

# Preface

Ordinary differential equations play a crucial role in providing answers to real-world problems and they continue to be an indispensable tool in scientific investigations. Although these equations are often first approximations to real-world systems, they can be refined into more accurate models by including information about past states in time into the equations, e.g., formulating them as delay differential equations, or more generally, functional differential equations. The rapid progress observed in this area is mainly due to their ability to capture the dynamics observed in real-world phenomena.

The literature on ordinary and delay differential equations is large and rapidly growing; results appear in a variety of journals, not just those in pure mathematics. This is due in part to their wide applicability. The domain of investigation and the type of contribution that is needed is often dictated by the application to be modeled. For example, it is most appropriate to study the existence and stability behavior of positive periodic solutions of an ecological system that is strongly influenced by periodic environmental variations.

The capability of functional differential equations to mimic the dynamics in a periodically fluctuating environment has been the driving force for researchers to make valuable contributions to such problems. There has also been great interest in finding conditions under which periodic functional differential equations admit multiple positive periodic solutions. Such problems have wide applications in biology.

In this context, models that attract the attention of researchers include the Hematopoiesis (red blood cell production) model, the Lasota-Ważewska model, Nicholson's Blowflies model, and models with Allee effects. These models have their roots in population dynamics and are extensively used to describe real-world problems. To the best of our knowledge, there is no book that systematically covers the dynamics and global aspects of these models in a periodic environment. Contributions to this area are quite recent. The necessity to bring together the theory and applications is the motivation for this monograph.

The basic tool we use here to prove the existence of multiple periodic solutions is the well-known Leggett-Williams multiple fixed point theorem. This theorem has been applied to problems on the existence of multiple solutions to boundary value problems. The approach is to transform the problem into an equivalent integral equation from which an integral operator is easily formed. The fixed points of the operator then correspond to solutions of the original problem. Of course, an appropriate mathematical setting for the operator must be constructed.

A brief description of the organization of this monograph is as follows; there are a total of five chapters. In [Chap. 1](#), we introduced the Leggett-Williams multiple fixed point theorems that guarantee the existence of two and three fixed points of operators. These fixed point theorems are used throughout the book in showing the existence of two or three positive periodic solutions of various equations and models.

[Chapter 2](#) is concerned with the existence of at least three positive periodic solutions of first-order nonlinear functional differential equations. The results are applied to several population models, such as those listed above. [Chapter 3](#) presents sufficient conditions for the existence of at least three positive periodic solutions for a system of nonlinear functional differential equations. An application to the Hematopoiesis model highlights the significance of the delay in the model.

Many interesting and easily verifiable conditions on the coefficient functions are given in [Chaps. 2](#) and [3](#) for the existence of at least three positive periodic solutions of first-order nonlinear functional differential equations where the nonlinear terms are normally unimodal. This excludes some interesting population models whose nonlinear term is either non-decreasing or non-increasing. Thus, an attempt has been made in [Chap. 4](#) to find existence of periodic solutions for nonlinear equations with non-decreasing nonlinear term and established existence of at least two positive periodic solutions.

[Chapter 5](#) concentrates on the existence of unique positive periodic solutions and their global asymptotic stability. Results on the existence of a unique globally asymptotically stable periodic solution for the fishing model, the Hematopoiesis model, Nicholson's Blowflies model, and the Lasota-Ważewska model are presented.

This book contains a large number of references and we hope this will be useful to readers in their future research work.

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