Preface to the Second Edition

Following the formulation of the laws of mechanics by Newton, Lagrange sought to clarify and emphasize their geometrical character. Poincaré and Liapunov successfully developed analytical mechanics further along these lines. In this approach, one represents the evolution of all possible states (positions and momenta) by the flow in phase space, or more efficiently, by mappings on manifolds with a symplectic geometry, and tries to understand *qualitative* features of this problem, rather than solving it explicitly.

One important outcome of this line of inquiry is the discovery that vastly different physical systems can actually be abstracted to a few *universal* forms, like Mandelbrot's fractal and Smale's horse-shoe map, even though the underlying processes are not completely understood. This, of course, implies that much of the observed diversity is only apparent and arises from different ways of looking at the same system. Thus, modern nonlinear dynamics[1] is very much akin to classical thermodynamics in that the ideas and results appear to be applicable to vastly different physical systems.

Chaos problem, which occupies a central place in modern nonlinear dynamics, refers to a deterministic development with chaotic outcome. Computers have contributed considerably to progress in chaos research via impressive complex graphics. However, this approach lacks organization and therefore does not afford complete insight into the underlying complex dynamical behavior.[2] Study of this dynamical behavior mandates concepts and methods from such areas of mathematics and physics as nonlinear differential equations, bifurcation theory, Hamiltonian dynamics, number theory, topology, fractals, and ergodic theory. Ergodic theory makes use of probabilistic methods to consider ensembles of trajectories rather than the detailed behavior of individual trajectories.

This book has grown out of my lecture notes for an interdisciplinary graduate level course on nonlinear dynamics I have taught for the past several years to a mix

[1]In view of the ubiquity of nonlinear phenomena (along with the scarcity of linear phenomena), it is rather awkward to call the subject nonlinear dynamics. As Stanislaus Ulam put it (Gleick 1987), "Calling the subject nonlinear dynamics is like calling zoology 'non-elephant studies' ".

[2]Indeed, Victor Weisskopf (1991) remarked, "We should not be content with computer data. It is important to find more direct insights into what a theory says…".

of students in applied mathematics, physics, and engineering. I have endeavored to describe the basic concepts, language and results of nonlinear dynamical systems. This book is accessible, therefore, to first-year graduate students in applied mathematics, physics, and engineering, and is useful, of course, to any theoretically inclined researcher in the physical sciences and engineering. In order to facilitate an interdisciplinary readership and highlight physical implications, I have adopted an informal style, kept the mathematical formalism to a minimum, and omitted much detail. Thus, I have abandoned the abstract language of bundles and connections and have stated a few theorems for which the proofs are only sketched or are replaced by illustrative examples which make the general principles plausible.

The book starts with a discussion of nonlinear ordinary differential equations, bifurcation theory and Hamiltonian dynamics. It then embarks on a systematic discussion of the traditional topics of modern nonlinear dynamics—integrable systems, Poincaré maps, chaos, fractals and strange attractors. The Baker's transformation, the logistic map and Lorenz system are discussed in detail in view of their central place in the subject. There is a detailed discussion of solitons centered around the Korteweg-de Vries equation in view of its central place in integrable systems. Then, there is a discussion of the Painlevé property of nonlinear differential equations which seems to provide a test of integrability. Finally, there is a detailed discussion of the application of fractals and multi-fractals to fully-developed turbulence—a problem whose understanding has been considerably enriched by the application of the concepts and methods of modern nonlinear dynamics. On the application side, one may discern a special emphasis on some aspects of fluid dynamics and plasma physics reflecting my involvement in these areas of physics. I have provided a few exercises that range from simple applications to occasional considerable extension of the theory. Finally, the list of references given at the end of the book contains primarily books and papers I have used in developing my lecture material (and this book).[3] This list is not to be construed as complete and some important papers may be missing.

The Second Edition, in addition to constituting an extensive rewrite of the text, incorporates the following features,

- refinement and enhancement of the clarity and precision of the text;
- extensive updating and amplification of several items in the book;
- addition of new material like theory of nonlinear differential equations, Lagrangian chaos in fluids, solitons and critical phenomena perspectives on the fluid turbulence problem;
- addition of new exercises.

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[3]One is reminded in this context of the observation made by Sheldon Glashow (Zee 2003)—"Tapestries are made by many artisans working together. The contributions of separate workers cannot be discerned in the completed work, and the loose and false threads have been covered over."

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