

Chapter 2

General Equilibrium Models: Usefulness and Techniques of Application

2.1 Introduction

The general equilibrium theory is a branch of theoretical economics that seeks to explain the behaviour of supply, demand and prices in an economy with many interacting markets. It intends to prove that there exist a set of prices that result in an overall (hence general) equilibrium, in contrast to partial equilibrium, which only analyses single markets. As is the case with all models, this is an abstraction from the real economy; nevertheless, it is depicted as a useful model that considers equilibrium prices as long-term prices and actual prices as deviations from equilibrium prices.

The effects of an FDI (perhaps of other parametric variations as well) in the developing economies are better studied in a general equilibrium framework rather than in a partial equilibrium framework. An FDI drives other resources towards the capital-receiving sector(s) from the other sectors including non-traded sectors of the economy, thereby affecting the prices of the non-tradables. The inherent interrelationships between different sectors determine which sectors would expand and which ones would contract. The sector that has a complementary relationship with the capital-receiving sector is likely to expand, while the sector that acts as a substitute should contract. An FDI is expected to affect all the key variables of the economy including social welfare, unemployment of labour, poverty and income inequalities, degrees of factor market distortions and human capital formation. It is not possible to study all these effects together by using a partial equilibrium framework, since it concentrates only on one market at a time.

Hence, for a comprehensive discussion in the subsequent chapters of the book, we follow the simple general equilibrium techniques as developed and popularized by R. W. Jones in his two highly influential articles, Jones (1965, 1971). While Jones (1965) deals with the 2×2 Heckscher–Ohlin–Samuelson (HOS) model, Jones (1971) is based on the 2×3 specific-factor, full-employment model. In this chapter, we present the essence of these two papers in the simplest possible manner. Later, we

shall discuss on extensions of the simple two-sector general equilibrium models, inclusion of non-traded goods, the techniques of measuring social welfare in a small open economy and its changes resulting from changes in policy parameters.

2.2 The 2×2 Heckscher–Ohlin–Samuelson Model

The basic assumptions of the 2×2 Heckscher–Ohlin–Samuelson (H–O–S) model of production are the following:

1. Two commodities, X_1 and X_2 , are produced using two primary factors of production, labour (L) and capital (K).
2. The production functions exhibit constant returns to scale (CRS) with positive but diminishing marginal returns to each factor.
3. The factors are fully employed and are perfectly mobile between the two sectors. The latter implies that factor prices are the same in both sectors.
4. Commodities can be classified in terms of relative factor intensities, which are irreversible, i.e. a commodity is intensive in the use of the same factor of production at all factor price-ratios. In other words, isoquants of the two commodities cut only once.
5. Perfect competition prevails in product as well as factor markets so that commodity prices, P_1 and P_2 , reflect unit costs of production in the two sectors.
6. Commodity prices and factor endowments are given exogenously.

The production functions are given by the following two equations:

$$X_i = F_i(L_i, K_i) \quad \text{for } i = 1, 2 \quad (2.1 \text{ \& } 2.2)$$

where L_i and K_i denote employment of labour and use of capital in the i th sector.

Since the production functions exhibit CRS, the equations of unit isoquants are obtained as follows:

$$1 = f_i(a_{L_i}, a_{K_i}) \quad (2.3)$$

where a_{L_i} and a_{K_i} denote, respectively, the labour and capital requirement per unit of X_i .

Now given the output level, profit maximization implies minimization of costs. In other words, the producers minimize cost along the unit isoquant. At the point of cost minimization, the iso-cost line, with slope $(-W/r)$, is tangent to the unit isoquant with slope (da_{K_i}/da_{L_i}) . Thus, cost minimization with respect to both the commodities implies

$$Wda_{L1} + rda_{K1} = 0 \quad (2.4)$$

$$Wda_{L2} + rda_{K2} = 0 \quad (2.5)$$

The above two equations are called the ‘envelope conditions’.

The input–output coefficients, a_{ji} s, are functions of the (W/r) ratio and the state of production technology, where W and r are the wage rate and return to capital, respectively.

The competitive profit conditions (equality between price and unit cost) in the two sectors are represented as follows

$$a_{L1}W + a_{K1}r = P_1 \quad (2.6)$$

$$a_{L2}W + a_{K2}r = P_2 \quad (2.7)$$

Now, the full-employment conditions of labour and capital are given by

$$a_{L1}X_1 + a_{L2}X_2 = L \quad (2.8)$$

$$a_{K1}X_1 + a_{K2}X_2 = K \quad (2.9)$$

The two zero-profit conditions, Eqs. (2.6) and (2.7), together are called the price system, while the two full-employment conditions, namely, Eqs. (2.8) and (2.9), comprise the output system of the model. So the model consists of four independent equations, Eqs. (2.6), (2.7), (2.8) and (2.9); four endogenous variables, W, r, X_1, X_2 ; and four parameters P_1, P_2, L and K .¹ For uniquely determining the factor prices and output levels, the factor intensities of production of the two commodities must differ. We assume that sector 1 (sector 2) is more labour-intensive (capital-intensive) than sector 2 (sector 1), i.e. $a_{L1}/a_{K1} > a_{L2}/a_{K2}$.

Now, sector 1 is more labour-intensive relative to sector 2 in physical sense if $\lambda_{L1}/\lambda_{K1} > \lambda_{L2}/\lambda_{K2}$,

where $\lambda_{ji} = (a_{ji}X_i/E_j)$ is the allocative share of the j th factor in the i th sector for $j = L, K, i = 1, 2$ and E_j is the endowment of factor j .

On the other hand, sector 1 is more labour-intensive relative to sector 2 in value sense if $\theta_{L1}/\theta_{K1} > \theta_{L2}/\theta_{K2}$ where $\theta_{ji} = (W_j a_{L_i}/P_i)$ is the distributive share of the j th factor in the total value of production of the i th commodity for $j = L, K$ and $i = 1, 2$. Besides, P_i denotes market price of the i th commodity, while W_j stands for price of the j th factor of production.

In the absence of any distortions in the factor markets, if sector 1 is more labour-intensive relative to sector 2 in physical sense, it is also labour-intensive in value sense.

Hence, when the factor intensities of the two sectors differ, the system is determinate and each variable can be uniquely determined. Given the commodity

¹There are, of course, two other parameters denoting the states of production technologies in the two sectors.

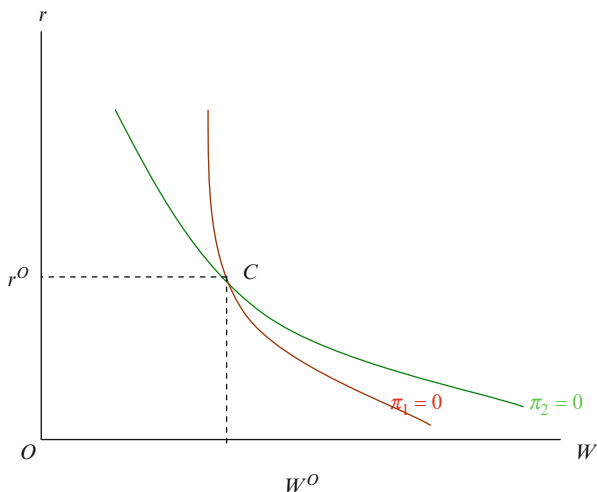


Fig. 2.1 Determination of equilibrium factor prices in the H–O–S model

prices, factor prices can be determined from the price system alone. Thus, any changes in factor endowments cannot affect factor prices. A production system like this where factor prices are independent of factor endowments is called a decomposable system.

The determination of factor prices can be shown in terms of Fig. 2.1. The two zero-profit curves, $\pi_1 = 0$ and $\pi_2 = 0$ in the (W, r) space, represent Eqs. (2.6) and (2.7), respectively. The slopes of the two curves are $-(a_{L1}/a_{K1})$ and $-(a_{L2}/a_{K2})$. The $\pi_1 = 0$ curve is steeper than the $\pi_2 = 0$ curve implying that sector 1 (sector 2) is labour-intensive (capital-intensive) with respect to the other input. The equilibrium W and r are obtained from the point of intersection (C in Fig. 2.1) of the two zero-profit curves.

Once the factor prices are obtained, the factor coefficients, a_{jis} , are also determined since these are functions of (W/r) ratio and technological parameters. Then, X_1 and X_2 are solved from Eqs. (2.8) and (2.9). The lines, L_1L_2 and K_1K_2 , in Fig. 2.2 represent the two full-employment conditions given by Eqs. (2.8) and (2.9). These are drawn as straight lines² given the two commodity prices, P_1 and P_2 . The slopes of the two curves are $-(a_{L1}/a_{L2})$ and $-(a_{K1}/a_{K2})$, respectively. As sector 1 (sector 2) is labour-intensive (capital-intensive) compared to the other sector, the L_1L_2 curve is steeper than the K_1K_2 curve. The equilibrium values of X_1 and X_2 are obtained from the point of intersection (D in Fig. 2.2) of the two curves.

²The a_{jis} depend only on the factor price ratio, (W/r) , which in turn depend on the commodity prices only. Therefore, so long as the commodity prices do not change, a_{jis} also do not change and the slopes of the L_1L_2 and K_1K_2 curves remain constant.

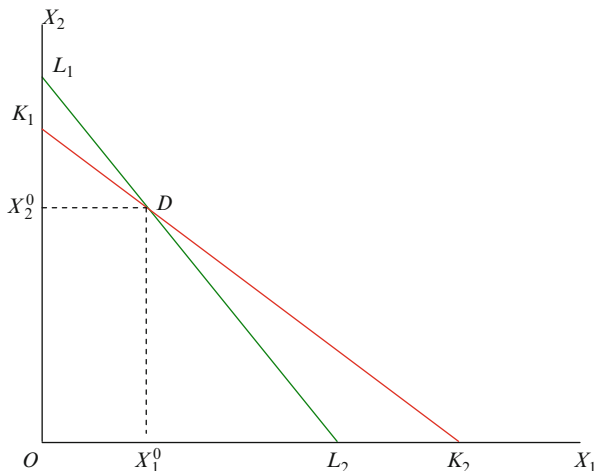


Fig. 2.2 Determination of equilibrium output composition in the H–O–S model

If P_1 rises (falls), *ceteris paribus*, the $\pi_1 = 0$ curve shifts upwards (downwards) thereby causing the wage rate to rise (fall) while the return to capital to fall (rise). Any change in P_2 leads to similar shifts of the $\pi_2 = 0$ curve. If the two prices change in the same proportion and in the same direction, the two curves shift equi-proportionately in the same direction. The two factor prices change in the same proportion and in the same direction producing no change in the wage–rental ratio and hence in $a_{ij}s$.

2.3 Comparative Statics

The stage is now ready for deriving comparative static results in the H–O–S model. The parameters of the system are P_1, P_2, L and K .³ We examine the consequences of changes in any of the parameter(s) on the four endogenous variables W, r, X_1 and X_2 .

2.3.1 Effects of Changes in Commodity Prices on Factor Prices

Totally differentiating Eqs. (2.6) and (2.7) in the price system, using the envelope conditions and the $\hat{}$ notation, we obtain

$$\theta_{L1}\hat{W} + \theta_{K1}\hat{r} = \hat{P}_1 \quad (2.10)$$

³The state of technology is also a parameter. But here we assume that it does not change.

$$\theta_{L2}\widehat{W} + \theta_{K2}\widehat{r} = \widehat{P}_2 \quad (2.11)$$

Here, ‘ $\widehat{}$ ’ means proportional change, e.g. $\widehat{x} = dx/x$.

The changes in factor prices can be determined uniquely by solving Eqs. (2.10) and (2.11). Thus, we find

$$\widehat{W} = \left(\frac{1}{|\theta|} \right) [\theta_{K2}\widehat{P}_1 - \theta_{K1}\widehat{P}_2] \quad (2.12)$$

and

$$\widehat{r} = \left(\frac{1}{|\theta|} \right) [\theta_{L1}\widehat{P}_2 - \theta_{L2}\widehat{P}_1] \quad (2.13)$$

where $|\theta|$ is given by

$$|\theta| = (\theta_{L1}\theta_{K2} - \theta_{K1}\theta_{L2}) \quad (2.14)$$

Now, subtraction of Eq. (2.13) from Eq. (2.12) yields

$$(\widehat{W} - \widehat{r}) = \left(\frac{1}{|\theta|} \right) (\widehat{P}_1 - \widehat{P}_2) \quad (2.15)$$

It is evident that \widehat{W} and \widehat{r} can be determined if both the commodities are produced and $|\theta| \neq 0$ which in turn implies that for uniquely determining the factor prices, the factor intensities for production of the commodities must differ. Since we have assumed that sector 1 (sector 2) is labour-intensive (capital-intensive), we have

$$|\theta| = (\theta_{L1}\theta_{K2} - \theta_{K1}\theta_{L2}) > 0$$

From (2.15) it follows that an increase in the price of labour-intensive good, X_1 , raises the wage-rental ratio in a magnified amount. If $\widehat{P}_1 > \widehat{P}_2$ and X_1 is labour-intensive, then from Eqs. (2.12), (2.13), (2.14) and (2.15), it is easy to show⁴ that

$$\widehat{W} > \widehat{P}_1 > \widehat{P}_2 > \widehat{r} \quad (2.16.1)$$

This is the essence of the Stolper–Samuelson theorem which states that a rise in the price of a commodity raises the real reward of its intensive factor and a decline in the real reward of its un-intensive factor.

Analogously, if X_1 is capital-intensive, we have $|\theta| < 0$. In this case, an increase in P_1 reduces the real wage and raises the real return to its intensive factor, capital.

⁴See Chaudhuri and Mukhopadhyay (2009), chapter 2 for the proof.

So in this case, we must have the following relationship:

$$\widehat{W} < \widehat{P}_2 < \widehat{P}_1 < \widehat{r} \quad (2.16.2)$$

2.3.2 Responses of Outputs to Changes in Commodity Prices and Factor Endowments

Any changes in factor endowments cannot affect factor prices since the latter depend only on commodity prices. However, output levels of the two sectors depend on both factor endowments and commodity prices.⁵

Totally differentiating Eqs. (2.8) and (2.9) we obtain

$$\lambda_{L1}\widehat{X}_1 + \lambda_{L2}\widehat{X}_2 = \widehat{L} - (\lambda_{L1}\widehat{a}_{L1} + \lambda_{L2}\widehat{a}_{L2}) \quad (2.8.1)$$

$$\lambda_{K1}\widehat{X}_1 + \lambda_{K2}\widehat{X}_2 = \widehat{K} - (\lambda_{K1}\widehat{a}_{K1} + \lambda_{K2}\widehat{a}_{K2}) \quad (2.9.1)$$

By definition, the elasticity of factor substitution in sector i is given by

$$\sigma_i = \frac{\widehat{a}_{Ki} - \widehat{a}_{Li}}{\widehat{W} - \widehat{r}} \quad \text{for } i = 1, 2 \quad (2.17.1 \text{ \& } 2.17.2)$$

Solving the two envelope conditions⁶ and Eqs. (2.17.1) and (2.17.2), one finds

$$\widehat{a}_{Ki} = \sigma_i \theta_{Li} (\widehat{W} - \widehat{r}) \quad \text{for } i = 1, 2 \quad (2.18.1 \text{ \& } 2.18.2)$$

$$\widehat{a}_{Li} = -\sigma_i \theta_{Ki} (\widehat{W} - \widehat{r}) \quad \text{for } i = 1, 2 \quad (2.19.1 \text{ \& } 2.19.2)$$

Using (2.18.1), (2.18.2), (2.19.1) and (2.19.2), Eqs. (2.8.1) and (2.9.1) can be rewritten as

$$\lambda_{L1}\widehat{X}_1 + \lambda_{L2}\widehat{X}_2 = \widehat{L} + \delta_L (\widehat{W} - \widehat{r}) \quad (2.8.2)$$

$$\lambda_{K1}\widehat{X}_1 + \lambda_{K2}\widehat{X}_2 = \widehat{K} - \delta_K (\widehat{W} - \widehat{r}) \quad (2.9.2)$$

⁵Outputs depend on commodity prices provided technologies of production are of variable-coefficient type, i.e. a_{ji} s are not fixed. This point has been explained in more details in a subsequent paragraph.

⁶ $\theta_{L1}\widehat{a}_{L1} + \theta_{K1}\widehat{a}_{K1} = 0$ and $\theta_{L2}\widehat{a}_{L2} + \theta_{K2}\widehat{a}_{K2} = 0$ are the two alternative expressions of the two envelope conditions, given by Eqs. (2.4) and (2.5).

where

$$\begin{cases} \delta_L = \lambda_{L1}\theta_{K1}\sigma_1 + \lambda_{L2}\theta_{K2}\sigma_2 \\ \delta_K = \lambda_{K1}\theta_{L1}\sigma_1 + \lambda_{K2}\theta_{L2}\sigma_2 \end{cases} \quad (2.20)$$

The changes in output levels can be determined by solving Eqs. (2.8.2) and (2.9.2) as follows:

$$\hat{X}_1 = \left(\frac{1}{|\lambda|} \right) \left[\lambda_{K2}\hat{L} - \lambda_{L2}\hat{K} + \{\lambda_{K2}\delta_L + \lambda_{L2}\delta_K\} (\hat{W} - \hat{r}) \right] \quad (2.21)$$

$$\hat{X}_2 = \left(\frac{1}{|\lambda|} \right) \left[\lambda_{L1}\hat{K} - \lambda_{K1}\hat{L} - \{\lambda_{L1}\delta_K + \lambda_{K1}\delta_L\} (\hat{W} - \hat{r}) \right] \quad (2.22)$$

where

$$|\lambda| = (\lambda_{L1}\lambda_{K2} - \lambda_{L2}\lambda_{K1}) \quad (2.23)$$

With the help of Eq. (2.15), Eqs. (2.21) and (2.22) can be rewritten as follows:

$$\hat{X}_1 = \left(\frac{1}{|\lambda|} \right) \left[\lambda_{K2}\hat{L} - \lambda_{L2}\hat{K} + \left\{ (\lambda_{K2}\delta_L + \lambda_{L2}\delta_K) (\hat{P}_1 - \hat{P}_2) \left(\frac{1}{|\theta|} \right) \right\} \right] \quad (2.21.1)$$

$$\hat{X}_2 = \left(\frac{1}{|\lambda|} \right) \left[\lambda_{L1}\hat{K} - \lambda_{K1}\hat{L} - \left\{ (\lambda_{L1}\delta_K + \lambda_{K1}\delta_L) (\hat{P}_1 - \hat{P}_2) \left(\frac{1}{|\theta|} \right) \right\} \right] \quad (2.22.1)$$

Subtraction of Eq. (2.22.1) from Eq. (2.21.1) yields

$$(\hat{X}_1 - \hat{X}_2) = \frac{(\hat{L} - \hat{K})}{|\lambda|} + \frac{(\delta_L + \delta_K)}{|\lambda||\theta|} (\hat{P}_1 - \hat{P}_2) \quad (2.24)$$

Owing to our assumption that sector 1 (sector 2) is labour-intensive (capital-intensive), we have $|\lambda| > 0$.

It has already been explained that in the absence of any factor market distortions if any sector is labour-intensive (capital-intensive) relative to the other in physical sense, it is also labour-intensive (capital-intensive) in value sense.

If X_1 is labour-intensive, both $|\lambda|$ and $|\theta|$ are positive, whereas if X_1 is assumed to be capital-intensive, both $|\lambda|$ and $|\theta|$ are negative, so that the product $|\lambda||\theta|$ is always positive.

Equation (2.24) shows the relationship between changes in outputs to changes in factor endowments and factor prices. The output response to changes in factor endowments is captured by the Rybczynski theorem which states that a rise in the

endowment of a factor at constant commodity prices leads to the expansion of the commodity that uses the factor intensively and contraction of the other commodity.

Substituting $\hat{P}_1 = \hat{P}_2 = 0$ in Eq. (2.24), it follows that if X_1 is labour-intensive, an increase in the labour endowment raises X_1 by a magnified amount and lowers X_2 . If \hat{L} exceeds \hat{K} , then

$$\hat{X}_1 > \hat{L} > \hat{K} > \hat{X}_2 \quad (2.25)$$

But if X_1 is capital-intensive, $|\lambda| < 0$. In this case, an increase in L leads to higher production of X_2 and a decline in X_1 .

Now for finding out the effects of changes in commodity prices on outputs, we keep the factor endowments unchanged so that $\hat{L}, \hat{K} = 0$. Thus, if $\hat{P}_1 > \hat{P}_2$, then $\hat{X}_1 > \hat{X}_2$. In particular, from (2.21.1) and (2.22.1) it follows that $\hat{X}_1 > 0$ and $\hat{X}_2 < 0$. If $\hat{P}_1 = \hat{P}_2 > 0$, then $\hat{W} = \hat{P}_1 = \hat{P}_2 = \hat{r} > 0$ so that $(\hat{W} - \hat{r}) = 0$ and $\hat{X}_1 = \hat{X}_2 = 0$.

Therefore, an increase in the price of a commodity leads to a rise in production of that commodity and a fall in that of the other commodity. If both the commodity prices change at the same rate, the production of both commodities remains unchanged.

If $\hat{P}_1 > \hat{P}_2 > 0$, the relative price of commodity 1 (commodity 2) rises (falls). This leads to an increase in the wage rate, W , and a fall in the return to capital, r , via the Stolper–Samuelson effect. As the (W/r) ratio increases, producers in both the sectors would substitute labour by cheaper capital. Consequently, in both the sectors, the producers adopt more capital-intensive techniques of production than before. Both a_{K1} and a_{K2} increase while a_{L1} and a_{L2} decrease. At given levels of output, there arises a shortage of capital and a surplus of labour. This leads to a Rybczynski-type effect thereby causing the labour-intensive sector (sector 1) to expand and the capital-intensive sector (sector 2) to contract.

So a change in the relative prices of the two commodities alters the product mix through changes in the input coefficients, a_{ji} s. If technologies of production are of the fixed-coefficient type, i.e. if $\sigma_1 = \sigma_2 = 0$, then δ_L and δ_K are also equal to zero. Then, from (2.21.1) and (2.22.1) it follows that $\hat{X}_1 = \hat{X}_2 = 0$. So changes in commodity prices have no effect on the composition of outputs. In this case, there is no Rybczynski-type effect that follows a Stolper–Samuelson effect. Therefore, any change in the price system is not transmitted into the output system.

2.4 The 2 × 3 Specific-Factor, Full-Employment Model

We now consider a two-sector, specific-factor model of production. Two commodities, X_1 and X_2 , are produced with three inputs, two sector-specific factors and one intersectorally mobile factor. Labour and capital of type 1 (say K_1) are used to produce X_1 , while labour and capital of type 2 (say K_2) are combined to produce X_2 . Each type of capital is used specifically in one sector while labour is mobile

between both the sectors. The three inputs are fully employed. The wage rate is denoted by W , while the returns to capital of type 1 and type 2 are represented by r_1 and r_2 , respectively. All the other assumptions of the H–O–S model are retained. It is to be noted that the two industries cannot be classified in terms of factor intensities because they use two different types of capital. However, according to Jones and Neary (1984), the two industries can still be classified in terms of the distributive shares of the intersectorally mobile factor, i.e. labour. If $\theta_{L1} > \theta_{L2}$, we can say that sector 1 is more labour-intensive than sector 2 and vice versa.

Under competitive conditions, the zero-profit conditions in the two sectors are given by

$$a_{L1}W + a_{K1}r_1 = P_1 \quad (2.26)$$

$$a_{L2}W + a_{K2}r_2 = P_2 \quad (2.27)$$

The full-employment conditions of labour and two types of capital are given by

$$a_{L1}X_1 + a_{L2}X_2 = L \quad (2.8)$$

$$a_{K1}X_1 = K_1 \quad (2.28)$$

$$a_{K2}X_2 = K_2 \quad (2.29)$$

Use of Eqs. (2.28) and (2.29) and substitution in (2.8) yield

$$\left(\frac{a_{L1}}{a_{K1}}\right) K_1 + \left(\frac{a_{L2}}{a_{K2}}\right) K_2 = L \quad (2.30)$$

This model consists of five independent equations, Eqs. (2.26), (2.27), (2.28), (2.29) and (2.30), and five endogenous variables, W, r_1, r_2, X_1 and X_2 . The parameters of the system are P_1, P_2, L, K_1 and K_2 . However, this model is indecomposable. The three unknown factor prices cannot be solved from the price system consisting of two equations. One has to derive an additional equation from the output system which is free of X_i s but contains terms, $a_{ji}s$, which are functions of factor prices. Equation (2.30) is such an equation. The values of W, r_1 and r_2 are obtained by solving Eqs. (2.26), (2.27) and (2.30). Therefore, in this 2×3 specific-factor, full-employment model, factor prices depend not only on commodity prices but also on factor endowments. Any changes in the factor endowments affect factor prices, which in turn affect the per unit input requirements, $a_{ji}s$, in each sector.⁷ The determination of factor prices can be shown in terms of Fig. 2.3.

⁷It is to be noted that the model loses its consistency if production technologies are of the fixed-coefficient type because Eq. (2.30) then does not implicitly contain factor prices.

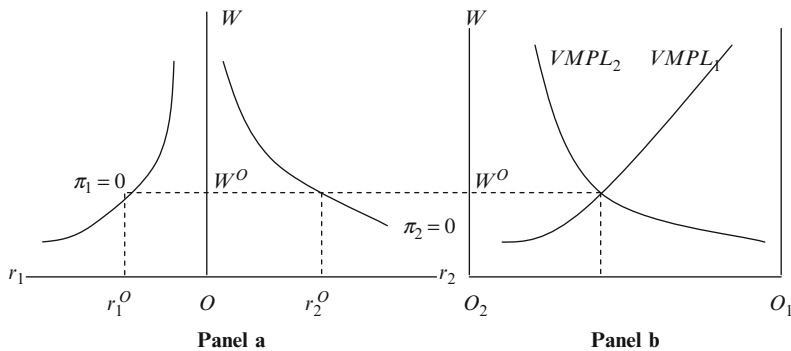


Fig. 2.3 Determination of factor prices in the 2×3 specific-factor, full-employment model

In Panel (b) of Fig. 2.3, the distance O_1O_2 measures the labour endowment of the economy. $VMPL_1$ and $VMPL_2$ are the labour demand curves of sector 1 and sector 2, respectively. The equilibrium wage rate is W^O . Panel (a) shows the two zero-profit curves representing Eqs. (2.26) and (2.27). The equilibrium returns to capital of type 1 and capital of type 2 are r_1^O and r_2^O , respectively. If the price of commodity 1, P_1 , rises, ceteris paribus, the zero-profit curve of sector 1 in the second quadrant of Panel (a) shifts upwards. Besides, the labour demand curve of sector 1 shown in Panel (b) in Fig. 2.3 shifts in the leftward direction. Consequently, the wage rate and the return to capital of type 1 increase while the return to capital of type 2 falls.⁸

2.4.1 Comparative Statics

Let us now study the consequences of any changes in the parameters of the system, namely, P_1, P_2, L, K_1 and K_2 , on the five endogenous variables, W, r_1, r_2, X_1 and X_2 .

Given that $a_{L1} = a_{L1}(W, r_1)$ and $a_{K1} = a_{K1}(W, r_1)$, total differentiation yields, respectively,

$$\left. \begin{aligned} \hat{a}_{L1} &= S_{LL}^1 \hat{W} + S_{LK}^1 \hat{r}_1 \\ \hat{a}_{K1} &= S_{KL}^1 \hat{W} + S_{KK}^1 \hat{r}_1 \end{aligned} \right\} \quad (2.18.3)$$

Similarly, from $a_{L2} = a_{L2}(W, r_2)$ and $a_{K2} = a_{K2}(W, r_2)$, we get

$$\left. \begin{aligned} \hat{a}_{L2} &= S_{LL}^2 \hat{W} + S_{LK}^2 \hat{r}_2 \\ \hat{a}_{K2} &= S_{KL}^2 \hat{W} + S_{KK}^2 \hat{r}_2 \end{aligned} \right\} \quad (2.19.3)$$

⁸The increase in r_1 may not be clear from Panel (a), Fig. 2.3. However, it can be proved mathematically. See the results presented in (2.37).

Here, S_{jk}^i is the degree of substitution between factors in the i th sector, $i = 1, 2$, for example, in sector 1, $S_{LL}^1 = (\partial a_{L1}/\partial W)(W/a_{L1})$, $S_{LK}^1 = (\partial a_{L1}/\partial r)(r/a_{L1})$. $S_{jk}^i > 0$ for $j \neq k$ and $S_{jj}^i < 0$. It should be noted that as the production functions are homogeneous of degree one, the factor coefficients, $a_{ji}s$, are homogeneous of degree zero in the factor prices. Hence, the sum of elasticities of any factor coefficient (a_{ji}) in any sector with respect to factor prices must be equal to zero. For example, in sector 1, for the labour coefficient, we have $(S_{LL}^1 + S_{LK}^1) = 0$, while for the capital coefficient, $(S_{KL}^1 + S_{KK}^1) = 0$. Similarly, in sector 2, $(S_{LL}^2 + S_{LK}^2) = 0$ and $(S_{KL}^2 + S_{KK}^2) = 0$.

Now, total differentiation of Eqs. (2.26) and (2.27) and use of ‘envelope conditions’ in sector 1 and sector 2 entail

$$\theta_{L1}\widehat{W} + \theta_{K1}\widehat{r}_1 = \widehat{P}_1 \quad (2.31)$$

$$\theta_{L2}\widehat{W} + \theta_{K2}\widehat{r}_2 = \widehat{P}_2 \quad (2.32)$$

Totally differentiating Eq. (2.30) gives

$$\lambda_{L1}(\widehat{a}_{L1} - \widehat{a}_{K1}) + \lambda_{L1}\widehat{K}_1 + \lambda_{L2}(\widehat{a}_{L2} - \widehat{a}_{K2}) + \lambda_{L2}\widehat{K}_2 = \widehat{L} \quad (2.30.1)$$

Using (2.18.3) and (2.19.3) and simplifying from Eq. (2.30.1), we can derive

$$A\widehat{W} + B\widehat{r}_1 + C\widehat{r}_2 = -\lambda_{L1}\widehat{K}_1 - \lambda_{L2}\widehat{K}_2 + \widehat{L} \quad (2.30.2)$$

Where

$$A = [\lambda_{L1}(S_{LL}^1 - S_{KL}^1) + \lambda_{L2}(S_{LL}^2 - S_{KL}^2)] < 0$$

$$B = \lambda_{L1}(S_{LK}^1 - S_{KK}^1) > 0$$

$$C = \lambda_{L2}(S_{LK}^2 - S_{KK}^2) > 0$$

Solving (2.31), (2.32) and (2.30.2), one finds

$$\begin{array}{c} \widehat{W} = \left(\frac{1}{\Delta}\right) \left[\begin{array}{ccc} -B\theta_{K2}\widehat{P}_1 & -C\theta_{K1}\widehat{P}_2 & +\theta_{K1}\theta_{K2} \left(\widehat{L} - \lambda_{L1}\widehat{K}_1 - \lambda_{L2}\widehat{K}_2 \right) \end{array} \right] \\ \quad (-) \quad (+) \quad (+) \end{array} \quad (2.33)$$

$$\begin{array}{c} \widehat{r}_1 = \left(\frac{1}{\Delta}\right) \left[\begin{array}{ccc} C\theta_{L1}\widehat{P}_2 & -\theta_{L1}\theta_{K2} \left(\widehat{L} - \lambda_{L1}\widehat{K}_1 - \lambda_{L2}\widehat{K}_2 \right) & - (C\theta_{L2} - A\theta_{K2})\widehat{P}_1 \end{array} \right] \\ \quad (-) \quad (+) \quad (+) \quad (-) \end{array} \quad (2.34)$$

$$\begin{array}{c} \widehat{r}_2 = \left(\frac{1}{\Delta}\right) \left[\begin{array}{ccc} (A\theta_{K1} - B\theta_{L1})\widehat{P}_2 & +B\theta_{L2}\widehat{P}_1 & +\theta_{K1}\theta_{L2} \left(\widehat{L} - \lambda_{L1}\widehat{K}_1 - \lambda_{L2}\widehat{K}_2 \right) \end{array} \right] \\ \quad (-) \quad (-) \quad (+) \quad (+) \end{array} \quad (2.35)$$

where

$$\Delta = \underset{(+)}{-\theta_{L1}\theta_{K2}B} - \underset{(+)}{\theta_{K1}} \left(\underset{(-)}{\theta_{L2}C} - \theta_{K2}A \right) < 0 \quad (2.36)$$

From (2.33), (2.34) and (2.35), the following results readily follow:

$$\left. \begin{array}{l} \text{(i) When } \hat{P}_1 > 0, \text{ then } \hat{W} > 0; \hat{r}_1 > 0 \text{ and } \hat{r}_2 < 0. \\ \text{(ii) When } \hat{P}_2 > 0, \text{ then } \hat{W} > 0; \hat{r}_1 < 0 \text{ and } \hat{r}_2 > 0. \\ \text{(iii) When } \hat{L} > 0, \text{ then } \hat{W} < 0; \hat{r}_1 > 0 \text{ and } \hat{r}_2 > 0. \\ \text{(iv) When } \hat{K}_1 > 0, \text{ then } \hat{W} > 0; \hat{r}_1 < 0 \text{ and } \hat{r}_2 < 0. \\ \text{(v) When } \hat{K}_2 > 0, \text{ then } \hat{W} > 0; \hat{r}_1 < 0 \text{ and } \hat{r}_2 < 0. \end{array} \right\} \quad (2.37)$$

At constant overall factor endowments, the relation between the changes in commodity prices and factor prices can be established by subtracting (2.32) from (2.31). Noting that $(\theta_{L1} + \theta_{K1}) = 1 = (\theta_{L2} + \theta_{K2})$, we get

$$\theta_{K2} (\hat{W} - \hat{r}_2) - \theta_{K1} (\hat{W} - \hat{r}_1) = (\hat{P}_1 - \hat{P}_2)$$

The above expression entails that if $\hat{P}_1 > \hat{P}_2$, then $\hat{r}_1 > \hat{P}_1 > \hat{W} > \hat{P}_2 > \hat{r}_2$. So, any changes in commodity prices drastically affect the returns to specific factors. The return to the mobile factor (labour) rises in terms of one sector and falls in terms of the other. From the relationships depicted in (2.37 – (iii), (iv) and (v)), it is evident that a rise in the endowment of the mobile factor brings about a fall in its return and augments the returns to both the specific factors, while an increase in the stock in one of the two specific factors lowers the returns to both the specific factors and raises the return to the mobile factor.

Total differentiation of (2.28) and (2.29), use of (2.33), (2.34) and (2.35) and simplification yield, respectively,

$$\begin{aligned} \hat{X}_1 = & \underset{(-)}{\left(\frac{\hat{P}_1}{\Delta}\right)} \left[\underset{(+)}{S_{KL}^1} \left\{ \underset{(-)}{\lambda_{L2}} \left(\underset{(-)}{S_{LL}^2} - \underset{(+)}{S_{KL}^2} \right) - \underset{(+)}{C\theta_{L2}} \right\} - \underset{(-)}{\left(\frac{\hat{P}_2}{\Delta}\right)} \underset{(+)}{CS_{KK}^1} - \underset{(-)}{\left(\frac{\hat{L}}{\Delta}\right)} \underset{(+)}{S_{KL}^1} \theta_{K2} \right] \\ & + \underset{(-)}{\left(\frac{\hat{K}_1}{\Delta}\right)} \left[-\underset{(+)}{\lambda_{L1}} \underset{(+)}{S_{LK}^1} \theta_{L1} \theta_{K2} - \underset{(+)}{C\theta_{K1}} \theta_{L2} + \left\{ \underset{(-)}{\lambda_{L1}} \underset{(-)}{S_{LL}^1} + \underset{(-)}{\lambda_{L2}} \left(\underset{(-)}{S_{LL}^2} - \underset{(+)}{S_{KL}^2} \right) \theta_{K1} \theta_{K2} \right\} \right] \\ & - \underset{(-)}{\left(\frac{\hat{K}_2}{\Delta}\right)} \underset{(-)}{S_{KK}^1} \lambda_{L2} \theta_{K2} \end{aligned} \quad (2.38)$$

and

$$\begin{aligned} \hat{X}_2 = & - \left(\frac{\hat{P}_1}{\Delta} \right) B S_{KK}^2 - \left(\frac{\hat{P}_2}{\Delta} \right) [S_{KK}^2 \{ \theta_{K1} \lambda_{L1} (S_{LL}^1 - S_{KL}^1) - B \theta_{L1} \}] - \left(\frac{\hat{L}}{\Delta} \right) S_{KL}^2 \theta_{K1} \\ & (-) (+) (-) \quad (-) \quad (-) \quad (-) \quad (+) \quad (+) \quad (-) (+) \\ & - \left(\frac{\hat{K}_1}{\Delta} \right) S_{KK}^2 \theta_{K1} \lambda_{L1} + \left(\frac{\hat{K}_2}{\Delta} \right) [-B \theta_{L1} \theta_{K2} - \theta_{K1} \lambda_{L2} S_{LK}^2 + A \theta_{K1} \theta_{K2}] \\ & (-) (-) \quad (-) (+) \quad (+) \quad (-) \end{aligned} \quad (2.39)$$

From (2.38) and (2.39) the following results readily follow:

$$\left. \begin{aligned} \text{(i) When } \hat{P}_1 > 0, \text{ then } \hat{X}_1 > 0 \text{ and } \hat{X}_2 < 0. \\ \text{(ii) When } \hat{P}_2 > 0, \text{ then } \hat{X}_1 < 0 \text{ and } \hat{X}_2 > 0. \\ \text{(iii) When } \hat{L} > 0, \text{ then } \hat{X}_1 > 0 \text{ and } \hat{X}_2 > 0. \\ \text{(iv) When } \hat{K}_1 > 0, \text{ then } \hat{X}_1 > 0 \text{ and } \hat{X}_2 < 0. \\ \text{(v) When } \hat{K}_2 > 0, \text{ then } \hat{X}_1 < 0 \text{ and } \hat{X}_2 > 0. \end{aligned} \right\} \quad (2.40)$$

Thus, an increase in the price of a commodity expands the production of that commodity and reduces that of the other. If the endowment of the mobile factor increases, the levels of production of both the commodities rise. An expansion in the stock of the specific factor raises the production of the commodity that uses the factor and reduces production of the other.

All the comparative static results obtained in the 2×3 specific-factor, full-employment model can be intuitively explained in the following fashion.

If P_1 rises (say, by 10 %), ceteris paribus, initially both W and r_1 in sector 1 increase by 10 %. As labour is the intersectorally mobile factor, labour moves out of sector 2 to sector 1 thereby partially offsetting the increase in W . Finally, W increases by less than 10 %. From the zero-profit condition in sector 1 (Eq. 2.26), it is evident that r_1 must rise by more than 10 %. The wage-rental ratio, W/r_1 , falls. Producers in sector 1 substitute capital of type 1 by labour as the latter has become relatively cheaper. The production technique in sector 1 becomes less capital (of type 1)-intensive. At given X_1 , there would be adequacy of capital of type 1. Besides, the supply of labour to this sector has already increased. As sector 1 gets higher supply of both resources, it expands. Sector 2 must contract as it now gets less labour than before. The demand for capital of type 2 decreases which in turn lowers the return to capital of type 2, i.e. r_2 . This is also clear from the zero-profit condition of sector 2 given by Eq. (2.27). The effects of an increase/decrease in P_2 on the factor prices and quantities of production can be explained in the similar line.

An increase in the endowment of capital of type 1 (K_1) lowers its return, r_1 . Sector 1 expands as K_1 is specific to this sector. It demands more labour for its expansion that raises the wage rate, W . Sector 1 draws the additional labour from sector 2 causing the latter to contract. So sector 2 contracts and the demand for

capital of type 2 falls that lowers the return to capital of this type, r_2 , since its supply is exogenously given. In Panel (b) of Fig. 2.3, the labour demand curve for sector 1, $VMPL_1$, shifts in the upward direction. Consequently, W rises and both r_1 and r_2 fall. The outcomes of an accumulation of capital of type 2 can also be explained in the similar fashion.

Finally, if the labour endowment grows (say, following an immigration of labour from neighbouring countries), the wage rate plummets. As labour is the intersectorally mobile factor, both the sectors expand. The demand for each type of capital (sector-specific input) goes up leading to increases in both r_1 and r_2 . In terms of Panel (b) of Fig. 2.3, the length of O_1O_2 in the horizontal axis, which measures the labour endowment, increases. The two labour demand curves, $VMPL_1$ and $VMPL_2$, must intersect each other at a lower wage rate. From Panel (a) of Fig. 2.3, one finds that both r_1 and r_2 increase.

2.5 Extensions of 2-Sector, Full-Employment General Equilibrium Model

Production Structure 1

Consider the following production structure for a small open economy which is a price-taker in the international market. Three commodities are produced and three inputs are used in production. Sector 1 is the agricultural sector that uses labour and land in the production process. Sectors 2 and 3 are the two manufacturing sectors which produce their outputs by means of labour and capital. All the commodities are internationally traded and their prices are given internationally. Commodity 1 is taken to be the numeraire. Other standard assumptions of the 2-sector, full-employment model hold.

Under competitive conditions, the three zero-profit conditions are

$$Wa_{L1} + Ra_{N1} = 1 \quad (2.41)$$

$$Wa_{L2} + ra_{K2} = P_2 \quad (2.42)$$

$$Wa_{L3} + ra_{K3} = P_3 \quad (2.43)$$

Here, R and r are the returns to land (N) and capital (K), respectively, and W is the wage rate.

The full-employment conditions of the three factors of production, namely, labour, land and capital, are as follows:

$$a_{L1}X_1 + a_{L2}X_2 + a_{L3}X_3 = L \quad (2.44)$$

$$a_{N1}X_1 = N \quad (2.45)$$

$$a_{K2}X_2 + a_{K3}X_3 = K \quad (2.46)$$

where L , N and K are the labour, land and capital endowments of the economy, respectively.

This is a decomposable production structure because there are three unknown factor prices, W , r and R , in the price system and the same number of independent equations, i.e. (2.41), (2.42) and (2.43). Equations (2.42) and (2.43) together look like the price system of the H–O–S model and display H–O properties. Thus, Eqs. (2.42) and (2.43) together form a Heckscher–Ohlin subsystem or HOSS. For uniquely determining the factor prices, relative factor intensities of the two sectors in the HOSS must differ. We assume that sector 2 is more labour-intensive than sector 3, i.e. $(a_{L2}/a_{K2}) > (a_{L3}/a_{K3})$. The two factor prices, W and r , are solved from Eqs. (2.42) and (2.43). R is obtained by plugging the value of W in Eq. (2.41). Once factor prices are known, the factor coefficients, a_{jis} , are also known. Then, from Eq. (2.45) one gets X_1 . Finally, solving Eqs. (2.44) and (2.46) simultaneously, we find out X_2 and X_3 . Owing to the decomposition property, we find that the factor prices do not depend on factor endowments. However, the production levels of the two commodities depend on both commodity prices and factor endowments.

Production Structure 2

It considers a three-sector economy with four factors of production: unskilled labour, skilled labour, capital of type N and capital of type K .⁹ The unskilled wage is fixed economy-wide at \bar{W} due to the minimum wage legislation of the government. In all the three sectors, both types of labour are used. Capital of type N is used in sector 1 which is the agricultural sector, while the other two sectors use capital of type K . Sectors 1 and 2 are the two export sectors, while sector 3 is the import-competing sector.

The usual zero-profit conditions for the three sectors are as follows:

$$\bar{W}a_{L1} + W_S a_{S1} + R a_{N1} = 1 \quad (2.47)$$

$$\bar{W}a_{L2} + W_S a_{S2} + r a_{K2} = P_2 \quad (2.48)$$

$$\bar{W}a_{L3} + W_S a_{S3} + r a_{K3} = P_3 \quad (2.49)$$

W_S is the skilled wage. As the unskilled wage is exogenously given, sectors 2 and 3 together form a HOSS. We assume that sector 3 is more capital-intensive relative to sector 2 with respect to skilled labour. This implies that $(a_{K3}/a_{S3}) > (a_{K2}/a_{S2})$.

⁹This production structure has been used in Beladi and Marjit (1992b).

Skilled labour and the two types of capital are fully utilized. The full-employment conditions for these resources are given as follows:

$$a_{S1}X_1 + a_{S2}X_2 + a_{S3}X_3 = S \quad (2.50)$$

$$a_{N1}X_1 = N \quad (2.51)$$

$$a_{K2}X_2 + a_{K3}X_3 = K \quad (2.52)$$

Since the unskilled wage is exogenously given at \overline{W} , there arises a possibility of unskilled unemployment. We assume that the supply of unskilled labour is greater than its aggregate demand in the three sectors at \overline{W} so that there is unemployment of unskilled labour in the economy. The aggregate employment of unskilled labour in the economy (L) is given by

$$L = a_{L1}X_1 + a_{L2}X_2 + a_{L3}X_3 \quad (2.53)$$

This production structure is also decomposable. W_S and r are determined from Eqs. (2.48) and (2.49). Plugging the value of W_S in (2.47), one can obtain R . So, here also factor prices do not depend on factor endowments. The three output levels are obtained from Eqs. (2.50), (2.51) and (2.52). The aggregate employment of unskilled labour is obtained from Eq. (2.53).

Production Structure 3

We consider another production structure with three sectors. Sector 2 produces a final manufacturing commodity, X_2 , with the help of labour and capital of type 1. Sector 1 is the agricultural sector that produces its output by using labour and fertilizer. Fertilizer is produced in sector 3 by means of labour and capital of type 2. Sector 3 is the import-competing sector of the economy. The domestic production of fertilizer falls short of its demand in sector 1. So, the remaining part is imported at the internationally given price, P_3 . All the three commodities are internationally traded, and hence, their prices are exogenously given. The equational structure of the model is as follows:

The usual three zero-profit conditions are given by

$$Wa_{L1} + P_3a_{31} = P_1 \quad (2.54)$$

$$Wa_{L2} + r_1a_{K2} = P_2 \quad (2.55)$$

$$Wa_{L3} + r_2a_{K3} = P_3 \quad (2.56)$$

The full-employment conditions are given by the following three equations:

$$a_{L1}X_1 + a_{L2}X_2 + a_{L3}X_3 = L \quad (2.57)$$

$$a_{K2}X_2 = K_1 \quad (2.58)$$

$$a_{K3}X_3 = K_2 \quad (2.59)$$

The volume of import of fertilizer (commodity 3) is

$$M = a_{31}X_1 - X_3 \quad (2.60)$$

It is to be noted that it is also a decomposable production structure but does not contain any HOSS. W is found from (2.54). Plugging the value of W in Eqs. (2.55) and (2.56), we respectively obtain r_1 and r_2 . Then, the output levels are obtained from Eqs. (2.57), (2.58) and (2.59).

Production Structure 4

Consider a small open economy with three sectors; two types of labour, skilled and unskilled; and two types of capital. Sector 1 is agriculture that uses unskilled labour and capital of type 1. Sector 2 is a low-skill manufacturing sector that produces its output by means of unskilled labour and capital of type 2. Finally, sector 3 produces a high-skill commodity like computer software by using skilled labour and capital of type 2. The three zero-profit conditions are given by the following equations:

$$Wa_{L1} + r_1a_{K1} = P_1 \quad (2.61)$$

$$Wa_{L2} + r_2a_{K2} = P_2 \quad (2.62)$$

$$W_Sa_{S3} + r_2a_{K3} = P_3 \quad (2.63)$$

All the inputs are fully employed and the full-employment conditions are given by the following four equations:

$$a_{L1}X_1 + a_{L2}X_2 = L \quad (2.64)$$

$$a_{K1}X_1 = K_1 \quad (2.65)$$

$$a_{K2}X_2 + a_{K3}X_3 = K_2 \quad (2.66)$$

$$a_{S3}X_3 = S \quad (2.67)$$

In the price system there are three equations, namely, Eqs. (2.61), (2.62) and (2.63), with four unknown factor prices, W, r_1, r_2 and W_S . This system does not satisfy the decomposition property. Factor prices cannot be determined from the price system alone. We shall have to derive an additional equation from the output system which is free of X_i s that can be used together with the three zero-profit conditions to solve for the four unknown factor prices. From (2.64) using (2.65), we get

$$\left(\frac{a_{L1}K_1}{a_{K1}} \right) + a_{L2}X_2 = L$$

or

$$X_2 = \left[L - \left(\frac{a_{L1}K_1}{a_{K1}} \right) \right] \left(\frac{1}{a_{L2}} \right) \quad (2.64.1)$$

Similarly, from (2.66) and (2.67)

$$a_{K2}X_2 + \left(\frac{a_{K3}S}{a_{S3}} \right) = K_2$$

or

$$X_2 = \left[K_2 - \left(\frac{a_{K3}S}{a_{S3}} \right) \right] \left(\frac{1}{a_{K2}} \right) \quad (2.66.1)$$

From Eqs. (2.64.1) and (2.66.1), one gets

$$\left[K_2 - \left(\frac{a_{K3}S}{a_{S3}} \right) \right] \left(\frac{1}{a_{K2}} \right) = \left[L - \left(\frac{a_{L1}K_1}{a_{K1}} \right) \right] \left(\frac{1}{a_{L2}} \right) \quad (2.68)$$

The four unknown factor prices are obtained by solving Eqs. (2.61), (2.62), (2.63) and (2.68) simultaneously. Equation (2.68) contains all endowment parameters, namely, L, S, K_1 and K_2 . So the equilibrium factor prices depend on P_1, P_2, P_3, L, S, K_1 and K_2 . X_1 and X_3 are obtained from Eqs. (2.65) and (2.67), respectively. Finally, X_2 is determined from either Eqs. (2.64.1) or (2.66.1).

2.5.1 Production Structures with Non-traded Goods

The commodities which are produced and consumed/used up within an economy and are not traded internationally are called non-traded goods or local goods. These goods cannot be traded internationally due to factors like the nature of the goods, political barriers and artificial trade barriers. However, these goods are traded

domestically and their prices are determined by demand–supply forces. Non-traded goods may either be intermediate inputs or final commodities.

Production Structure 5

If we introduce a third sector, in the 2×2 full-employment model, that produces a non-traded intermediate good for another sector, the production structure would look like the following:

$$Wa_{L1} + ra_{K1} = P_1 \quad (2.69)$$

$$Wa_{L2} + ra_{K2} = P_2 \quad (2.70)$$

$$Wa_{L3} + ra_{K3} + P_2 a_{23} = P_3 \quad (2.71)$$

In all the three sectors of the economy, labour and capital are used as inputs. Sector 1 produces the export commodity, while sector 3 is the import-competing sector. Both the traded sectors produce final commodities. Apart from labour and capital, sector 3 uses a non-traded input which is produced in sector 2. Equations (2.69), (2.70) and (2.71) are the three competitive equilibrium conditions. a_{23} is the amount of the non-traded good required to produce one unit of good, X_3 . P_1 and P_3 are given by the small open economy assumption, while P_2 being the price of the non-traded good is determined endogenously.

The other equations of the model are as follows:

$$a_{L1}X_1 + a_{L2}X_2 + a_{L3}X_3 = L \quad (2.72)$$

$$a_{K1}X_1 + a_{K2}X_2 + a_{K3}X_3 = K \quad (2.73)$$

$$a_{23}X_3 = X_2 \quad (2.74)$$

Equations (2.72) and (2.73) are the full-employment conditions for labour and capital, respectively. Finally, Eq. (2.74) states that in equilibrium the demand for the non-traded input in sector 3 is exactly equal to its production in sector 2.

There are six endogenous variables, W, r, P_2, X_1, X_2 and X_3 , and the same number of independent equations, namely, Eqs. (2.69), (2.70), (2.71), (2.72), (2.73) and (2.74). In the price system there are three variables and the same number of equations. So this is a decomposable system and factor prices depend on commodity prices only. From Eqs. (2.69) and (2.70), W and r are obtained as functions of P_2 . Plugging the values of W and r in Eq. (2.71), we can solve for P_2 . Once the factor prices are known, the factor coefficients, a_{jis} , are also known. Then, by solving Eqs. (2.72), (2.73) and (2.74), the output levels are obtained. Sector 1 and sector 2 together form a HOSS.

Production Structure 6

We consider a small open economy with a 3×3 full-employment production structure. Sectors 1 and 3 are the two traded sectors which use two different types of capital apart from labour for production. Sector 2 produces a non-traded final commodity using labour and capital of N type. Capital of N type is completely mobile between sectors 1 and 2, while capital of K type is specific to sector 3. The three competitive zero-profit conditions are as follows:

$$Wa_{L1} + Ra_{N1} = P_1 \quad (2.75)$$

$$Wa_{L2} + Ra_{N2} = P_2 \quad (2.76)$$

$$Wa_{L3} + ra_{K3} = P_3 \quad (2.77)$$

Here also sectors 1 and 2 form a HOSS. Labour, capital of type N and capital of type K are fully utilized in production. The full-employment conditions for these three inputs are respectively given by the following equations:

$$a_{L1}X_1 + a_{L2}X_2 + a_{L3}X_3 = L \quad (2.78)$$

$$a_{N1}X_1 + a_{N2}X_2 = N \quad (2.79)$$

$$a_{K3}X_3 = K \quad (2.80)$$

There are six equations to solve for seven unknowns, W, R, r, P_2, X_1, X_2 and X_3 , which means that there is indeterminacy problem. In order to make the system consistent, we shall have to include the demand–supply equality condition for the non-traded final commodity.

The demand function for the non-traded final commodity (good 2) depends on the relative prices of the commodities, P_2/P_1 and P_3/P_1 , and the national income, Y , and is given by the following:

$$X_2^D = D \left(\frac{P_2}{P_1}, \frac{P_3}{P_1}, Y \right) \quad (2.81)$$

(−) (+) (+)

Commodity 2 is assumed to be a normal good with negative and positive own price and income elasticities of demand, respectively. The cross-price elasticity is positive.

Now, the national income, which is equal to the aggregate factor income in the present context, is expressed as

$$Y = WL + RN + rK \quad (2.82)$$

Finally, the demand–supply equality condition for good 2 is given by

$$D \left(\underset{(-)}{\frac{P_2}{P_1}}, \underset{(+)}{\frac{P_3}{P_1}}, \underset{(+)}{Y} \right) = X_2 \quad (2.83)$$

From Eqs. (2.75), (2.76) and (2.77), the three unknown factor prices, W , R and r , are determined as functions of P_2 . Then, X_1 , X_2 and X_3 are solved from (2.78), (2.79) and (2.80) as functions of P_2 . Y is found from Eq. (2.82). Finally, the equilibrium price of the non-traded good, P_2 , is solved from Eq. (2.83). Once P_2 is determined, all other endogenous variables are automatically determined.

2.6 Measurement of Social Welfare

The optimum social welfare depends on the commodity prices faced by the consumers and national income. When the commodity prices change, there are two effects on welfare – price effect and income effect. In such cases, national welfare should ideally be measured in terms of a strictly quasi-concave social welfare function since both the price and income effects can be captured by this function. However, in a small open economy which is a price-taker at the international market and where there is no non-traded final commodity, national income at domestic or world prices can be used as a good proxy for social welfare as it can capture the income effect. So, national income at world prices or domestic prices may be used for measuring social welfare only when the commodity prices do not change.¹⁰

2.6.1 National Income at World Prices as Measurement of Social Welfare

We consider a production structure where two goods, X_1 and X_2 , are produced with the help of labour (L) and capital (K). There is international trade, and X_1 is the export good, while X_2 is the importable good. Commodity 1 is chosen as the numeraire. The world price of good 2, P_2 , is determined in the international market. There is a tariff at the ad valorem rate, t , on the import-competing sector so that the domestic price of commodity 2 is $P_2(1 + t)$. Both the factors are fully employed and

¹⁰If there is a tariff on the import-competing sector, the domestic price of the commodity that the consumers face is different for its international price. Now, if the tariff rate changes, the domestic price of the commodity also changes which would alter the consumption levels of the final commodities due to price effect. Consequently, there would be a corresponding change in social welfare which remains unaccounted for if one attempts to analyse the welfare consequence of a change in the tariff through a change in national income.

are mobile between the sectors producing the two goods. The total capital stock in the economy consists of domestic capital (K_D) and foreign capital (K_F) and these are perfect substitutes.¹¹ Foreign capital income, rK_F , is completely repatriated where r is the return to capital. It is assumed that X_1 is more labour-intensive than X_2 so that $(a_{L1}/a_{K1}) > (a_{L2}/a_{K2})$.

The competitive profit conditions imply

$$a_{L1}W + a_{K1}r = 1 \quad (2.84.1)$$

$$a_{L2}W + a_{K2}r = P_2(1 + t) \quad (2.84.2)$$

The full-employment conditions of labour and capital are depicted by

$$a_{L1}X_1 + a_{L2}X_2 = L \quad (2.85.1)$$

$$a_{K1}X_1 + a_{K2}X_2 = K = K_D + K_F \quad (2.85.2)$$

The expression for national income at international prices, I , is given by

$$I = X_1 + P_2X_2 - rK_F \quad (2.86)$$

Differentiating Eq. (2.86) and assuming the initial stock of foreign capital to be equal to zero, the change in national income at world prices is given by

$$dI = [dX_1 + P_2dX_2 - rdK_F] = [dX_1 + P_2^*dX_2 - tP_2dX_2 - rdK_F]$$

or

$$dI = [F_L^1dL_1 + F_K^1dK_1 + P_2^*F_L^2dL_2 + P_2^*F_K^2dK_2 - tP_2dX_2 - rdK_F] \quad (2.86.1)$$

[Here, note that $X_1 = F^1(L_1, K_1)$ and $X_2 = F^2(L_2, K_2)$ are the two production functions.]

$$\begin{aligned} dI &= [WdL_1 + rdK_1 + WdL_2 + rdK_2 - rdK_F - tP_2dX_2] \\ &= [W(dL_1 + dL_2) + r(dK_1 + dK_2) - rdK_F - tP_2dX_2] \end{aligned} \quad (2.86.2)$$

¹¹This simplified assumption has been made in Brecher and Alejandro (1977), Khan (1982), Grinols (1991), Chandra and Khan (1993), Gupta (1997), etc. However, in the papers of Beladi and Marjit (1992a, b) and Marjit and Beladi (1996), foreign capital has been treated differently from domestic capital, and these two types of capital are not engaged in the same sector of the economy.

In Eq. (2.86.2), tP_2dX_2 measures the change in the distortionary cost of tariff protection of the supply side.

Also note that the full-employment conditions for the two inputs, labour and capital, are $L_1 + L_2 = L$ and $K_1 + K_2 = K_D + K_F = K$.

When there occurs an inflow of foreign capital, given the labour endowment, we have $[(dL_1 + dL_2 = dL = 0 \text{ and, } dK_1 + dK_2 = dK = dK_F)]$. Then, from (2.86.2) we find that

$$dI = -tP_2dX_2 \quad (2.86.3)$$

2.6.2 National Income at Domestic Prices

The expression for national income at domestic prices, Y , is given by

$$Y = X_1 + P_2^*X_2 + tP_2M - rK_F \quad (2.87)$$

where M denotes the volume of import of commodity 2 and is given by

$$M = D_2(P_2^*, Y) - X_2 \quad (2.88)$$

So, tP_2M is the tariff revenue collected by the government which is transferred to the consumers in a lump-sum manner.

Differentiating Eq. (2.87) and keeping t and P_i s unchanged, one gets

$$dY = [dX_1 + P_2^*dX_2 + tP_2dM - rdK_F] \quad (2.87.1)$$

Differentiating (2.88) (holding t and P_2 constant) and using (2.87.1), we find

$$dM = \left(\frac{\partial D_2}{\partial Y} \right) [dX_1 + P_2^*dX_2 + tP_2dM - rdK_F] - dX_2 \quad (2.88.1)$$

Differentiating the production functions and the full-employment conditions, Eq. (2.87.1) may be expressed as follows:

$$\begin{aligned} dY &= [F_L^1dL_1 + F_K^1dK_1 + P_2^*F_L^2dL_2 + P_2^*F_K^2dK_2 + tP_2dM - rdK_F] \\ &= [WdL_1 + rdK_1 + WdL_2 + rdK_2 - rdK_F + tP_2dM] \\ &= [W(dL_1 + dL_2) + r(dK_1 + dK_2) - rdK_F + tP_2dM] \end{aligned}$$

or

$$dY = tP_2dM \quad (2.87.2)$$

Using (2.87.2), Eq. (2.88.1) may be expressed as

$$dM = tP_2 (\partial D_2 / \partial Y) dM - dX_2$$

$$\text{or } dM[1 - tP_2(\partial D_2/\partial Y)] = -dX_2$$

or

$$dM = -VdX_2 \quad (2.88.2)$$

where $m = P_2^*(\partial D_2/\partial Y)$ is the marginal propensity to consume commodity 2 with $1 > m > 0$ and $V = [(1+t)/\{1+t(1-m)\}] > 1$.

Using (2.88.2) from Eq. (2.87.2), one can easily derive the following expression for change in national income at domestic prices:

$$dY = -tP_2 V dX_2 \quad (2.87.3)$$

2.6.3 Social Welfare Function

Each individual in the society derives positive utility from consumption of the two goods produced in the economy. It is assumed that the individuals are homogeneous in their preferences, so that the strictly quasi-concave social welfare function is given by

$$U = U(D_1, D_2) \quad (2.89)$$

where D_i denotes the demand for the i th commodity for $i = 1, 2$.

Given that international trade occurs, trade balance requires

$$D_1 + P_2^* D_2 = X_1 + P_2^* X_2 + tP_2 (D_2 - X_2) - rK_F \quad (2.90)$$

where $(X_1 - D_1)$ is the amount of export of X_1 and $(D_2 - X_2)$ denotes the amount of X_2 that is imported.

Differentiating Eqs. (2.89) and (2.90), the production functions and the import demand function, it can be shown that¹²

$$\left(\frac{dU}{U_1} \right) = tP_2 V [HP_2 dt - dX_2] \quad (2.91)$$

where $H = [(\partial D_2/\partial P_2^*) + D_2(\partial D_2/\partial Y)] < 0$ is the Slutsky's pure substitution term. m and V have already been defined in Sect. 2.6.2.

¹²This has been done in details in Chapter 2 of Chaudhuri and Mukhopadhyay (2009).

From (2.91) we find that

$$\left(\frac{1}{U_1}\right) \left(\frac{dU}{dK_F}\right) = -tP_2V \left(\frac{dX_2}{dK_F}\right) \quad (2.91.1)$$

2.6.4 Labour Market Imperfection and Welfare

Now, let us assume imperfection in the labour market. Let sector 2 producing X_2 be the formal sector with unionized wage, while sector 1 that produces X_1 be the informal sector offering the competitive wage.

Let the unionized wage function in the formal sector be as follows:

$W^* = W^*(W, Z)$ with $W^* > W$, $(\partial W^*/\partial W) > 0$ and $(\partial W^*/\partial Z) > 0$.

Here, Z denotes the bargaining power of the trade unions in sector 2.

Equation (2.84.2) has to be replaced by the following:

$$a_{L2}W^* + a_{K2}r = P_2(1+t) \quad (2.84.3)$$

In this case $P_2^*F_L^2 = W^*$, and the expressions (2.86.2), (2.87.3) and (2.91) would respectively become

$$dI = (W^* - W) dL_2 - tP_2 dX_2 = [(W^* - W) a_{L2} - tP_2] dX_2 \quad (2.86.4)$$

$$dY = V [(W^* - W) a_{L2} - tP_2] dX_2 \quad (2.87.4)$$

and,

$$\left(\frac{dU}{U_1}\right) = V \left[tH(P_2)^2 dt + \{(W^* - W) a_{L2} - tP_2\} dX_2 \right] \quad (2.91.2)$$

In the presence of labour market distortions, the expressions for changes in national income at world prices, national income at domestic prices and in social welfare, with respect to a change in foreign capital stock, are given as follows:

$$\left(\frac{dI}{dK_F}\right) = [(W^* - W) a_{L2} - tP_2] (dX_2/dK_F) \quad (2.86.5)$$

$$\left(\frac{dY}{dK_F}\right) = V [(W^* - W) a_{L2} - tP_2] \left(\frac{dX_2}{dK_F}\right) \quad (2.87.5)$$

and

$$\left(\frac{1}{U_1}\right) \left(\frac{dU}{dK_F}\right) = V [(W^* - W) a_{L2} - tP_2] \left(\frac{dX_2}{dK_F}\right) \quad (2.91.3)$$

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