Preface

In 1926, Lukasiewicz proposed the following problem in his seminar at the University of Warsaw: to find a new formalization of classical predicate logic that could mirror the use of assumptions in real life mathematical arguments and reasoning. Jaskowski took up the task and formulated for the first time a system of Natural Deduction. The system was presented in 1927 at a meeting whose proceedings were published in 1929. The journal paper was published in 1934 under the title "On the Rules of Suppositions in Formal Logic".

In the same year Gentzen published his classical paper "Untersuchungen über das logische Schliessen" where we can find a different Natural Deduction system for both classical logic and intuitionistic logic. In Gentzen's Natural Deduction systems each logical connective has an *introduction* rule showing how to introduce an instance of the connective into a proof, and an *elimination* rule showing how to obtain a consequence of a sentence with that connective as main sign in a proof. These paired introduction and elimination rules provide a self-contained and intuitive explanation of the meaning of each connective. But these paired rules could also produce some "roundabouts" in proofs and Gentzen's *Hauptsatz* idea was that these roundabouts could be eliminated from proofs: Gentzen himself described how this idea could be carried out for intuitionistic predicate logic. But Gentzen's main goal was the consistency of classical arithmetic and for this reason the theoretical fate of natural deduction systems could have been sealed by Gentzen's prescient commentary:

> The Hauptsatz holds both for classical and for intuitionistic predicate logic. In order to be able to enunciate and prove the Hauptsatz in a convenient form, I had to provide a logical calculus especially suited to the purpose. For this the natural calculus proved unsuitable. For, although it already contains the properties essential to the validity of the Hauptsatz, it does so only with respect to its intuitionistic form, in view of the fact that the law of the excluded middle, as pointed out earlier, occupies a special position in relation to these properties.

As Gentzen said, although in the intuitionistic case the natural calculus had the desired properties for the formulation and proof of the Hauptsatz, classical logic formulated with the rule of double negation elimination or through the addition of the axiom of the excluded middle did not seem to possess the essential properties "for the validity of the Haupsatz". This was the main reason for the development of a new calculus, the *sequent calculus*, for which the Haupsatz was formulated and proved for both classical and intuitionistic logic. From the founding fathers (Gentzen and Jaskowski) to 1965, studies on natural deduction systems were basically restricted to (1) their use as a good deductive system for teaching logic, and (2) to problems related to the formulation of the rules for the quantifiers, especially to the rules corresponding to universal generalization and to existential instantiation. This situation changed dramatically 30 years later, when in 1965 Dag Prawitz and Andrés Raggio proved, independently of each other, the normalization theorem for classical first order logic.

This process of proof normalization, and its later extensions to higher order logics, was a breakthrough that paved the way for numerous fruitful applications of proof theory and logic to computer science. In particular, the Curry-Howard isomorphism showing the intimate connections between Natural Deduction and Type Theory, and especially proof normalization and type equivalence, laid the groundwork for the development of type systems in programming languages. This not only led to the creation of new programming languages, but also fed back into the development of new logics. The Extended Curry-Howard Correspondence furthered the connection between types, proofs, and programs to include morphisms in appropriate categories between programs and proofs. This algebraic dimension lent new mathematical insight into computational and syntactic issues in computer science, and stands to open up new areas of study in classical proof theory.

Natural Deduction is very dear to all of us involved with this book. The book grew out of the organization of a conference entitled simply "Natural Deduction" that took place in the Pontifical Catholic University of Rio de Janeiro (PUC-Rio) in 2001. The conference was conceived as a celebration of Dag Prawitz's work in logic, in particular of his work on proving normalization for logical systems and his careful philosophical discussion of the importance of proofs. Due to circumstances beyond our control, the project was shelved for several years, but as the possibility of resuming it presented itself, we jumped at the opportunity, as interdisciplinary and deep-meaning interactions between philosophers and computer scientists need to be fostered by all means at our disposal.

We have selected 12 papers associated with Natural Deduction, in an extended sense, as the theme of the conference and of this volume that we now briefly describe. The first contribution, by Schröder-Heister, reminds us that in 1979, Prawitz proposed a kind of operational completeness for the set of intuitionistic logical constants: he first defined abstract introduction and elimination schemes and claimed that any logical operator whose introduction and elimination rules were instances of these abstract schemes would be defined by the intuitionistic logical constants. This problem was solved by Peter Schröder-Heister who introduced for the first time higher-level natural deduction rules and in particular higher-level generalized elimination rules. In his contribution Schröder-Heister investigates some interesting relations between generalized higher-level elimination rules and standard-level generalized elimination rules and shows how some results for generalized higher-level elimination rules can produce new results in the standard-level, in particular a new left-introduction schema for implication can be defined from the interpretation of implication as rules.

Urban's paper re-addresses the problem of the correspondence between normalization steps in Natural Deduction and the corresponding reductions in Sequent Calculus, an issue going back to Kreisel's observations in 1971, heavily developed in Zucker's 1974 doctoral work. Returning to Zucker's investigation, Urban reproves Zucker's result that in the fragment of intuitionistic logic consisting of formulae build up from $\exists$ and $\forall$ only, every reduction sequence in natural deduction corresponds to a reduction sequence in the sequent calculus and vice versa. But this is proved in a much simpler way than Zucker's via terms associated to the sequent derivations, with the additional benefit that the cut-elimination procedure (from Urban 2000) is strongly normalising for all connectives, while the cut-elimination procedure in Zucker's work is not strongly normalising when $\bigvee$ and $\exists$ are included. Urban's work also shows that the negative result when $\bigvee$ and $\exists$ are included, is not because cut-elimination fails to be strongly normalising for these connectives, as asserted by Zucker, rather it is because there are cut-elimination reductions that do not correspond to any normalisation reduction.

Joinet's paper provides an insightful discussion on how logic changed in the twentieth century from a philosophical enterprise concerned with reasoning to a mathematical enterprise concerned with proofs, and more recently, to a computer science enterprise concerned with the dynamics of proofs. In particular he discusses the dynamic character of reasoning versus the static character of proofs; the referential dimension of reasoning versus the inferential nature of proofs and the indeterminacy of reasoning versus the correctness of proofs and reflects that this change of concerns (the metamorphosis in the title), especially the last one, from Mathematics to Computer Science, seen as a consequence of the Curry-Howard isomorphism, provides us with a *Computational Foundation for Logic*, which amazingly enough bring us back to a dynamic theory of reasoning as originally conceived by philosophers.

De Queiroz and de Oliveira's paper also discusses the origins of Natural Deduction, going back to Frege's introduction of variable-binding, and his idea of having terms representing incomplete "objects" whenever they contain free variables, as motivating their discussion of equality rules in first-order logic. De Queiroz and De Oliveira wish to see in first-order logic with equality a greater separation into two independent levels that they call the functional and the logical levels of the calculus. An analysis of how to keep these two levels of calculus independent, and yet harmonious, lead the authors to describe a system of Natural Deduction rules for propositional equality that sits between the extensional and the intensional versions of Martin-Loef's type theory. The paper contains a cogent discussion of why one would like to have a system which is a middle ground solution between the intensional and the extensional accounts of Martin-Loef's propositional equality. And then presents a system satisfying these required properties, via a notion of “rewrite reasons”.

Legris’ paper has a historical-conceptual nature. Its aim is to examine Paul Hertz logical theory and to show its influence and impact on Gentzen’s logical systems. According to Legris, Hertz’s work can be seen as an anticipation of a theory of proofs in the current sense. Legris also shows how Hertz’s work can be considered as a kind of bridge that would connect traditional logic to Gentzen’s systems. The paper concludes with an analysis of Hertz’s philosophical ideas concerning the relation between the essence of logic and more general epistemological issues.

In the literature logics modelling the idea of “generally” have been considered as extensions of First-Order Logic. Deductive systems for these logics can be obtained either by a usual system for first-order plus a set of axioms introducing the quantifiers as a kind of defined symbol, or by tailoring specific inference rules to deal with these new quantifiers. The article by Vana, Veloso and Veloso describes how to develop good Natural Deduction systems for the quantifiers corresponding to “generally”, “most” and “several”. This article is a comprehensive tract on the techniques used to deal with quantification notions derived from filters and ultra-filters. The systems presented are shown to be normalizable and the article discusses some consequences of normalization.

Anyone using a theorem prover or a proof assistant wants to have confidence that the system implemented will not allow the derivation of false results. Theorem provers (and systems used in their implementation) must ensure that it is not possible to derive contradictions and that the implementation itself is correct with respect to the original system. Professor Seldin’s article discusses clearly how the use of logical systems based on typed lambda-calculus are a most adequate basis for theorem provers when one wants to have confidence on the correctness of their implementation. Taking each of the many vertices presented in Barendregt’s lambda-cube, his article discusses how consistency and confidence on a given implementation can be obtained by means of basic observations on the main meta-properties of the lambda-calculi considered. The article is also a nice step-by-step presentation of how to improve the expressive power of a theorem prover by following paths along the edges of the lambda-cube.

Very traditional philosophical questions concerning the nature of propositional logic are taken up in Chateaubriand’s paper. Is logic a theory about something? Can one conceive of propositional logic as an ontological theory? What is the (epistemological) role of deductive systems? Do propositions have a structure in the same way that we can say that sentences have a structure? These questions become especially interesting and acute with respect to the connection between the material implication and the material conditional, a central topic in the development of natural deduction systems.

Since the work of Lambek, category theory has been used to provide semantics to substructural logics. This semantics is proof-theoretical, since proofs are incorporated in the models. Linear logic is a logic that interprets substructural implicational into intuitionistic implication by means of exponentials. There have been plenty of categorical models for providing semantics for linear logic. In this article, Valeria de Paiva compares several notions of categorical model of intuitionistic linear logic in the literature. Her article explains why choices can be made and what they amount to. Her conclusion is that one of the oldest and more complicated notions, namely linear categories are still the best way to describe models of linear logic, if one wants the correspondence between syntax and semantics to be as tight as possible.

Assertions and hypothesis are key notions in the Logic for Pragmatics introduced by Carlo Dalla Pozza and Claudio Garola. In his contribution to the present volume, Bellin reconsiders and revises some results that are based on the modal translation of his proposal for an intuitionistic logic for assertions and conjectures into classical S4: while assertions require epistemic necessity of the truth of the propositional content, doubts require the epistemic possibility of falsity and conjectures are justified by the possibility of epistemic necessity. In the final part of the paper, Bellin investigates the proof theory of a fragment of co-intuitionistic logic.

Anyone who has studied proof theory has already been exposed to strong normalization proofs. Compared with weak normalization, they are more complicated. They fall into to main proof patterns: (1) Using reducibility candidates, or; (2) Providing a direct proof that any sequence of reductions is terminating. The first case is known to be more elegant and more famous indeed. It was used in the case of stronger theories such as System F and Gödel's system T, for example. The second case is more informative in general, and has two forms: (1) Provide a special kind of reduction sequence that dominates any other reduction sequence and prove weak normalization for this (dominating) reduction sequence, or; (2) Prove directly by induction, using a tricky induction measure, that any reduction sequence is terminating. The first approach is also known as the worst-sequence approach, since the special sequence definition involved considers the longest possible reduction sequence for each term. Proofs of confluence can be combinatorial or semantical, in the case a stronger than termination/confluence inductive hypothesis is used. Zimmerman's article introduces a method of decomposition of reductions. He uses the terminology and main results from rewriting theory to show how one can decompose a reduction in such a way that its confluence and termination can be obtained from the termination and confluence of its constituents. This method is applied to Natural Deduction systems for Intuitionistic Logic and Intuitionistic Linear Logic. Zimmerman obtains termination and confluence in a rather elegant way by means of a general theory of reductions.

Forty years ago, Prawitz formulated a very interesting conjecture about the completeness of intuitionistic logic: are Gentzen's elimination rules for intuitionistic logic the strongest possible rules? This conjecture was formulated in the framework of a certain semantic approach to general proof theory. In his contribution to the present volume, Prawitz reconsiders this old conjecture and the very semantic approach that served as the base for the formulation of the conjecture. A revised notion of validity is defined and given that Gentzen's elimination rules are valid, the conjecture now assumes the form: are all valid inferences rules derivable in Gentzen's natural deduction system for intuitionistic logic?

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