Preface

There has been a renewed interest in the study of nonlinear dynamical systems during the last two decades. This has been partly because of "Chaotic Behavior" which is associated with the study of Dynamical Systems. Researchers from varied disciplines have seen the power of the various techniques invented in this period, both geometric and qualitative. With these new techniques stunning results have been achieved. They could visualize extremely difficult problems, from an analytic point of view using geometric and qualitative methods. Many nonlinear systems have randomness and chaos built into them. This feature has been brought out very well using geometric techniques.

The study of the Qualitative Theory of Dynamical systems began with Poincaré in his study of Celestial Mechanics (1899). For Poincaré a study of the global behaviour was more important than their local behaviour. Birkhoff adopted many of Poincaré's views and developed the theory further in the beginning of the twentieth century. Lately, many striking results have emerged mostly because of the pioneering work of Smale and the "Smale Horseshoes".

This theory is with regard to a set S which is diffeomorphic to a rectangle in a two-dimensional manifold M and a diffeomorphism $f : S \to M$. Starting with the first the iterate of f on S, conclusions about all iterates of f can be made. Of particular interest is the conclusion of Smale about the existence of a compact invariant set in S which is homeomorphic to a shift on M symbols. In this book we will call M the "crossing number".[1] The definition of this number is a fundamental concept in our analysis of chaos present in dynamical systems.

[1] Kennedy, J., Yorke, J.A.: Topological horseshoes. Trans. Amer. Math. Soc. **353**, 2513–2530 (2001).

However, Smale's assumption of hyperbolicity of f on S is in practice slightly difficult. It is our contention that even without the hyperbolicity assumption, the dynamics remain just as rich as before. This is with respect to the existence of a compact invariant set in S that factors over a shift on M symbols, although the map is not one-to-one. Some authors[2] refer to these as "geometric horseshoes" and the term "topological horseshoes" is also often used. (The authors discuss situations where stable and unstable manifolds cross but not necessarily transversally. They come to the conclusion that there is a geometric horseshoe for some iterate of the map.)

Hence, more general and less stringent definitions of a horseshoe have been suggested so as to reproduce some geometrical features typical of the Smale horseshoe while leaving out the hyperbolicity conditions associated with it. This led to the study of the so-called *topological horseshoes*.

This is the approach used in this monograph. In this work, phase-plane analysis, combined with results from the theory of topological horseshoes and linked twist maps are used to prove the existence of chaotic dynamics.

A small comment on the Linked Twist Maps (for short, LTMs): This is a geometric configuration characterized by the alternation of two planar homeomorphisms (or diffeomorphisms) which twist two circular annuli instead of a single map that expands the arcs along a domain homeomorphic to a rectangle, intersecting in two disjoint generalized rectangles A and B. Both the maps act in their domain so that a twist effect is produced. The presence of chaos-like dynamics for a vertically driven planar pendulum, a pendulum of variable length, and other, more general, related equations is proved.

This monograph emphasizes some mathematical aspects of the theory of dynamical systems. A certain level of mathematical sophistication would be useful throughout the text. During the last few years, several good textbooks on nonlinear dynamics have appeared for graduate students in applied mathematics. A majority of books are with a theoretical approach. Several practical issues remain unclear for application of the theory to particular research problems. This book is oriented towards advanced undergraduate or graduate students in mathematics doing applied research. It is also useful to professional researchers in physics, biology, engineering, and economics who use dynamical systems as modeling tools in their studies. Hence a moderate mathematical background in geometry, linear algebra, analysis, differential equations, and dynamical systems is required. Wherever necessary simple mathematical tools are used.

[2] Burns, K., Weiss, H.: A geometric criterion for positive topological entropy. Comm. Math. Phys. **172**, 95–118 (1995).

The book intends to provide the student (or researcher) with a solid basis in dynamical systems theory. It would be also useful to researchers in Chaotic Dynamical Systems as it offers a different route to Chaos. Graduate students could profit from it.

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