

Preface

The approximation theory, optimization theory, theory of variational inequalities, and fixed point theory constitute some of the core topics of nonlinear analysis. These topics provide most elegant and powerful tools to solve the problems from diverse branches of science, social science, engineering, management, etc.

The theory of best approximation is applicable in a variety of problems arising in nonlinear functional analysis. The well-posedness and stability of minimization problems are topics of continuing interest in the literature on variational analysis and optimization. The variational inequality problem, complementarity problem, and fixed point problem are closely related to each other. However, they have their own applications within mathematics and in diverse areas of science, management, social sciences, engineering, etc. The split feasibility problem is a general form of the inverse problem which arises in phase retrievals and in medical image reconstruction. This book aims to provide the current, up-to-date, and comprehensive literature on different topics from approximation theory, variational inequalities, fixed point theory, optimization, complementarity problem, and split feasibility problem. Each chapter is self-contained and contributed by different authors. All chapters contain several examples and complete references on the topic.

Ky Fan's best approximation theorems, best proximity pair theorems, and best proximity point theorems have been studied in the literature when the fixed point equation $Tx = x$ does not admit a solution. "[Best Proximity Points](#)" contains some basic results on best proximity points of cyclic contractions and relatively non-expansive maps. An application of a best proximity point theorem to a system of differential equations has been discussed here. Although, it is not possible to include all the available interesting results on best proximity points, an attempt has been made to introduce some results involving best proximity points and references of the related work have been indicated.

"[Semi-continuity Properties of Metric Projections](#)" presents some selected results regarding semi-continuity of metric projections onto closed subspaces of normed linear spaces. Though there are several significant results relevant to this topic, only a limited coverage of the results is undertaken, as an extensive survey is beyond our scope. This exposition is divided into three parts. The first one deals with results from finite dimensional normed linear spaces. The second one deals with results connecting semi-continuity of metric projection maps and duality

maps. The third one deals with subspaces of finite codimension of infinite dimensional normed linear spaces.

The purpose of “[Convergence of Slices, Geometric Aspects in Banach Spaces and Proximality](#)” is to discuss some notions of convergence of sequence of slices and relate these notions with certain geometric properties of Banach spaces and also to some known proximality properties in best approximation theory. The results which are presented here are not new and in fact they are scattered in the literature in different formulations. The geometric and proximality results discussed in this chapter are presented in terms of convergence of slices, and it is observed that several known results fit naturally in this framework. The presentation of the results in this framework not only unifies several results in the literature, but also it allows us to view the results as geometric results and understand some problems, which remain to be solved in this area. The chapter is in two parts. The first part begins from the classical works of Banach and Šmulian on the characterizations of smooth spaces and uniformly smooth spaces (or uniformly convex spaces) and present similar characterizations for other geometric properties including some recent results. Similarly, the second part begins from the classical results of James and Day on characterizations of reflexivity and strict convexity in terms of some proximality properties of closed convex subsets and present similar characterizations for other proximality properties including some recent results.

“[Measures of Noncompactness and Well-Posed Minimization Problems](#)” is devoted to present some facts concerning the theory of well-posed minimization problems. Some classical results obtained in the framework of that theory are presented but the focus here is mainly on the detailed presentation of the application of the theory of measures of noncompactness to investigations of the well-posedness of minimization problem.

“[Well-Posedness, Regularization, and Viscosity Solutions of Minimization Problems](#)” is divided into two parts. The first part surveys some classical notions for well-posedness of minimization problems. The main aim here is to synthesize some known results in approximation theory for best approximants, restricted Chebyshev centers and prox points from the perspective of well-posedness of these problems. The second part reviews Tikhonov regularization of ill-posed problems. This leads us to revisit the so-called viscosity methods for minimization problems using the modern approach of variational convergence. Lastly, some of these results are particularized to convex minimization problems, and also to ill-posed inverse problems.

In “[Best Approximation in Nonlinear Functional Analysis](#),” some results from fixed point theory, variational inequalities, and optimization theory are presented. At the end, convergence of approximating sequences and the sequence of iterative process are also given.

In “[Hierarchical Minimization Problems and Applications](#),” several iterative methods for solving fixed point problems, variational inequalities and zeros of monotone operators are presented. A generalized mixed equilibrium problem is considered. The hierarchical minimization problem over the set of intersection of

fixed points of a mapping and the set of solutions of a generalized mixed equilibrium problem are considered. A new unified hybrid steepest-descent-like iterative algorithm for finding a common solution of a generalized mixed equilibrium problem and a common fixed point problem of uncountable family of nonexpansive mappings is presented and analyzed.

“[Triple Hierarchical Variational Inequalities](#)” is devoted to the theory of variational inequalities. A brief introduction of variational inequalities is given. The hierarchical variational inequalities are considered, and several iterative methods are presented. The triple hierarchical variational inequalities are discussed in detail along with several examples. Several solution methods are presented.

“[Split Feasibility and Fixed Point Problems](#)” is devoted to the theory of split feasibility problems and fixed point problems. The split feasibility problems and multisets split feasibility problems are described. Several solution methods, namely, CQ methods, are presented for these two problems. Mann-type iterative methods are given for finding the common solution of a split feasibility problem and a fixed point problem. Some methods and results are illustrated by examples.

The last chapter is devoted to the study of nonlinear complementarity problems in a Hilbert space. A notion of \ast -isotone is discussed in relation with solvability of nonlinear complementarity problems. The problem of finding nonzero solution of these problems is also presented.

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