mu_0 i_a l}{6\pi} \left(\frac{1}{2} - \frac{x^2}{2r^2} + \ln \frac{D_1}{r} \right) x \mathrm{d}x \mathrm{d}\theta = \frac{\mu_0 i_a l}{6\pi} \ln \frac{D_1}{r'} \tag{2.32}$$

where $r' \triangleq \mathrm{e}^{-1/4} r$.

The flux density due to i_b is

$$B_b = \frac{\mu_0 i_b}{2\pi D'} \quad \text{if } D' \ge r \tag{2.33}$$

where D' is the distance from the axis of conductor b. Then,

$$\psi_{fb} = \frac{l}{3} \int_{D}^{D_2} B_b \mathrm{d}D' = \frac{\mu_0 i_b l}{6\pi} \ln \frac{D_2}{D} \tag{2.34}$$

where $D = \left(D_{12}^2 + x^2 - 2D_{12} x \cos\theta \right)^{1/2}$. From (2.28) and (2.34),

$$\psi_{ab} = \frac{1}{\pi r^2} \int_{\theta=0}^{2\pi} \int_{x=0}^{r} \frac{\mu_0 i_b l}{6\pi} \ln \frac{D_2}{D} x \mathrm{d}x \mathrm{d}\theta = \frac{\mu_0 i_b l}{6\pi} \ln \frac{D_2}{D_{12}} \tag{2.35}$$

Similarly,

$$\psi_{ac} = \frac{\mu_0 i_c l}{6\pi} \ln \frac{D_3}{D_{31}} \tag{2.36}$$

The flux linkage of phase a in section 1 is

$$\psi_{a1} = \psi_{aa} + \psi_{ab} + \psi_{ac} = \frac{\mu_0 l}{6\pi} \left(i_a \ln \frac{D_1}{r'} + i_b \ln \frac{D_2}{D_{12}} + i_c \ln \frac{D_3}{D_{31}} \right) \tag{2.37}$$

Similarly, the flux linkage of phase a in sections 2 and 3, ψ_{a2} and ψ_{a3}, respectively, are given by

$$\psi_{a2} = \frac{\mu_0 l}{6\pi} \left(i_a \ln \frac{D_2}{r'} + i_b \ln \frac{D_3}{D_{23}} + i_c \ln \frac{D_1}{D_{12}} \right) \tag{2.38}$$

$$\psi_{a3} = \frac{\mu_0 l}{6\pi} \left(i_a \ln \frac{D_3}{r'} + i_b \ln \frac{D_1}{D_{31}} + i_c \ln \frac{D_2}{D_{23}} \right) \tag{2.39}$$

The flux linkage of phase a is

$$\psi_a = \psi_{a1} + \psi_{a2} + \psi_{a3} \tag{2.40}$$

From (2.37) to (2.40),

$$\psi_a = \frac{\mu_0 l}{2\pi} \left[i_a \ln \frac{(D_1 D_2 D_3)^{1/3}}{r'} + i_b \ln \left(\frac{D_1 D_2 D_3}{D_{12} D_{23} D_{31}} \right)^{1/3} + i_c \ln \left(\frac{D_1 D_2 D_3}{D_{12} D_{23} D_{31}} \right)^{1/3} \right] \tag{2.41}$$

The coefficient of i_a is self inductance and the coefficients of i_b and i_c are mutual inductances. Using (2.23), the self and mutual inductances can be replaced by an equivalent self inductance L.

$$L = \frac{\mu_0 l}{2\pi} \ln \frac{(D_{12} D_{23} D_{31})^{1/3}}{r'} \tag{2.42}$$

2.2.1.2 Composite Conductors

A composite conductor consists of two or more individual conductors. Examples of composite conductor are bundled conductor, stranded conductor, and conductor of a multi-circuit transmission line. Figures 2.9, 2.10, and 2.11 show a double circuit transmission line, a transmission line with bundled conductors, and a stranded conductor, respectively.

Fig. 2.9 Double circuit transmission line

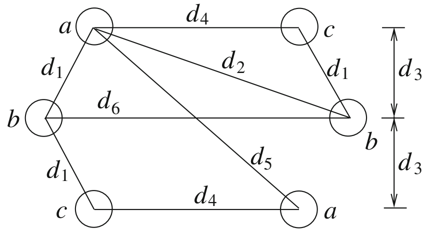


Fig. 2.10 Transmission line with bundled conductors

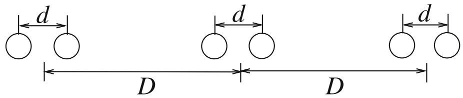


Fig. 2.11 Stranded conductor

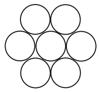
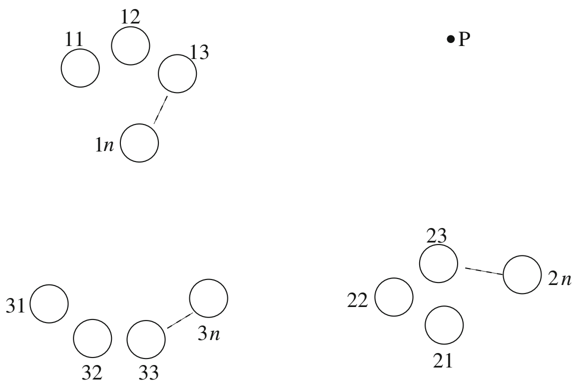


Fig. 2.12 Transmission line with composite conductors



Consider the transmission line consisting of a composite conductor in each phase, as shown in Fig. 2.12 [2]. Let each phase consist of n individual conductors of radius r.

It is assumed that the three phases are transposed if not placed symmetrically; the transmission line is divided into three sections of equal lengths and each phase occupies each of the three positions 1, 2, and 3 for one third of the transmission line length. Let phases a, b, c occupy positions 1, 2, 3, respectively, in the first section, positions 2, 3, 1, respectively, in the second section, and positions 3, 1, 2, respectively, in the third section. The position of each individual conductor is identified by two numbers as in Fig. 2.12; the first number is that of the position of the phase and the second number is that of the position of the individual conductor. Let the current in the individual conductors of phases a, b, c be i_a/n, i_b/n, i_c/n, respectively. This is true if the individual conductors in each phase are transposed so that each individual conductor occupies each of the n positions for equal lengths along a section. i_a, i_b, and i_c satisfy (2.23). Consider the closed path consisting of the individual conductor of phase a at position $1k$ for length $l/(3n)$, and the straight line (at P) parallel to the conductors. Similar to (2.37), the flux linkage of this closed path is

$$\psi_{a1k} = \frac{\mu_0 l}{6\pi n^2} \sum_{m=1}^{n} \left[i_a \ln \frac{D_{1m}}{D_{1k1m}} + i_b \ln \frac{D_{2m}}{D_{1k2m}} + i_c \ln \frac{D_{3m}}{D_{1k3m}} \right] \tag{2.43}$$

where D_{pkqm} (p and q are 1, 2, or 3, and $pk \neq qm$) is the distance between the axes of conductors at positions pk and qm, $D_{pkpk} = r'$, and D_{pk} is the distance between point P and the axis of conductor at position pk. It is evident from (2.27) to (2.29) that the flux linkage of a phase is the average of the flux linkages of the closed paths formed by individual conductors in that phase. Therefore, the flux linkage of phase a in section 1 is

$$\psi_{a1} = \frac{\mu_0 l}{6\pi n^2} \sum_{k=1}^{n} \sum_{m=1}^{n} \left[i_a \ln \frac{D_{1m}}{D_{1k1m}} + i_b \ln \frac{D_{2m}}{D_{1k2m}} + i_c \ln \frac{D_{3m}}{D_{1k3m}} \right] \tag{2.44}$$

Similarly, the flux linkage of phase a in sections 2 and 3, ψ_{a2} and ψ_{a3}, respectively, are given by

$$\psi_{a2} = \frac{\mu_0 l}{6\pi n^2} \sum_{k=1}^{n} \sum_{m=1}^{n} \left[i_a \ln \frac{D_{2m}}{D_{2k2m}} + i_b \ln \frac{D_{3m}}{D_{2k3m}} + i_c \ln \frac{D_{1m}}{D_{2k1m}} \right] \tag{2.45}$$

$$\psi_{a3} = \frac{\mu_0 l}{6\pi n^2} \sum_{k=1}^{n} \sum_{m=1}^{n} \left[i_a \ln \frac{D_{3m}}{D_{3k3m}} + i_b \ln \frac{D_{1m}}{D_{3k1m}} + i_c \ln \frac{D_{2m}}{D_{3k2m}} \right] \tag{2.46}$$

The flux linkage of phase a is

$$\psi_a = \psi_{a1} + \psi_{a2} + \psi_{a3} \tag{2.47}$$

From (2.23) and (2.44) to (2.47), the equivalent self inductance of each phase is

$$L = \frac{\mu_0 l}{2\pi} \ln \frac{D_m}{D_s} \tag{2.48}$$

where

$$D_m \triangleq \left(\prod_{k=1}^{n} \prod_{m=1}^{n} D_{1k2m} D_{2k3m} D_{3k1m} \right)^{1/(3n^2)} \tag{2.49}$$

$$D_s \triangleq \left(\prod_{k=1}^{n} \prod_{m=1}^{n} D_{1k1m} D_{2k2m} D_{3k3m} \right)^{1/(3n^2)} \tag{2.50}$$

D_m is known as mutual geometric mean distance (GMD) and D_s is known as self GMD.

For the double circuit transmission line shown in Fig. 2.9,

$$D_m = \left(2d_1^2 d_2^2 d_3 d_4\right)^{1/6} \tag{2.51}$$

$$D_s = \left(\mathrm{e}^{-3/4} r^3 d_5^2 d_6\right)^{1/6} \tag{2.52}$$

where r is the radius of the individual conductors. For hexagonal spacing ($d_4 = d_1$ and $d_6 = d_5$), transposition is not necessary.

For the transmission line with bundled conductors shown in Fig. 2.10, where each bundle (composite conductor) consists of two individual conductors,

$$D_m = \left[4D^6\left(D^2 - d^2\right)^2\left(4D^2 - d^2\right)\right]^{1/12} \tag{2.53}$$

$$D_s = \left(\mathrm{e}^{-1/4} r d\right)^{1/2} \tag{2.54}$$

where r is the radius of the individual conductors.

For the stranded conductor shown in Fig. 2.11,

$$D_s = 2\,(364.5)^{1/49}\,\mathrm{e}^{-1/28} r \tag{2.55}$$

where r is the radius of the strands.

It is to be noted that the transposition of the individual conductors in the composite conductor is not necessary for the three phases to be symmetrical; but the assumption of transposition helps in easily obtaining the expression for inductance.

2.2.2 Capacitance

2.2.2.1 Transmission Line with Three Conductors

Consider the transmission line with three conductors shown in Fig. 2.8. Let the charge per unit length on the conductors of phases a, b, and c be q_a, q_b, and q_c, respectively, such that

$$q_a + q_b + q_c = 0 \tag{2.56}$$

The radius of the conductors is assumed to be very small compared to the distance between any two conductors. Therefore, the potential of conductor a with respect to the point P is

$$v_{aP} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{D_1}{r} + q_b \ln\frac{D_2}{D_{12}} + q_c \ln\frac{D_3}{D_{31}}\right) \tag{2.57}$$

ε_0 is the permittivity of free space; permittivity of air is almost equal to that of free space. The potential of the conductor is obtained by allowing P to recede to infinity. As P recedes to infinity, using (2.56), the potential of conductor a for symmetrical spacing of conductors ($D_{12} = D_{23} = D_{31} = D$) is

$$v_a = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{1}{r} + q_b \ln\frac{1}{D} + q_c \ln\frac{1}{D}\right) \tag{2.58}$$

From (2.56) and (2.58), the capacitance in each phase is

$$C = \frac{2\pi\varepsilon_0 l}{\ln(D/r)} \tag{2.59}$$

If the conductors are not spaced symmetrically, it is assumed that transposition is done in order to have a symmetrical system. Let the conductors a, b, c occupy positions 1, 2, 3, respectively, in the first section, positions 2, 3, 1, respectively, in the second section, and positions 3, 1, 2, respectively, in the third section. The charge per unit length is not same in all three sections for any phase whereas the potential is same. It is assumed that the charge per unit length is same in all the three sections [3]; let the charge per unit length on the conductors of phases a, b, and c be q_a, q_b, and q_c, respectively, which satisfy (2.56). With this assumption, the potential of conductor a in sections 1, 2, and 3, v_{a1}, v_{a2}, and v_{a3}, respectively, are given by

$$v_{a1} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{1}{r} + q_b \ln\frac{1}{D_{12}} + q_c \ln\frac{1}{D_{31}}\right) \tag{2.60}$$

$$v_{a2} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{1}{r} + q_b \ln\frac{1}{D_{23}} + q_c \ln\frac{1}{D_{12}}\right) \tag{2.61}$$

$$v_{a3} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{1}{r} + q_b \ln\frac{1}{D_{31}} + q_c \ln\frac{1}{D_{23}}\right) \tag{2.62}$$

The potential of conductor a is assumed to be given by the following equation [3].

$$v_a = \frac{1}{3}(v_{a1} + v_{a2} + v_{a3}) \tag{2.63}$$

From (2.56) and (2.60) to (2.63), the capacitance in each phase is

$$C = \frac{2\pi\varepsilon_0 l}{\ln\left[(D_{12}D_{23}D_{31})^{1/3}/r\right]} \tag{2.64}$$

2.2.2.2 Composite Conductors

Consider the transmission line with composite conductors shown in Fig. 2.12. It is assumed that the three phases are transposed if not placed symmetrically. Let the composite conductors of phases a, b, c occupy positions 1, 2, 3, respectively, in the first section, positions 2, 3, 1, respectively, in the second section, and positions 3, 1, 2, respectively, in the third section. Let each individual conductor of a composite conductor occupy each of the n positions for equal lengths along a section. Let the charge per unit length on the individual conductors of phases a, b, and c be q_a/n, q_b/n, and q_c/n, respectively, which satisfy (2.56). It is assumed that the charge per unit length on all individual conductors is same along the entire length of the transmission line. The potential of phase a individual conductor at position $1k$ is

$$v_{a1k} = \frac{1}{2\pi\varepsilon_0 n}\sum_{m=1}^{n}\left[q_a \ln\frac{1}{D'_{1k1m}} + q_b \ln\frac{1}{D'_{1k2m}} + q_c \ln\frac{1}{D'_{1k3m}}\right] \tag{2.65}$$

where D'_{pkqm} (p and q are 1, 2, or 3, and $pk \neq qm$) is the distance between the axes of conductors at positions pk and qm; D'_{pkpk} is the radius of the individual conductors. The potential of phase a composite conductor in section 1, v_{a1}, is assumed to be equal to the average of the potentials of the individual conductors.

$$v_{a1} = \frac{1}{2\pi\varepsilon_0 n^2}\sum_{k=1}^{n}\sum_{m=1}^{n}\left[q_a \ln\frac{1}{D'_{1k1m}} + q_b \ln\frac{1}{D'_{1k2m}} + q_c \ln\frac{1}{D'_{1k3m}}\right] \tag{2.66}$$

The potential of phase a composite conductor in sections 2 and 3, v_{a2} and v_{a3}, respectively, are given by

$$v_{a2} = \frac{1}{2\pi\varepsilon_0 n^2}\sum_{k=1}^{n}\sum_{m=1}^{n}\left[q_a \ln\frac{1}{D'_{2k2m}} + q_b \ln\frac{1}{D'_{2k3m}} + q_c \ln\frac{1}{D'_{2k1m}}\right] \tag{2.67}$$

$$v_{a3} = \frac{1}{2\pi\varepsilon_0 n^2}\sum_{k=1}^{n}\sum_{m=1}^{n}\left[q_a \ln\frac{1}{D'_{3k3m}} + q_b \ln\frac{1}{D'_{3k1m}} + q_c \ln\frac{1}{D'_{3k2m}}\right] \tag{2.68}$$

The potential of phase a composite conductor is assumed to be given by

$$v_a = \frac{1}{3}(v_{a1} + v_{a2} + v_{a3}) \tag{2.69}$$

From (2.56) and (2.66) to (2.69), the capacitance in each phase is

$$C = \frac{2\pi\varepsilon_0 l}{\ln\left(D_m/D'_s\right)} \tag{2.70}$$

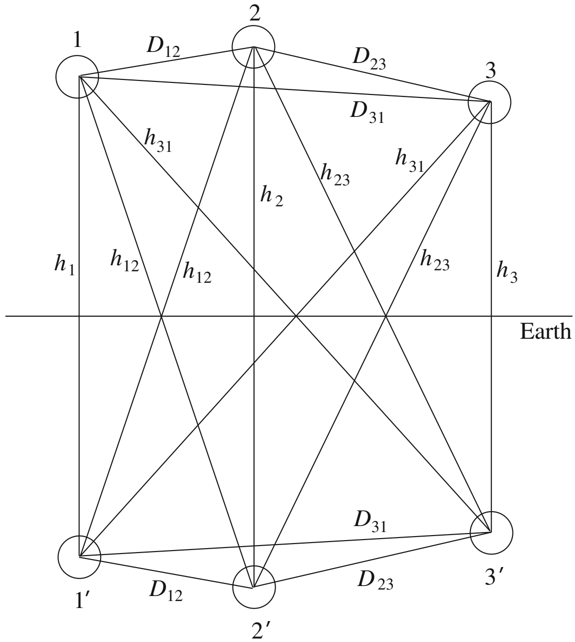


Fig. 2.13 Conductors and their images

where

$$D_m \triangleq \left(\prod_{k=1}^{n} \prod_{m=1}^{n} D'_{1k2m} D'_{2k3m} D'_{3k1m} \right)^{1/(3n^2)} \tag{2.71}$$

$$D'_s \triangleq \left(\prod_{k=1}^{n} \prod_{m=1}^{n} D'_{1k1m} D'_{2k2m} D'_{3k3m} \right)^{1/(3n^2)} \tag{2.72}$$

2.2.2.3 Effect of Earth

The earth affects the distribution of the electric field due to charge on a conductor. The earth is at zero potential. The effect of earth is same as that of the image of the conductor [3]. The image of the conductor with charge q_a per unit length is a conductor with charge $-q_a$ per unit length located at the same distance from the earth's surface below it as shown in Fig. 2.13. The images of conductors at positions 1, 2, and 3 are at positions 1′, 2′, and 3′, respectively.

Transposition of conductors is assumed. Let the conductors a, b, c occupy positions 1, 2, 3, respectively, in the first section, positions 2, 3, 1, respectively, in the second section, and positions 3, 1, 2, respectively, in the third section. Let the charge per unit length on the conductors a, b, and c be q_a, q_b, and q_c, respectively, which satisfy (2.56). It is assumed that the charge per unit length is same in all the three sections. The potential of conductor a in sections 1, 2, and 3, v_{a1}, v_{a2}, and v_{a3}, respectively, are given by

$$v_{a1} = \frac{1}{2\pi\varepsilon_0}\left[q_a\left(\ln\frac{1}{r} - \ln\frac{1}{h_1}\right) + q_b\left(\ln\frac{1}{D_{12}} - \ln\frac{1}{h_{12}}\right) + q_c\left(\ln\frac{1}{D_{31}} - \ln\frac{1}{h_{31}}\right)\right] \tag{2.73}$$

$$v_{a2} = \frac{1}{2\pi\varepsilon_0}\left[q_a\left(\ln\frac{1}{r} - \ln\frac{1}{h_2}\right) + q_b\left(\ln\frac{1}{D_{23}} - \ln\frac{1}{h_{23}}\right) + q_c\left(\ln\frac{1}{D_{12}} - \ln\frac{1}{h_{12}}\right)\right] \tag{2.74}$$

$$v_{a3} = \frac{1}{2\pi\varepsilon_0}\left[q_a\left(\ln\frac{1}{r} - \ln\frac{1}{h_3}\right) + q_b\left(\ln\frac{1}{D_{31}} - \ln\frac{1}{h_{31}}\right) + q_c\left(\ln\frac{1}{D_{23}} - \ln\frac{1}{h_{23}}\right)\right] \tag{2.75}$$

where r is the radius of the conductors. The potential of conductor a is assumed to be given by

$$v_a = \frac{1}{3}(v_{a1} + v_{a2} + v_{a3}) \tag{2.76}$$

From (2.56) and (2.73) to (2.76), the capacitance in each phase is

$$C = \frac{2\pi\varepsilon_0 l}{\ln\dfrac{(D_{12}D_{23}D_{31})^{1/3}}{r} - \ln\left(\dfrac{h_{12}h_{23}h_{31}}{h_1h_2h_3}\right)^{1/3}} \tag{2.77}$$

Since $h_{12}h_{23}h_{31} > h_1h_2h_3$, the effect of earth is to increase the capacitance.

2.2.3 Transmission Line Model

Let v_a, v_b, and v_c be the voltages with respect to the neutral, i_a, i_b, and i_c be the currents, at the point which is at distance x from the receiving end, as shown in Fig. 2.14. The currents satisfy (2.23). Then, the voltages and currents are related by the following equations.

$$\frac{\partial v_a}{\partial x} = \frac{R}{l}i_a + \frac{L}{l}\frac{\partial i_a}{\partial t} \tag{2.78}$$

$$\frac{\partial v_b}{\partial x} = \frac{R}{l}i_b + \frac{L}{l}\frac{\partial i_b}{\partial t} \tag{2.79}$$

$$\frac{\partial v_c}{\partial x} = \frac{R}{l}i_c + \frac{L}{l}\frac{\partial i_c}{\partial t} \tag{2.80}$$

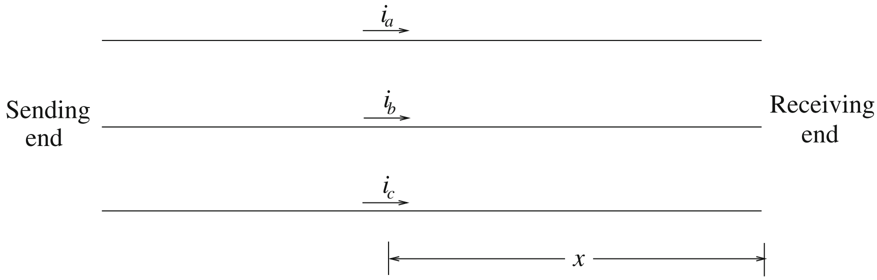


Fig. 2.14 Transmission line

$$\frac{\partial i_a}{\partial x} = \frac{G}{l} v_a + \frac{C}{l} \frac{\partial v_a}{\partial t} \tag{2.81}$$
$$\frac{\partial i_b}{\partial x} = \frac{G}{l} v_b + \frac{C}{l} \frac{\partial v_b}{\partial t} \tag{2.82}$$
$$\frac{\partial i_c}{\partial x} = \frac{G}{l} v_c + \frac{C}{l} \frac{\partial v_c}{\partial t} \tag{2.83}$$

The equations for the three phases are decoupled.

Dividing (2.78)–(2.80) by V_B, and (2.81)–(2.83) by I_B gives the equations in per unit quantities.

$$\frac{\partial \bar{v}_a}{\partial x} = \frac{\bar{R}}{l} \bar{i}_a + \frac{\bar{X}}{l\omega_B} \frac{\partial \bar{i}_a}{\partial t} \tag{2.84}$$
$$\frac{\partial \bar{v}_b}{\partial x} = \frac{\bar{R}}{l} \bar{i}_b + \frac{\bar{X}}{l\omega_B} \frac{\partial \bar{i}_b}{\partial t} \tag{2.85}$$
$$\frac{\partial \bar{v}_c}{\partial x} = \frac{\bar{R}}{l} \bar{i}_c + \frac{\bar{X}}{l\omega_B} \frac{\partial \bar{i}_c}{\partial t} \tag{2.86}$$
$$\frac{\partial \bar{i}_a}{\partial x} = \frac{\bar{G}}{l} \bar{v}_a + \frac{\bar{B}}{l\omega_B} \frac{\partial \bar{v}_a}{\partial t} \tag{2.87}$$
$$\frac{\partial \bar{i}_b}{\partial x} = \frac{\bar{G}}{l} \bar{v}_b + \frac{\bar{B}}{l\omega_B} \frac{\partial \bar{v}_b}{\partial t} \tag{2.88}$$
$$\frac{\partial \bar{i}_c}{\partial x} = \frac{\bar{G}}{l} \bar{v}_c + \frac{\bar{B}}{l\omega_B} \frac{\partial \bar{v}_c}{\partial t} \tag{2.89}$$

where $X \triangleq \omega_B L$, $B \triangleq \omega_B C$, base admittance $Y_B \triangleq 1/Z_B$, and base capacitance $C_B \triangleq Y_B/\omega_B$. Two special cases: lossless transmission line and sinusoidal operation, are considered.

2.2.3.1 Lossless Transmission Line

If $R = G = 0$, then (2.78)–(2.83) can be written as follows.

$$\frac{\partial v_a}{\partial x} = \frac{L}{l}\frac{\partial i_a}{\partial t} \tag{2.90}$$

$$\frac{\partial i_a}{\partial x} = \frac{C}{l}\frac{\partial v_a}{\partial t} \tag{2.91}$$

$$\frac{\partial v_b}{\partial x} = \frac{L}{l}\frac{\partial i_b}{\partial t} \tag{2.92}$$

$$\frac{\partial i_b}{\partial x} = \frac{C}{l}\frac{\partial v_b}{\partial t} \tag{2.93}$$

$$\frac{\partial v_c}{\partial x} = \frac{L}{l}\frac{\partial i_c}{\partial t} \tag{2.94}$$

$$\frac{\partial i_c}{\partial x} = \frac{C}{l}\frac{\partial v_c}{\partial t} \tag{2.95}$$

If the subscripts a, b, and c are not shown, the solution for any phase is

$$i(x,t) = -f_1(x - v_p t) - f_2(x + v_p t) \tag{2.96}$$

$$v(x,t) = Z_c f_1(x - v_p t) - Z_c f_2(x + v_p t) \tag{2.97}$$

where $v_p \triangleq l/\sqrt{LC}$ and $Z_c \triangleq \sqrt{L/C}$ [4]. v_p is called phase velocity and Z_c is called characteristic impedance. f_1 and f_2 are functions of x and t. Let subscripts S and R denote sending end quantities and receiving end quantities, respectively. If only terminal response is of interest, the following method known as Bergeron's method is used. From (2.96) and (2.97),

$$i_R(t) = i(0,t) = -f_1(-v_p t) - f_2(v_p t) \tag{2.98}$$

$$v_R(t) = v(0,t) = Z_c f_1(-v_p t) - Z_c f_2(v_p t) \tag{2.99}$$

$$i_S\left(t - \frac{l}{v_p}\right) = i\left(l, t - \frac{l}{v_p}\right) = -f_1\left(2l - v_p t\right) - f_2\left(v_p t\right) \tag{2.100}$$

$$v_S\left(t - \frac{l}{v_p}\right) = v\left(l, t - \frac{l}{v_p}\right) = Z_c f_1\left(2l - v_p t\right) - Z_c f_2\left(v_p t\right) \tag{2.101}$$

Elimination of $f_1\left(-v_p t\right)$, $f_1\left(2l - v_p t\right)$, and $f_2(v_p t)$ from (2.98) to (2.101) gives

$$i_R(t) = i_S\left(t - \frac{l}{v_p}\right) + \frac{1}{Z_c} v_S\left(t - \frac{l}{v_p}\right) - \frac{1}{Z_c} v_R(t) \tag{2.102}$$

This equation relates the receiving end current and voltage. Similarly, one can obtain the following equation which relates the sending end current and voltage.

$$i_S(t) = i_R\left(t - \frac{l}{v_p}\right) - \frac{1}{Z_c}v_R\left(t - \frac{l}{v_p}\right) + \frac{1}{Z_c}v_S(t) \tag{2.103}$$

Dividing (2.102) and (2.103) by I_B gives the equations in per unit quantities.

$$\bar{i}_R(t) = \bar{i}_S\left(t - \frac{l}{v_p}\right) + \frac{1}{\bar{Z}_c}\bar{v}_S\left(t - \frac{l}{v_p}\right) - \frac{1}{\bar{Z}_c}\bar{v}_R(t) \tag{2.104}$$

$$\bar{i}_S(t) = \bar{i}_R\left(t - \frac{l}{v_p}\right) - \frac{1}{\bar{Z}_c}\bar{v}_R\left(t - \frac{l}{v_p}\right) + \frac{1}{\bar{Z}_c}\bar{v}_S(t) \tag{2.105}$$

2.2.3.2 Sinusoidal Operation

Let the voltages and currents be sinusoidal with angular frequency ω_o. Then voltages and currents can be represented by phasors. Let $\mathbf{V}$ and $\mathbf{I}$ be the notations for phasor representation of v and i, respectively. If subscripts a, b, and c are not shown, (2.78)–(2.83) can be written in the following form for each phase.

$$\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}x} = \frac{R + \mathrm{j}\omega_o L}{l}\mathbf{I} \tag{2.106}$$

$$\frac{\mathrm{d}\mathbf{I}}{\mathrm{d}x} = \frac{G + \mathrm{j}\omega_o C}{l}\mathbf{V} \tag{2.107}$$

If $\mathbf{V}_R$ and $\mathbf{I}_R$ are the receiving end voltage and current, respectively, the solution of (2.106) and (2.107) is

$$\mathbf{V} = \cosh(\gamma x)\mathbf{V}_R + Z_c \sinh(\gamma x)\mathbf{I}_R \tag{2.108}$$

$$\mathbf{I} = \frac{1}{Z_c}\sinh(\gamma x)\mathbf{V}_R + \cosh(\gamma x)\mathbf{I}_R \tag{2.109}$$

where Z_c is called characteristic impedance and γ is called propagation constant.

$$Z_c \triangleq \sqrt{\frac{R + \mathrm{j}\omega_o L}{G + \mathrm{j}\omega_o C}},\ \gamma \triangleq \frac{\sqrt{(R + \mathrm{j}\omega_o L)(G + \mathrm{j}\omega_o C)}}{l} \tag{2.110}$$

If $\mathbf{V}_S$ and $\mathbf{I}_S$ are the sending end voltage and current, respectively, then from (2.108) and (2.109),

$$\mathbf{V}_S = \cosh(\gamma l)\mathbf{V}_R + Z_c \sinh(\gamma l)\mathbf{I}_R \tag{2.111}$$

$$\mathbf{I}_S = \frac{1}{Z_c}\sinh(\gamma l)\mathbf{V}_R + \cosh(\gamma l)\mathbf{I}_R \tag{2.112}$$

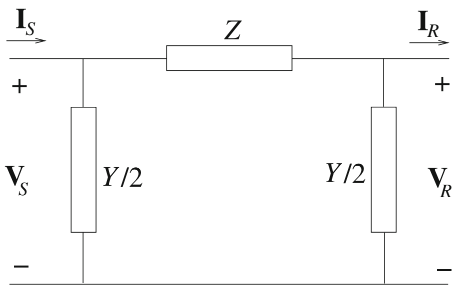


Fig. 2.15 Equivalent π circuit of transmission line

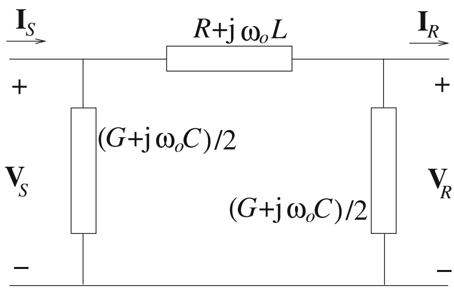


Fig. 2.16 Nominal π circuit of transmission line

Each phase of the transmission line can be represented by the equivalent π circuit shown in Fig. 2.15, where Z is impedance and Y is admittance. For this circuit, the following equations can be written.

$$\mathbf{V}_S = \left(1 + \frac{YZ}{2}\right)\mathbf{V}_R + Z\mathbf{I}_R \tag{2.113}$$

$$\mathbf{I}_S = Y\left(1 + \frac{YZ}{4}\right)\mathbf{V}_R + \left(1 + \frac{YZ}{2}\right)\mathbf{I}_R \tag{2.114}$$

Equating the coefficients of $\mathbf{V}_R$ and $\mathbf{I}_R$ in (2.111) and (2.113) gives

$$Z = (R + \mathrm{j}\omega_o L)\frac{\sinh(\gamma l)}{\gamma l}, Y = (G + \mathrm{j}\omega_o C)\frac{\tanh(\gamma l/2)}{\gamma l/2} \tag{2.115}$$

It is to be noted that

$$\lim_{l\to 0} Z = R + \mathrm{j}\omega_o L, \lim_{l\to 0} Y = G + \mathrm{j}\omega_o C \tag{2.116}$$

If Z and Y in Fig. 2.15 are replaced by the values of their respective limits as $l \to 0$, the circuit shown in Fig. 2.16 is obtained. This circuit is called nominal π circuit. For transmission lines of length less than 240 km, the nominal π circuit shown in Fig. 2.16 is a good approximation [3].

2.3 Kron's Transformation

Kron's transformation does a transformation of the three-phase voltages and currents as follows [5].

$$\begin{bmatrix} v_D \\ v_Q \\ v_0 \end{bmatrix} \triangleq T_K \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}, \begin{bmatrix} i_D \\ i_Q \\ i_0 \end{bmatrix} \triangleq T_K \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{2.117}$$

where

$$T_K \triangleq \frac{1}{\sqrt{3}} \begin{bmatrix} \sqrt{2}\cos(\omega_o t) & \sqrt{2}\cos(\omega_o t - 2\pi/3) & \sqrt{2}\cos(\omega_o t + 2\pi/3) \\ \sqrt{2}\sin(\omega_o t) & \sqrt{2}\sin(\omega_o t - 2\pi/3) & \sqrt{2}\sin(\omega_o t + 2\pi/3) \\ 1 & 1 & 1 \end{bmatrix} \tag{2.118}$$

where ω_o is the operating frequency. It can be verified that $T_K^{-1} = T_K^T$.

If $v_0 = i_0 = 0$, then (2.117) can be written as

$$\begin{bmatrix} v_D \\ v_Q \end{bmatrix} = T'_K \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}, \begin{bmatrix} i_D \\ i_Q \end{bmatrix} = T'_K \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{2.119}$$

where

$$T'_K \triangleq \frac{1}{\sqrt{3}} \begin{bmatrix} \sqrt{2}\cos(\omega_o t) & \sqrt{2}\cos(\omega_o t - 2\pi/3) & \sqrt{2}\cos(\omega_o t + 2\pi/3) \\ \sqrt{2}\sin(\omega_o t) & \sqrt{2}\sin(\omega_o t - 2\pi/3) & \sqrt{2}\sin(\omega_o t + 2\pi/3) \end{bmatrix} \tag{2.120}$$

In certain studies, high-frequency transients in the transformer and the transmission line are neglected. Then, Kron's transformation results in simplification of equations. Kron's transformation also enables generalization of the definitions of certain electrical quantities.

2.3.1 Definitions

There are quantities such as voltage magnitude, phase angle, frequency, reactive power etc. which are well defined in steady state when voltages and currents are sinusoidally varying and balanced. The definition of these quantities will be generalized so that they can be used even in the presence of harmonics and during a transient when the voltage and current are not sinusoidal; however, these definitions are made with the assumption that $v_0 = i_0 = 0$.

Consider the shunt-connected equipment shown in Fig. 2.17. Let v_a, v_b, and v_c be the voltages of terminals a, b, and c, respectively, with respect to the neutral.

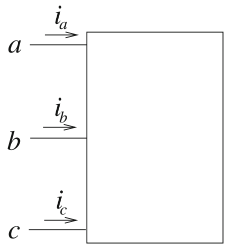


Fig. 2.17 Shunt-connected equipment

The magnitude V and phase angle ϕ of the voltage of the three-phase bus, at which the equipment in Fig. 2.17 is connected, are defined as

$$V \triangleq \sqrt{v_D^2 + v_Q^2} \tag{2.121}$$

$$\phi \triangleq \tan^{-1} \frac{v_D}{v_Q} \tag{2.122}$$

In other words, $V\angle\phi = v_Q + \mathrm{j}v_D$. v_Q and v_D can be expressed in terms of V and ϕ as follows.

$$v_Q = V \cos\phi,\ v_D = V \sin\phi \tag{2.123}$$

If v_a, v_b, and v_c are obtained from these expressions for v_Q and v_D using (2.119), then

$$v_a = \sqrt{\frac{2}{3}} V \sin(\omega_o t + \phi) \tag{2.124}$$

$$v_b = \sqrt{\frac{2}{3}} V \sin\left(\omega_o t + \phi - \frac{2\pi}{3}\right) \tag{2.125}$$

$$v_c = \sqrt{\frac{2}{3}} V \sin\left(\omega_o t + \phi + \frac{2\pi}{3}\right) \tag{2.126}$$

Therefore, if v_a, v_b, and v_c are sinusoidal with angular frequency ω_o and balanced, V is the rms value of the line-to-line voltage and ϕ is the phase angle of v_a.

The frequency at the three-phase bus f is defined as

$$f \triangleq f_o + \frac{1}{2\pi} \frac{\mathrm{d}\phi}{\mathrm{d}t} \tag{2.127}$$

where $f_o \triangleq \omega_o/(2\pi)$.

Similar to voltage magnitude and phase angle definitions, the magnitude I and phase angle ψ of the current drawn by the equipment in Fig. 2.17 are defined as

$$I \triangleq \sqrt{i_D^2 + i_Q^2} \tag{2.128}$$

$$\psi \triangleq \tan^{-1} \frac{i_D}{i_Q} \tag{2.129}$$

If i_a, i_b, and i_c are sinusoidal with angular frequency ω_o and balanced, then I is $\sqrt{3}$ times the rms value of i_a, i_b, or i_c, and ψ is the phase angle of i_a.

The power drawn by the equipment in Fig. 2.17 is

$$P = v_a i_a + v_b i_b + v_c i_c \tag{2.130}$$

From (2.119),

$$P = v_D i_D + v_Q i_Q \tag{2.131}$$

It can be seen that

$$P = \text{Re}[V\angle\phi I\angle(-\psi)] \tag{2.132}$$

P is also known as active power. The reactive power Q drawn by the equipment in Fig. 2.17 is defined as

$$Q \triangleq \text{Im}[V\angle\phi I\angle(-\psi)] = v_D i_Q - v_Q i_D \tag{2.133}$$

The active current i_A and the reactive current i_R drawn by the equipment in Fig. 2.17 are defined as

$$i_A \triangleq I\cos(\phi - \psi) = i_Q \cos\phi + i_D \sin\phi \tag{2.134}$$

$$i_R \triangleq I\sin(\phi - \psi) = i_Q \sin\phi - i_D \cos\phi \tag{2.135}$$

It is to be noted that $i_A > 0 \Leftrightarrow P > 0$, and $i_R > 0 \Leftrightarrow Q > 0$. The reactive current is said to be inductive if it is positive, and is said to be capacitive if it is negative.

Consider the series-connected equipment shown in Fig. 2.18. The magnitude V and phase angle ϕ of the voltage across the equipment in Fig. 2.18 are given by (2.119)–(2.122) using v_a, v_b, and v_c of Fig. 2.18. Similarly, the magnitude I and phase angle ψ of the current through the equipment in Fig. 2.18 are given by (2.119), (2.120), (2.128), and (2.129) using i_a, i_b, and i_c of Fig. 2.18. The active voltage v_A and the reactive voltage v_R across the equipment in Fig. 2.18 are defined as

$$v_A \triangleq V\cos(\phi - \psi) \tag{2.136}$$

$$v_R \triangleq V\sin(\phi - \psi) \tag{2.137}$$

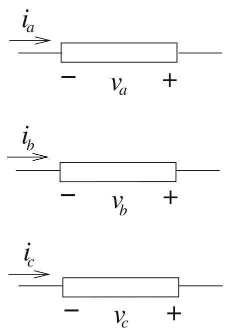


Fig. 2.18 Series-connected equipment

If v_A is positive, active power is supplied by the equipment, otherwise, active power is drawn by the equipment. The reactive voltage is said to be capacitive if it is positive and inductive if it is negative.

2.3.2 Application to Transformer

Equations (2.18)–(2.20) of the wye-wye-connected transformer can be written as

$$\begin{bmatrix} \bar{v}_{2a} \\ \bar{v}_{2b} \\ \bar{v}_{2c} \end{bmatrix} = \begin{bmatrix} \bar{v}_{1a} \\ \bar{v}_{1b} \\ \bar{v}_{1c} \end{bmatrix} - \left(\bar{R}_1 + \bar{R}_2\right) \begin{bmatrix} \bar{i}_a \\ \bar{i}_b \\ \bar{i}_c \end{bmatrix} - \frac{1}{\omega_B} \left(\bar{X}_1 + \bar{X}_2\right) \begin{bmatrix} \mathrm{d}\bar{i}_a/\mathrm{d}t \\ \mathrm{d}\bar{i}_b/\mathrm{d}t \\ \mathrm{d}\bar{i}_c/\mathrm{d}t \end{bmatrix} \tag{2.138}$$

By Kron's transformation,

$$T_K'^T \begin{bmatrix} \bar{v}_{2D} \\ \bar{v}_{2Q} \end{bmatrix} = T_K'^T \begin{bmatrix} \bar{v}_{1D} \\ \bar{v}_{1Q} \end{bmatrix} - \left(\bar{R}_1 + \bar{R}_2\right) T_K'^T \begin{bmatrix} \bar{i}_D \\ \bar{i}_Q \end{bmatrix} - \frac{1}{\omega_B} \left(\bar{X}_1 + \bar{X}_2\right) \frac{\mathrm{d}}{\mathrm{d}t} \left(T_K'^T \begin{bmatrix} \bar{i}_D \\ \bar{i}_Q \end{bmatrix} \right) \tag{2.139}$$

where $\begin{bmatrix} \bar{v}_{1D} \\ \bar{v}_{1Q} \end{bmatrix} \triangleq T_K' \begin{bmatrix} \bar{v}_{1a} \\ \bar{v}_{1b} \\ \bar{v}_{1c} \end{bmatrix}$, $\begin{bmatrix} \bar{v}_{2D} \\ \bar{v}_{2Q} \end{bmatrix} \triangleq T_K' \begin{bmatrix} \bar{v}_{2a} \\ \bar{v}_{2b} \\ \bar{v}_{2c} \end{bmatrix}$ and $\begin{bmatrix} \bar{i}_D \\ \bar{i}_Q \end{bmatrix} \triangleq T_K' \begin{bmatrix} \bar{i}_a \\ \bar{i}_b \\ \bar{i}_c \end{bmatrix}$.
Pre-multiplying (2.139) by T_K' gives

$$\bar{v}_{2D} = \bar{v}_{1D} - (\bar{R}_1 + \bar{R}_2)\bar{i}_D - \frac{\omega_o}{\omega_B} \left(\bar{X}_1 + \bar{X}_2\right) \bar{i}_Q - \frac{1}{\omega_B} \left(\bar{X}_1 + \bar{X}_2\right) \frac{\mathrm{d}\bar{i}_D}{\mathrm{d}t} \tag{2.140}$$

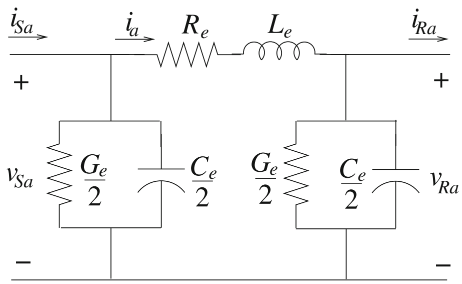


Fig. 2.19 Equivalent π circuit of phase a of transmission line

$$\overline{v}_{2Q} = \overline{v}_{1Q} - (\overline{R}_1 + \overline{R}_2)\overline{i}_Q + \frac{\omega_o}{\omega_B}\left(\overline{X}_1 + \overline{X}_2\right)\overline{i}_D - \frac{1}{\omega_B}\left(\overline{X}_1 + \overline{X}_2\right)\frac{\mathrm{d}\overline{i}_Q}{\mathrm{d}t} \quad (2.141)$$

If the high-frequency transients are to be neglected, then the last term on the right-hand side of (2.140) and (2.141) are set to zero. For balanced sinusoidal operation at angular frequency ω_o, all transformed variables are constant and hence the last term on the right-hand side of (2.140) and (2.141) is equal to zero. The factor ω_o/ω_B in one of the terms of (2.140) and (2.141) is usually approximated to 1. Therefore,

$$\overline{v}_{2D} = \overline{v}_{1D} - (\overline{R}_1 + \overline{R}_2)\overline{i}_D - \left(\overline{X}_1 + \overline{X}_2\right)\overline{i}_Q \quad (2.142)$$

$$\overline{v}_{2Q} = \overline{v}_{1Q} - (\overline{R}_1 + \overline{R}_2)\overline{i}_Q + \left(\overline{X}_1 + \overline{X}_2\right)\overline{i}_D \quad (2.143)$$

2.3.3 Application to Transmission Line

For sinusoidal operation, the equivalent π circuit of the transmission line shown in Fig. 2.15 is applicable. Let $Z = R_e + \mathrm{j}\omega_o L_e$ and $Y = G_e + \mathrm{j}\omega_o C_e$. The circuit of Fig. 2.15 can be redrawn as shown in Fig. 2.19 for phase a.

From the circuit diagram in Fig. 2.19,

$$v_{Sa} - v_{Ra} = R_e i_a + L_e\frac{\mathrm{d}i_a}{\mathrm{d}t} \quad (2.144)$$

$$i_{Sa} - i_a = \frac{G_e}{2}v_{Sa} + \frac{C_e}{2}\frac{\mathrm{d}v_{Sa}}{\mathrm{d}t} \quad (2.145)$$

$$i_a - i_{Ra} = \frac{G_e}{2}v_{Ra} + \frac{C_e}{2}\frac{\mathrm{d}v_{Ra}}{\mathrm{d}t} \quad (2.146)$$

Similarly, for phases b and c,

$$v_{Sb} - v_{Rb} = R_e i_b + L_e\frac{\mathrm{d}i_b}{\mathrm{d}t} \quad (2.147)$$

$$v_{Sc} - v_{Rc} = R_e i_c + L_e\frac{\mathrm{d}i_c}{\mathrm{d}t} \quad (2.148)$$

$$i_{Sb} - i_b = \frac{G_e}{2} v_{Sb} + \frac{C_e}{2} \frac{\mathrm{d}v_{Sb}}{\mathrm{d}t} \tag{2.149}$$

$$i_{Sc} - i_c = \frac{G_e}{2} v_{Sc} + \frac{C_e}{2} \frac{\mathrm{d}v_{Sc}}{\mathrm{d}t} \tag{2.150}$$

$$i_b - i_{Rb} = \frac{G_e}{2} v_{Rb} + \frac{C_e}{2} \frac{\mathrm{d}v_{Rb}}{\mathrm{d}t} \tag{2.151}$$

$$i_c - i_{Rc} = \frac{G_e}{2} v_{Rc} + \frac{C_e}{2} \frac{\mathrm{d}v_{Rc}}{\mathrm{d}t} \tag{2.152}$$

By Kron's transformation, (2.144)–(2.152) can be written as

$$v_{SD} - v_{RD} = R_e i_D + \omega_o L_e i_Q + L_e \frac{\mathrm{d}i_D}{\mathrm{d}t} \tag{2.153}$$

$$v_{SQ} - v_{RQ} = R_e i_Q - \omega_o L_e i_D + L_e \frac{\mathrm{d}i_Q}{\mathrm{d}t} \tag{2.154}$$

$$i_{SD} - i_D = \frac{G_e}{2} v_{SD} + \omega_o \frac{C_e}{2} v_{SQ} + \frac{C_e}{2} \frac{\mathrm{d}v_{SD}}{\mathrm{d}t} \tag{2.155}$$

$$i_{SQ} - i_Q = \frac{G_e}{2} v_{SQ} - \omega_o \frac{C_e}{2} v_{SD} + \frac{C_e}{2} \frac{\mathrm{d}v_{SQ}}{\mathrm{d}t} \tag{2.156}$$

$$i_D - i_{RD} = \frac{G_e}{2} v_{RD} + \omega_o \frac{C_e}{2} v_{RQ} + \frac{C_e}{2} \frac{\mathrm{d}v_{RD}}{\mathrm{d}t} \tag{2.157}$$

$$i_Q - i_{RQ} = \frac{G_e}{2} v_{RQ} - \omega_o \frac{C_e}{2} v_{RD} + \frac{C_e}{2} \frac{\mathrm{d}v_{RQ}}{\mathrm{d}t} \tag{2.158}$$

where $\begin{bmatrix} v_{SD} \\ v_{SQ} \end{bmatrix} \triangleq T'_K \begin{bmatrix} v_{Sa} \\ v_{Sb} \\ v_{Sc} \end{bmatrix}$, $\begin{bmatrix} v_{RD} \\ v_{RQ} \end{bmatrix} \triangleq T'_K \begin{bmatrix} v_{Ra} \\ v_{Rb} \\ v_{Rc} \end{bmatrix}$, $\begin{bmatrix} i_{SD} \\ i_{SQ} \end{bmatrix} \triangleq T'_K \begin{bmatrix} i_{Sa} \\ i_{Sb} \\ i_{Sc} \end{bmatrix}$, $\begin{bmatrix} i_{RD} \\ i_{RQ} \end{bmatrix} \triangleq T'_K \begin{bmatrix} i_{Ra} \\ i_{Rb} \\ i_{Rc} \end{bmatrix}$, and $\begin{bmatrix} i_D \\ i_Q \end{bmatrix} \triangleq T'_K \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$. For balanced sinusoidal operation at frequency ω_o, all transformed variables are constant. Therefore, (2.153)–(2.158) can be written as

$$v_{SD} - v_{RD} = R_e i_D + \omega_o L_e i_Q \tag{2.159}$$

$$v_{SQ} - v_{RQ} = R_e i_Q - \omega_o L_e i_D \tag{2.160}$$

$$i_{SD} - i_D = \frac{G_e}{2} v_{SD} + \omega_o \frac{C_e}{2} v_{SQ} \tag{2.161}$$

$$i_{SQ} - i_Q = \frac{G_e}{2} v_{SQ} - \omega_o \frac{C_e}{2} v_{SD} \tag{2.162}$$

$$i_D - i_{RD} = \frac{G_e}{2} v_{RD} + \omega_o \frac{C_e}{2} v_{RQ} \tag{2.163}$$

$$i_Q - i_{RQ} = \frac{G_e}{2} v_{RQ} - \omega_o \frac{C_e}{2} v_{RD} \tag{2.164}$$

Dividing (2.159) and (2.160) by V_B, and (2.161)–(2.164) by I_B gives the following equations in per unit quantities.

$$\overline{v}_{SD} - \overline{v}_{RD} = \overline{R}_e \overline{i}_D + \frac{\omega_o}{\omega_B} \overline{X}_e \overline{i}_Q \tag{2.165}$$

$$\overline{v}_{SQ} - \overline{v}_{RQ} = \overline{R}_e \overline{i}_Q - \frac{\omega_o}{\omega_B} \overline{X}_e \overline{i}_D \tag{2.166}$$

$$\overline{i}_{SD} - \overline{i}_D = \frac{\overline{G}_e}{2} \overline{v}_{SD} + \frac{\omega_o}{\omega_B} \frac{\overline{B}_e}{2} \overline{v}_{SQ} \tag{2.167}$$

$$\overline{i}_{SQ} - \overline{i}_Q = \frac{\overline{G}_e}{2} \overline{v}_{SQ} - \frac{\omega_o}{\omega_B} \frac{\overline{B}_e}{2} \overline{v}_{SD} \tag{2.168}$$

$$\overline{i}_D - \overline{i}_{RD} = \frac{\overline{G}_e}{2} \overline{v}_{RD} + \frac{\omega_o}{\omega_B} \frac{\overline{B}_e}{2} \overline{v}_{RQ} \tag{2.169}$$

$$\overline{i}_Q - \overline{i}_{RQ} = \frac{\overline{G}_e}{2} \overline{v}_{RQ} - \frac{\omega_o}{\omega_B} \frac{\overline{B}_e}{2} \overline{v}_{RD} \tag{2.170}$$

where $X_e \triangleq \omega_B L_e$ and $B_e \triangleq \omega_B C_e$. The factor ω_o/ω_B in one of the terms in all equations is usually approximated to 1. As an approximation, (2.165)–(2.170) are used even during transients.

2.4 Load

In many system studies, the effects of the subtransmission and the distribution networks along with the connected load devices are represented by an aggregated load at a transmission substation. The load model is given by the expressions for active power P and reactive power Q drawn, in terms of voltage magnitude and/or frequency [5, 6]. Two commonly used models are:

-
$$P = P_o \left(\frac{V}{V_o}\right)^a \left[1 + k_{pf}(f - f_o)\right] \tag{2.171}$$

$$Q = Q_o \left(\frac{V}{V_o}\right)^b \left[1 + k_{qf}(f - f_o)\right] \tag{2.172}$$

-
$$P = P_o \left[p_1 \left(\frac{V}{V_o}\right)^2 + p_2 \frac{V}{V_o} + p_3\right] \left[1 + k_{pf}(f - f_o)\right] \tag{2.173}$$

$$Q = Q_o \left[q_1 \left(\frac{V}{V_o}\right)^2 + q_2 \frac{V}{V_o} + q_3\right] \left[1 + k_{qf}(f - f_o)\right] \tag{2.174}$$

Subscript o identifies the values of the respective variables at the operating point. a, b, p_1, p_2, p_3, q_1, q_2, q_3, k_{pf}, and k_{qf} are constants; $p_1 + p_2 + p_3 = 1$ and $q_1 + q_2 + q_3 = 1$. If frequency dependence is not to be considered, k_{pf} and k_{qf} are set to zero.

Equations (2.171)–(2.174) in per unit quantities are

$$\overline{P} = \frac{P_o V_B^a}{S_B V_o^a} \overline{V}^a \left[1 + k_{pf}(f - f_o)\right] \tag{2.175}$$

$$\overline{Q} = \frac{Q_o V_B^b}{S_B V_o^b} \overline{V}^b \left[1 + k_{qf}(f - f_o)\right] \tag{2.176}$$

$$\overline{P} = \frac{P_o}{S_B} \left[p_1 \frac{V_B^2}{V_o^2} \overline{V}^2 + p_2 \frac{V_B}{V_o} \overline{V} + p_3 \right] \left[1 + k_{pf}(f - f_o)\right] \tag{2.177}$$

$$\overline{Q} = \frac{Q_o}{S_B} \left[q_1 \frac{V_B^2}{V_o^2} \overline{V}^2 + q_2 \frac{V_B}{V_o} \overline{V} + q_3 \right] \left[1 + k_{qf}(f - f_o)\right] \tag{2.178}$$

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