Curriculum Design and Systemic Change

Hugh Burkhardt

Abstract This chapter describes and comments on the large qualitative differences between curriculum intentions and outcomes, within and across countries. It is not a meta-analysis of research on international comparisons; rather the focus is the relationship between what a government intends to happen in its society's mathematics classrooms and what actually does. Is there a mismatch? In most countries there is. Why? This leads us into the dynamics of school systems, in a steady state and when change is intended—and, finally, to what might be done to bring classroom outcomes closer to policy intentions. Two areas are discussed in more detail: problem solving and modeling, and the roles of computer technology in mathematics classrooms.

Keywords Curriculum change · Curriculum design · Curriculum goals · Curriculum implementation · Pushback · Modeling · Systemic change · Technology

"Curriculum" and Curriculum Change

The term "curriculum" is used with many different meanings. In the US it often means a textbook series, in the UK the set of experiences a child has in school classrooms. Neither of these fits the purposes of this chapter, which is concerned with the interrelations and differences between the variant definitions set out, for example, by the Second International Mathematics Study (Travers and Westbury 1989). I want to distinguish and compare the:

"*intended curriculum*": that described in official documents carrying the status of policy;

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"*tested curriculum*": the range of performances covered by the official tests, particularly when the results have serious consequences for students' or teachers' future lives;
"*implemented curriculum*": what is actually taught in most classrooms.

The "achieved curriculum", what most students actually learn, would take us into much too large a field of research. Other chapters address this.

Thus the focus in this paper is on the path from government intentions, usually set out in policy documents, to the actual pattern of teaching and learning activities in classrooms—some typical, some that are unusually innovative.

As always, studying the steady state tells you little about causation. Accordingly, I look at two areas where there has long been general international agreement on the need for change in mathematics curricula: problem solving and modeling, and the roles of computer technology. I have benefited from special issues of the *Zentralblatt für Didaktik der Mathematik*, in which distinguished authors from around the world describe what has happened over recent decades to problem solving and to modeling in their own curricula.[1]

Curriculum Goals in Mathematics

Around the world people seem to have much the same goals for the outcomes of a mathematics education. Students should emerge with a reliable command of a wide range of mathematical skills, a deep understanding of the concepts that underlie them, and an ability to use them, flexibly and effectively, to tackle problems that arise—within mathematics and in life and work beyond the classroom. Students should, as far as possible, find learning and using mathematics interesting and enjoyable.

If all these "goods" were commonly achieved, mathematics education would be just an interesting academic field of study, rather than a centre of social concern and political disputation. Far from that *nirvana*, we are still much closer to the historical picture of school mathematics. 100 years ago, there was good middle class employment for all those who could "do mathematics". Command of the procedures of arithmetic was enough for employment as a clerk or bookkeeper. Command of algebra, a rare accomplishment, gave access to professions like engineering or teaching. But in today's world, those skills are far from enough; arithmetic is largely done with technology, while jobs in finance require higher-level skills involving analysis of data and of risks, using prediction based on models—hence the widely agreed goals summarized in the first paragraph.

In seeking to get closer to these goals different groups have very different priorities—shown, for example, by the "as far as possible" in the sentence on

[1] My thanks to Kaye Stacey, Michel Doorman, Berinderjeet Kaur, Akihiko Takahashi and, particularly, Gabriele Kaiser, the editor of ZDM.

student feelings about mathematics. Indeed, one of the striking results from the Third International Mathematics and Science Study (TIMSS) is the *anti-correlation* between attitude and performance: East Asian students appear to combine high-achievement with a dislike of mathematics, stronger in both respects than those in lower-achieving countries.[2] It is fair to say that student enjoyment of mathematics, while seen as desirable, if only for motivation, is rarely given high priority. The next few paragraphs set out attitudes characteristic of various groups that promote their priorities for teaching and learning mathematics, more or less effectively.

"*Basic skills people*" focus on the importance of students' building fluency and accuracy in standard mathematical procedures, moving over time through the four operations of arithmetic on whole numbers, fractions and decimals to manipulating algebraic expressions. Calculators are for use in other subjects. This group recognizes the ultimate importance and satisfaction of being able to use these skills in solving problems that arise outside the classroom but they are happy to defer this until the procedural skills have been "mastered". For most students this gratification is deferred indefinitely. This curriculum consists of routine exercises, supplemented by routine "word problems". "Basic skills people" cannot understand why students find these problems so difficult.

"*Mathematical literacy people*" occupy the opposite end of the spectrum of priorities. They see mathematics as primarily a toolkit of concepts and skills that, learned and used properly, can help people understand the world better and make better decisions. They want students to develop their mathematics with close links to real world problems. They believe skills need to be rooted in solid conceptual understanding, so those that are not used every day can be refreshed when needed. They accept the research evidence (see e.g. Brown and Burton 1978) that successful performers do not remember *precisely* the procedures they have been taught but have the understanding to reconstruct and check them. Calculators and computers should be used freely. Understanding should be consolidated through concrete illustrations of the concept in action. This curriculum spends time on the development of modeling skills: formulation of mathematical models of new problem situations, transforming them to give solutions, the interpretation of solutions and of data, and explanation of what has been learnt.

"*Technology people*" start from the way mathematics is done outside the classroom—with the unquestioning use of computing devices. They believe that mental arithmetic is important for estimation but would only use pencil and paper for sketching diagrams and graphs, for formulating models, and for recording results. They accept the research that shows that concepts can be learned faster and understood more deeply through carefully designed uses of technology. They also believe that this research justifies the return of programming to the math curriculum.

[2]This, like every other statement in this chapter, is a trend; it is not true for everyone in each group.

They, too, would focus curriculum on rich problem situations, particularly from the real world.

"*Investigation people*" focus on mathematical reasoning and see the beauty of mathematics itself as the main driver for students to develop conceptual understanding and reasoning skill. They are less concerned with the real world, seeing the inexactitude of modeling as clouding that beauty. Their curriculum is dominated by a rich variety of mathematical microworlds that students are led to explore, discovering properties of and patterns in such systems—from "odd and even numbers" through "the 10 by 10 multiplication table" to non-commuting algebras. Skills are learned as they are needed and fluency built by their repeated use in diverse situations.

There is general acceptance that each of these aspects of learning mathematics should have a place; the balance of the intended curriculum in each school system reflects the tensions among these groups. Those mainly influenced by their own education tend to the first of the positions listed; more sophisticated thinkers about mathematical education tend to the later views.

The curriculum areas that I will discuss are examples where the mismatch between policy intentions and what happens in most classrooms is stark. "Problem solving" and "modeling" are suitable choices because, over many decades, the difference between declared curriculum intentions and the classroom outcomes has been not just large, but qualitative. "Technology" shows a striking double mismatch, both between aspirations and practice and between the real world and mathematics classrooms.

I will not discuss a universal priority area: the development of concepts and skills. Even here, there are mismatches: for example, all intended curricula recognize that conceptual understanding is important while, in contrast, learning procedural skills dominates in many classrooms. These matters are discussed in other chapters.

Problem Solving and Modeling

I have chosen "problem solving" and modeling[3] as the first area to study because for many decades these have been widely accepted goals for curriculum improvement in mathematics across much of the developed world. The need seems unanswerable; yet, observing at random in classrooms in any country, one is unlikely to see students engaged in tackling rich non-routine problems requiring substantial chains of autonomous reasoning by the student. In this section we outline something of the history in this area, looking for plausible explanations of the limited progress that has been made.

[3]Modeling, the now-standard term for the use of mathematics in tackling problems from the world outside mathematics, uses the same practices as mathematical problem solving—plus a few more.

What Is Problem Solving?

Why have I put "problem solving" in quotes? Because, even within mathematics curricula, the phrase is used by different people with different meanings. At its most basic level it is commonly used for "word problems" that are intended to be routine exercises presented in the form of a sentence or two; such word exercises normally appear in the curriculum unit where the method of solution is taught. I use "problem solving" in the very different sense, illustrated in Figs. 1 and 2,[4] that is now widely accepted in the international mathematics education community.

This defines a "problem" as a task that is:

Non-routine: A substantial part of the challenge is working out *how* to tackle the task. (If the student is expected to *remember* a well-defined method from prior teaching, the task is routine—an exercise not a problem.)

Mathematically rich: Substantial chains of reasoning, involving more than a few steps, are normally needed to solve a task that is worth calling a problem.

Well-posed: Both the problem context and the kind of solution required are clearly specified. (In an "investigation" the problem context is defined but the student is expected to *pose* questions as well as to answer them; investigations are implicit in the following discussion.)

Reasoning-focused: Answers are not enough; in problem solving students are also expected to *explain* the reasoning that led to their solutions and *why* the result is true.

These properties make a problem *more difficult* than a well-defined exercise on the same mathematical content. So, for a problem to present a challenge that is comparable to a routine exercise it must be *technically* simpler, involving mathematics that was taught in earlier grades and has been well-absorbed by the students. Problem solving depends on building and using *connections* to other contexts and to other parts of mathematics.

Various problem solving approaches to *Boomerangs* are shown in the samples of student work in Fig. 3, two of which show students "inventing" standard graphical and algebraic approaches to linear programming.[5]

From the above it will be clear that what is a problem depends on a student's prior experience. A problem becomes an exercise if the student has seen, or been taught, a solution. Equally, some rich curricula regularly present as problems some tasks

[4]These examples were developed by the Shell Centre/Berkeley *Mathematics Assessment Project*, see http://map.mathshell.org.uk/materials/index.php. The "expert tasks" under the "Tasks" tab epitomize problem solving. *Boomerangs*, Figs. 2 and 3, is from a MAP formative assessment lesson-lesson on problem solving.

[5]None of the solutions in Fig. 3 is fully correct and complete—a design choice that makes them a better stimulus for classroom discussion, because the students are put into a *critiquing* "teacher role", which is more proactive than merely *understanding* someone else's solution. The more sophisticated solutions are beyond most students' problem solving at this level, but are there to show the potential of more powerful mathematics.

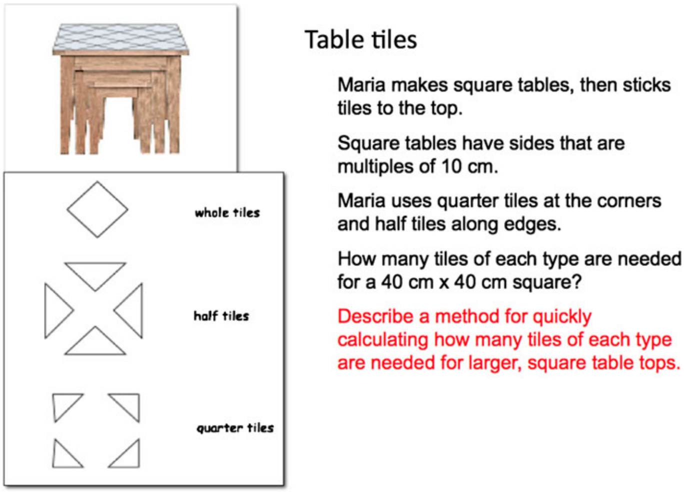

Fig. 1 *Table tiles*—a problem solving task

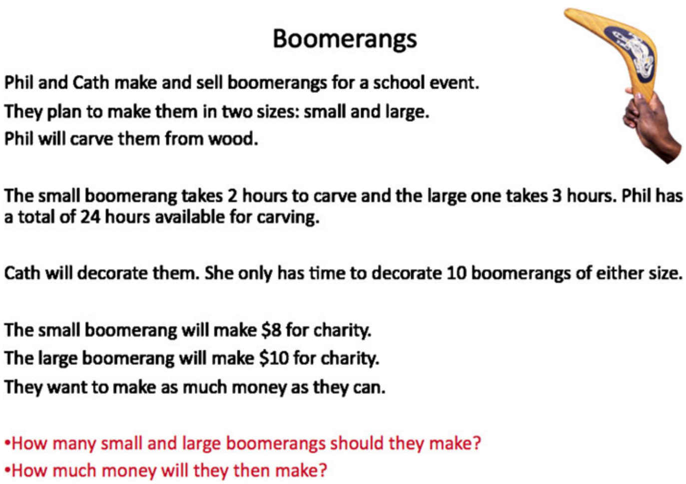

Fig. 2 *Boomerangs*—a problem solving task

that will become exercises when new techniques are taught in later years. For example, pattern generalization tasks like *Table Tiles* in Fig. 1 become exercises if and when students have learned the "method of differences". Similarly, the *Boomerangs* task in Fig. 2 becomes a straightforward exercise when you have been taught linear programming.

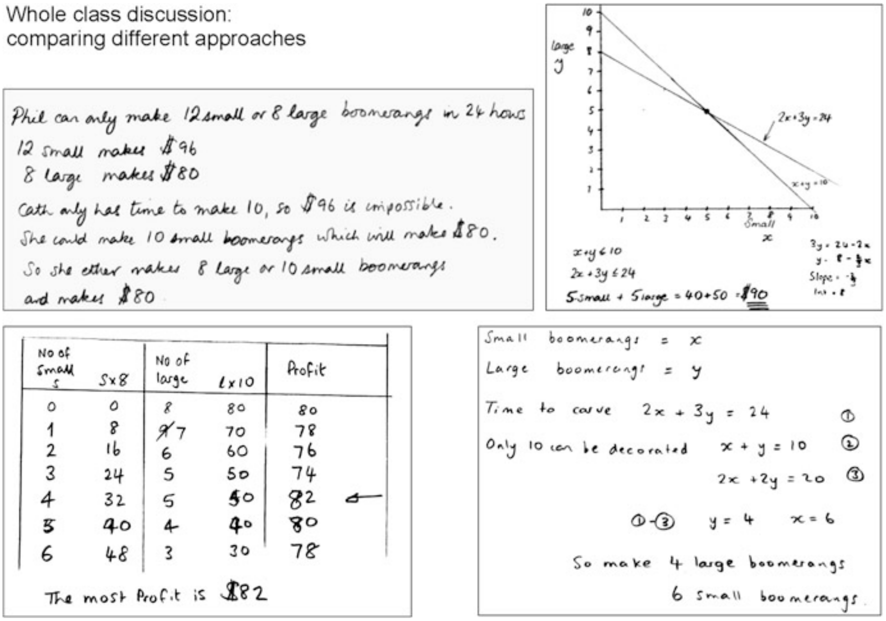

Fig. 3 Sample student solutions from the *Boomerangs* lesson

Problem Solving around the World

In 2007 ZDM produced special issues (Törner et al. 2007) in which contributors from around the world described the position of problem solving in their country's curriculum. The pictures presented were broadly similar. Problem solving is recognized as an element that *should* have a substantial place in the mathematics curriculum but, in practice, it plays little or no part in the pattern of learning activities in most classrooms. This subsection gives a flavour of my reading of the articles. These extracts are no substitute for reading these rich pictures of history, research and practice.

For **England**, Alan Bell and I offer a rather gloomy picture of a current situation, largely driven by the 1989 National Curriculum, which the then-government required to be based on "levels" described in terms of detailed content criteria. Because the difficulty of a task depends on many factors, the wish to give students their best chance of achieving a higher "level" led inevitably (see Burkhardt 2009, Sect. 2B) to testing the criteria in their simplest form—as short items on each criterion. Problem solving, still seen as important in principle, disappeared from the high-stakes public examinations and, consequently, from most classrooms.

We noted some hope of improvement through recent changes in the National Curriculum with an emphasis on "key processes" and an explicit recognition that non-routine problems have various sources of difficulty, as listed above. Improved examinations have now appeared in pilot form but performance on the problem solving tasks has been weak—not surprising since teachers have little experience in this area. The 1980s remain the high-point for problem solving and modeling in England,[6] though even then implementation was patchy.

From **Australia**, Clarke, Goos and Morony noted that the collaboration between states has, at various times, produced position statements that represent a form of national curricular consensus, including the view that

> Problem solving is the process of applying previously acquired knowledge in new and unfamiliar situations. Being able to use mathematics to solve problems is a major reason for studying mathematics at school. Students should have adequate practice in developing a variety of problem solving strategies so they have confidence in their use.

And yet

> video studies of grade 8 mathematics classrooms in Australia show little evidence of an active culture of problem solving.

Again the 1980s saw an outstanding development in problem solving through the VCE (Victoria Certificate of Education) school leaving examinations in Mathematics, which produced significant change throughout secondary schools (Clarke and Stephens 1996; Burkhardt 2009).

Current examinations (addressing students at different levels) are innovative and of high quality, containing tasks that probe concepts and skills. The lower level has a strong applications emphasis while all students have access to computer-algebra systems for part of the examination. But the tasks are essentially routine.

From the **Netherlands**, Doorman, Drijvers, Dekker, van den Heuvel-Panhuizen, de Lange, and Wijers present a similar picture.

> As in primary education, problem solving in secondary mathematics education has only a marginal position. In the introduction to this paper, it has already been pointed out that even an application and modeling-oriented curriculum like the one for Mathematics A tends to standardize problem-solving tasks into routine assignments. The national examination does not encourage paying much attention to problem solving skills. Textbooks usually do not address problem solving as a result of examination demands, designing teacher and student proof activities, and the time need for designing problem solving activities.

They report some exceptional textbooks and initiatives outside the mainstream, such as the national "Mathematics A-lympiad: an experimental garden for problem solving" (Freudenthal Institute 2010) which, in many schools, plays a role in the school-based component of national assessment. Many mathematics tasks are set in more realistic contexts than in other countries.

These authors make an important point—that, to stimulate and sustain problem solving in a curriculum, "an important challenge is the design of good problem solving tasks that are original, non-routine and new to the students". This is an ongoing challenge, at least until a population of tasks has been developed that is large enough for teaching them all to be an ineffective strategy (Daro and Burkhardt 2012).

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[6]Equally, until the 1950s the Geometry examinations for the highest achieving 20 % of 16 year old students included proofs of standard Euclidean theorems, each followed by a non-routine application of the theorem—an example of solving problems with a well-controled "transfer distance".

There are some interesting variations on the global trend sketched above.

From **Hungary**, Julianna Szendrei paints a more encouraging picture, albeit a mixed one. The examination at the end of secondary schools includes a non-routine problem as one of seven tasks. This influences some secondary school teachers to include such problems in the classroom as well. Lower secondary teachers prefer to use routine problems in the classroom. However, the government requires assessments at ages 10, 12 and 14 that contain problem solving as well. Though the results are not public, this motivates teachers to prepare children for problem solving.

Problem solving in the culture of Hungarian teachers also involves an approach to teaching: "not to show routine problems directly but to hide them a little".

> Let us prepare all the three digit numbers using the digits 2, 3, 5. Let us choose one of these numbers randomly. What is the probability of the event that the number will be odd?

Almost all Hungarian teachers know how to teach in this way but only about 10 % of them will do so in their classroom.

This looks rather like the picture from **China**, where Jinfa Cai and Bikai Nie write:

> The purpose of teaching problem solving in the classroom is to develop students' problem solving skills, help them acquire ways of thinking, form habits of persistence, and build their confidence in dealing with unfamiliar situations. Second, problem-solving activities in the classroom are used as an instructional approach that provides a context for students to learn and understand mathematics. In this way, problem solving is valued not only for the purpose of learning mathematics but also as a means to achieve learning goals.

They describe as typical the "teaching with variation" approach, in that the transition from routine problems is supported by gently increasing the transfer distance in various ways, including "...three problem-solving activities: one problem, multiple solutions; multiple problems, one solution; and one problem, multiple changes".[7]

> Situation. A factory is planning to make a billboard. A master worker and his apprentice are employed to do the job. It will take 4 days by the master worker alone to complete the job, but it takes 6 days for the apprentice alone to complete the job. Please create problems based on the situation. Students may add conditions for problems they create.
> Posed problems:
>
> 1. How many days will it take the two workers to complete the job together?
> 2. If the master joins the work after the apprentice has worked for 1 day, how many additional days will it take the master and the apprentice to complete the job together?
> 3. After the master has worked for 2 days, the apprentice joins the master to complete the job. How many days in total will the master have to work to complete the job?
> 4. If the master has to leave for other business after the two workers have worked together on the job for 1 day, how many additional days will it take the apprentice to complete the remaining part of the job?
> 5. If the apprentice has to leave for other business after the two workers have worked together for 1 day, how many additional days will it take the master to complete the remaining part of the job?

[7] The authors add "However, there is little empirical data available to confirm the promise of 'teaching with variation' ".

6. The master and the apprentice are paid 450 Yuans after they completed the job. How much should the master and the apprentice each receive if each worker's payment is determined by the proportion of the job the worker completed?

The picture presented here reflects an approach to teaching concepts and skills that can be found in other countries (see e.g. Swan 2006); it is a long way from the holistic problems exemplified in Figs. 1 and 2.

From **Japan**, Keiko Hino focuses on how ideals are reflected in approaches to lesson structure at a research level, partly reflected in lesson study, but reports on some evidence on its scale of implementation:

> The TIMSS video study identified the lesson patterns as cultural scripts for teaching in Germany, Japan, and the US (Stigler and Hiebert 1999). They identified the Japanese pattern of teaching a lesson as a series of five activities: reviewing the previous lesson; presenting the problem for the day; students working individually or in groups; discussing solution methods; and highlighting and summarizing the major points (p. 79). Here, a distinct feature of the Japanese lesson pattern, compared with the other two countries, was that presenting a problem set the stage for students to work on developing solution procedures. In contrast, in the US and in Germany, students work on problems after the teacher demonstrates how to solve the problem (U.S.) or after the teacher directs students to develop procedures for solving the problem (Germany). This pattern, or the motto of Japanese teaching, has been called "structured problem solving" by Stigler & Hiebert.

School leaving examinations are replaced by entrance examinations, set by different universities, that vary in difficulty. I find no suggestion that they involve non-routine problem solving.

From **Germany** Reiss and Törner describe an active program of curriculum and professional development on problem solving and, particularly, modeling that is "work in progress".

> The situation in Germany now parallels that of the United States some years ago. Stanic and Kilpatrick (1989, p. 1) get to the point when stating: Problems have occupied a central place in the school mathematics curriculum since antiquity, but problem solving has not. Only recently have mathematics educators accepted the idea that the development of problem-solving ability deserves special attention.

Finally, from the **USA**, as well as the implementation challenges, conflict over the *intended* curriculum has been a major factor. In the "math wars" a politically active group from outside mathematics education demand a curriculum focused on students' developing fluent manipulative skills. Alan Schoenfeld summarizes it thus:

> What optimism one might have regarding the re-infusion of problem solving into the US curriculum in meaningful ways must come from taking a long-term perspective.

The recent widespread adoption of Common Core State Standards that emphasize *mathematical practices* featuring reasoning, problem solving and modeling gives some grounds for hope—but, given all the political and institutional barriers, not for holding one's breath.

In reviewing these extracts, it is notable that the countries that give the most optimistic picture of implementation describe a relatively unambitious form of problem solving. The "teaching with variation" problems from China, for example, are rather like the "exercises with a twist that makes you think" that we see in England. Problems like the examples at the beginning of this section, involving more substantial chains of problem solving and reasoning, are still rare.

Problem Solving: The Challenges

Why is this pattern the way it is? What are the factors that impede the implementation of problem solving?

Testing traditions have a role, at least in those countries that have high-stakes tests. These have a strong influence on what is taught and valued in classrooms. Some people feel it is "unfair" to give students non-routine problems in tests, though evidence shows that score distributions for well-engineered tasks are similar to those for exercises. Designing non-routine tasks, year after year, presents a challenge to examination providers that they are happy to avoid; it is much easier to recycle minor variants of standard problems. However, since many countries have no high-stakes tests, this cannot be the main factor in the absence of problem solving.

Equity concerns play a role in most advanced societies. "We must give all kids the best chance to reach high standards". Since 'high standards' are usually seen in terms of the mathematical content covered, this supports the focus on short routine exercises. Further, since this fragmentation obscures the meaning of mathematics, it does not help disadvantaged students whose parents may not pressure them into persisting with, to them, meaningless activities in pursuit of long-term goals.

Difficulty Complex non-routine problems, which must be technically easier, make some people concerned that in problem solving "the math is not up to grade". They want students to be learning more techniques rather than "wasting time on stuff they already know". This issue is sometimes referred to as "acceleration" versus "enrichment".

Teaching challenges Handling non-routine problems in the classroom presents teachers with substantial challenges, both mathematical and pedagogical, that are not met in a traditional curriculum. Concepts and skills can be taught in the standard "XXX" approach: *explanation* by the teacher or the book, a worked *example*, then multiple imitative *exercises*. This teacher-centered approach cannot be used for problem solving, where students must work out their own approach to each problem.

Early materials to support teachers of problem solving simply provided teachers with some interesting problems and general guidance, based on the Polya (1945) "strategies" for problem solving. Schoenfeld (1985) showed that this is not enough; effective problem solvers need more detailed "tactics", elaborating the strategies for specific types of problem. For example, the strategy "Try some simple cases" is more powerful if you know what is "a simple case": perhaps "low *n*" in pattern generalization problems, but "end games" in game problems. More sophisticated and supportive materials have been developed over succeeding decades. We developed more powerful support for problem solving in "The Blue Box" (*Problems*

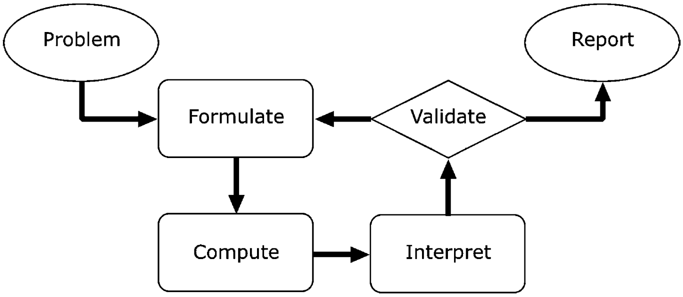

Fig. 4 The modeling process

with Patterns and Numbers, Swan et al. 1984), the first package to integrate examples of *examination tasks* with *teaching materials* and do-it-yourself *professional development materials* for teachers. This approach proved popular and effective. The sophistication of materials to support teachers in facing these challenges has developed over the last 30 years (see, for example, Swan et al. 2011). It is now fair to say that, as a field, we know how to enable typical teachers to handle non-routine problem solving in their classrooms.

System change challenges and how they might be more effectively tackled will be discussed below.

Modeling: the Further Challenges

Modeling justifies a separate coda to this section for three main reasons: it is the activity at the core of the *utility* of mathematics; its history has been rather different from that of pure mathematical problem solving; it is the focus of PISA, the now-dominant measure for international comparisons.

Modeling is problem solving that involves the processes, summarized in Fig. 4, that are involved in taking the world outside mathematics seriously. Real world problems are often messy—you can't address everything. Part of the modeling challenge is to identify the features of the situation that you need to analyze, select the essential relevant variables, and represent the relationships between them with mathematics. Only then will you have a well-posed mathematical problem to solve.[8] The ability to interpret the solution and evaluate the model requires an understanding of the practical situation and the ability to select what data is most relevant, to collect and analyze it.

All countries say that they want students to be able to use their mathematics in everyday life situations; yet the special issues of ZDM on modeling (Kaiser et al.

[8]The earlier examples, though they are related to practical problems, have been taken to this "well-posed" stage—the reason to call them problem solving not modeling.

2006) present a kaleidoscopic picture of work in the community of innovators (see also www.ICTMA.net) but little on modeling in typical classrooms. It is not unreasonable to infer that the situation is, at best, no better than for problem solving.

This fits the picture from other sources. For example, in ZDM Henry Pollak and I (Burkhardt and Pollak 2006) report on the history of modeling in England and the US over the last 50 years—from the diverse early explorations that we and others began in the 1960s, through the development of exemplar courses to the present day. We noted some hopeful signs: the growing awareness of the importance of *mathematical literacy* and the growth of PISA. Nonetheless, the impact in typical UK and US classrooms is minimal.

What are the factors, beyond those listed for problem solving, that impede implementation?

The real world is an unwelcome intruder in many mathematics classrooms. "I'm a math teacher, not a teacher of ....".[9] The clean abstraction of mathematics is something that attracted many mathematics teachers, particularly at the higher levels. Teaching mathematics, they say, is demanding enough without the messiness of modeling reality. This attitude also reflects concern about their ability to handle other areas of knowledge, at least with the same authority and control as for mathematics itself. In modeling, there are rarely "right answers". (There are wrong ones!)

Concerns about getting and handling real data which require skills that are new to many mathematics teachers.

The time modeling takes is a cause for concern for teachers facing curricula that are usually already too full. While all problem solving involves a time-scale longer than the few-minute exercises that dominate in many classrooms, it is possible to work through some interesting well-posed problems in 15 minutes or so. Modeling an interesting problem situation with attention to reality usually needs longer than this.

Again, the 1980s was a high point, with successes like *Numeracy through Problem Solving* (Swan et al. 1987-89), which supported students modeling real life problems in group projects.

Why "Technology" Remains Peripheral

Mathematics has long been done with microprocessor-based technology everywhere —except in the mathematics classroom. While a doctor from a century ago would be astonished and bemused in a hospital today, a teacher would be quite at home watching most current mathematics classrooms.

[9]This contrasts with the attitude of teachers of English, who welcome the opportunity to link the technical and stylistic aspects of language with the student's world.

In business, accounting, scheduling and stock control, not to mention check-out tills, are all computer-based. In industry, CAD-CAM systems are at the heart of design and manufacturing. Most routine repetitive tasks are done by computer-controlled machines. In research, where it all began, computers are everywhere. Why has school mathematics not changed to reflect this?

In addition to the roles of technology in *doing* mathematics, the last half-century has seen the development of a huge range of educationally powerful software for *learning* mathematics. The early efforts were behaviorist "learning machines", building fluency through simple exercises with instant feedback (and built-in testing). These "integrated learning systems" are still around, but the reinforcement they provide doesn't help those many children whose conceptual understanding is somewhat "dis-integrated". In contrast, the variety of software designed as a *supportive* learning resource is impressive. Perhaps most important for stimulating learning are the "microworlds" that offer domains for investigation by students.

The best known of these, because they cover a large domain, are the Euclidean Geometry programs: Judah Schwartz's *Geometric Supposer*, Jean-Michel Laborde's *Cabri Geometre*, Nick Jackiw's *Geometer's Sketchpad* and their followers. These enable an investigative approach to the learning of Euclidean Geometry in which the students play a much more active role than in the traditional learning of theorems and their proofs.

There are many smaller investigative microworlds that simulate specific situations in mathematics or science. From the late 1970s the British project ITMA, *Investigations on Teaching with Microcomputers as an Aid*, developed a wide range of such software. Rosemary Fraser's *Jane* is a "function machine" that invites conjecture and the weighing of evidence; this work showed that the concept of function as a consistent input-output process is natural for children no older than seven, providing a natural route into algebra through functions. Richard Phillip's *Eureka* is about a man taking a bath. It links a cartoon, a 4-command programming language[10] that controls the bath sequence, and a line graph of the depth of water against time. A research program on teachers use of these and other classics showed their power (Burkhardt et al. 1988). Teachers with little experience of handling non-routine problem solving in their classroom moved quite naturally from the traditional directive roles (manager, explainer, task setter) into the supportive roles that are essential for teaching problem solving (counselor, fellow student, resource). The single computer screen took on some of the traditional roles, hence the "teaching aid" name for this mode of use.

More familiar are computer game modes, some of which have significant mathematics content beyond behaviourist skills training.

The mathematical software tools themselves can be used to promote learning. Spreadsheets and programming languages provide environments that help students explore problems, and learn to design algorithms for modeling the real world, and

[10] T $\sim$ tap on/off, P $\sim$ plug in/out, M $\sim$ man in/out, and S $\sim$ sings/stops singing—because it is important to recognize that there are some variables that do not affect the quantity of interest, in this case water level.

for investigations in pure mathematics. As a curriculum element, such activities equip students with tools that will be used in life beyond the classroom.

Why are these powerful tools for doing and for learning mathematics still only used in a small minority of classrooms in most countries?

How effective these learning activities are depends on the teacher and, one would expect, on the textbooks that embody the intended curriculum. This brings us to a big surprise: there are no published mathematics curricula that exploit the potential of technology. Why? At least three powerful forces have contributed to this: cost, equity and, perhaps most important, mismatched timescales.

Timescale mismatch: The timescale of change for computer technology is short, with new devices appearing every few years; in contrast, curriculum changes take a decade or two from initial discussions to widespread implementation.[11]

Cost: When a new technology is introduced, it is expensive.[12] The cost of equipping every child in a class seems prohibitive. As the price comes down, new technologies appear that offer much greater educational possibilities. Each implies a substantial curriculum and professional development program, if teachers are to learn how to exploit its potential.

Equity: Students and school systems give a high priority to fairness, to trying to give all kids the same opportunities. The challenge of equipping all schools in a short time makes it difficult for school systems to require any specific technology as part of the intended curriculum.

And there is always conservatism. While the importance of technology in mathematics is accepted at a rhetorical level, when it comes to deciding on the intended curriculum, politicians are reluctant to abandon traditional goals.[13] Perhaps they find traditional values in education play well with electorates, particularly for mathematics where many parents feel insecure. “Look what it did for me”. But is fluency in pencil-and-paper arithmetic still a sensible priority for children, particularly those who struggle? In this sense, the mismatch for technology is different from that for problem solving and modeling in that, even in the intended curriculum, its place is far from clear.

There are signs that the situation may be changing. The basic tablet computer is getting cheaper and offers a stable platform that can offer a very wide range of support for learning and teaching. School systems are talking of “the post-textbook era” and publishers are responding by supporting the development of technology-based curricula.

[11]For example, in the 1980s the National Council of Teachers of Mathematics developed “standards”, setting out curriculum goals, the National Science Foundation funded the development of curricula and assessment in the 1990s, while substantial impact in classrooms began from 2000 onwards.

[12]The first 4-function calculator I used cost \$450 $\sim$ several thousand dollars in current money.

[13]Against the advice of the mathematics education experts, the British Government insisted on retaining fluent “long division” as an essential skill in the National Curriculum.

On the other hand, the timescale mismatch continues to present problems. Within a few years the focus has moved from laptops to tablets and smartphones. The design opportunities for any of these platforms are immense but they are rather different—and realizing their potential will take many times longer than publishers' deadlines usually allow.

The fundamental challenge remains: to move school mathematics closer to the way math is done in the world outside. Whatever happens next, this will remain an exciting area.

Systemic Change: Failures and Ways Forward

Why does this mismatch persist? Why are the activities in mathematics classrooms still so like those of a century ago? I have listed some of the factors that help to explain. In this section I will argue that the problems lie mainly at school system level, describe some important causes of failure in implementation, and suggest possible ways forward. Key failures include: underestimating the challenge; misalignment and mixed messages; unrealistic pace of change; pressure with inadequate support; inadequate evaluation in depth; and inadequate design and "engineering". A challenging list.

These deep-seated problems, involving as they do multiple constituencies with well-grooved attitudes and modes of working, have no well-established solutions. However, we know enough to set out a path that has real prospects of improving the convergence between intentions and outcomes. In the following, I shall discuss each of them in turn.

Underestimating the Challenge

When countries are concerned about education, there are intense and ongoing debates about *what* should go into the curriculum. There is much less discussion as to *how* to get it to happen. It is assumed that once the decisions have been made and a process of implementation specified, things will work out as intended.[14] Curriculum changes of the kinds discussed in this chapter involve fairly profound changes in the professional practice of many people across a range of constituencies: textbook writers, test designers, professional development leaders and, particularly, teachers. All these need to be not just *motivated* but *enabled* to meet the new challenges. Further, some may feel threatened, producing "pushback", overt or covert, against the change. To have a reasonable chance of realization, a change must have (at least)

[14] As the Mathematics Working Group finished its design of the original 1989 National Curriculum in England, I asked a senior civil servant why we should expect it to happen; she replied "But it's the law of the land"!

the consent of teachers, principals, curriculum directors, superintendents, the relevant professions, and the public. Some groups, within or outside the school system, may disapprove of the change and work to undermine it—the US "math wars" being an extreme example.[15]

Misalignment and Mixed Messages

It is important to avoid mixed messages by ensuring close alignment of learning goals, curriculum, teaching materials, professional development support, and assessment. A common problem arises when the curriculum intentions are broad and deep, the textbooks and professional development only partly reflect that, and narrow official tests have consequences for teachers or students. It is not difficult to guess which message is likely to influence teaching most strongly. Yet it is common to ignore the effect of high-stakes tests on the implemented curriculum, seeing them as "just measurement", and to underfund key elements, notably professional development. Progress will depend on enhancing awareness of the central importance of alignment and of the engineering needed to achieve it.

Unrealistic Pace of Change

The design of an implementation program has many aspects that clearly need attention, including all those mentioned above. One that is commonly ignored is the planned pace of change. This is often grotesquely misjudged, again due to a mismatch of timescales. Politicians feel the need to be seen to "solve" problems—and before the next election. As we have seen, the timescales for the design, development and implementation of new curriculum elements, assessments and professional development programs are much longer than this.

There is much to be said for an *ongoing* program of improvements of the kind that is seen as normal in other spheres of public policy: health care and the military, for example. There are many advantages in the incremental introduction of small but significant steps that address major weaknesses in the curriculum. Unlike "big bang" changes, this approach does not fundamentally call in question the established practice of the professionals, be they teachers, principals or the leadership of the school system. Professional development can be focused on the few weeks of challenging new teaching and learning involved. Most teachers find innovation on this scale stimulating and enjoyable; though many will be relieved to get back to the comfort zone of their established practice, they usually welcome the next increment when it comes along, six months or a year later. Most important, a qualitative

[15]Paul Black (2008) describes the process of consensus building across communities behind a successful curriculum innovation, Nuffield A-level Physics.

change that is modest in scale can be done well, in contrast with major changes that so often degenerate back into "business as usual". Burkhardt (2009) describes a successful example of this approach: the introduction of new task-types into a high-stakes examination, supported by teaching and professional development materials. The materials came to be known as The Blue Box (Swan et al. 1984) and The Red Box (*The Language of Functions and Graphs*, Swan et al. 1985).[16] They included exemplar test tasks, materials for the three weeks of teaching, and a do-it-yourself professional development package.

Gradual change approaches have been used in various ways. "Replacement units" have been used in California and elsewhere. The introduction of "coursework" into British examinations was of this kind: 25% of the examination score was based on student performances in class. Portfolio assessment was introduced in some US states. It is important to note that these and other successful initiatives have often not survived, often for unconnected reasons arising from systemic changes.

Pressure without Support

Pressure and support need to be balanced if improvement is to happen as intended. That both are important is widely accepted but the amount of each is often determined by financial and political considerations that are not guided by likely cost-effectiveness. Normally pressure costs less than support, so "accountability" systems, largely based on tests, are a favourite tool of policy makers. Conversely, effective support systems normally involve teachers and other professionals regularly working outside the classroom on their professional development.

Professional development support is recognized rhetorically as essential but implementation is almost always inadequate, constrained by politically-determined financial limits. Typically, a few sessions will be specifically funded, or it may simply be left to existing structures to fit new demands into their current programs, themselves usually inadequate. Yet the timescale for becoming an accomplished teacher of problem solving and modeling, or for learning about how to exploit the multiple opportunities that technology affords, is decade-long, with an ongoing need for professional development support.

Regular time for professional development in the teacher's week has financial and logistic implications. The main cost of an education system is the cost of having a teacher in every classroom, which reinforces the simplistic view that other activities are "time off" from a teacher's job. To an administrator an hour a week is 4 % increase in this major cost. Average class size, the complementary variable, is so

[16] The Blue and Red Boxes are still widely regarded as classics. In 2008, one of the first "Eddies", the $10,000 prizes for excellence in educational design of the *International Society for Design and Development in Education*, was awarded to Malcolm Swan, its lead designer, for The Red Box. (The other went to an Editor of this book.)

controversial that a small increase to compensate for professional development time is rarely discussed.[17]

In contrast to this attitude, "continuing professional development" for doctors is a requirement of their continuing license to practice, taken into account in financial planning.

Evaluation in Depth

The standards for evaluation of the outcomes of interventions are abysmal. Curriculum materials are reviewed by inspection, only rarely using evidence on their effect on student learning and attitude. Professional development programs are evaluated by the perceptions of those who took part, not on evidence of change in the teachers' classroom practice—presumably the key goal. Studies of effects on student learning often use tests that cover only a subset of the stated learning goals, usually using narrow state tests.

In education, there are no equivalents of consumer magazines like *Consumer Reports* that test products systematically, let alone government bodies like the US *Food and Drug Administration* (*FDA*) or the British *National Institute for Clinical Excellence* (*NICE*) which evaluate medicines. This reflects the limited acceptance that education can be a research-based field. Making it so depends on improving evaluation in both range and depth.

This situation reflects various factors. Studies in depth are expensive, involving observation and analysis of what happens in many classrooms, as well as the learning outcomes.[18] Yet it is only such studies that provide a sound basis for choosing curriculum materials and, even more important, the formative feedback to inform for the next phase of improvement.

Equally, there are not yet enough good instruments for such a program to provide a sound research basis for such judgments. Broad spectrum tests of mathematical concepts, skills and practices, including problem solving, modeling and other forms of mathematical reasoning have been developed, but there is no accepted set that most studies use. For professional development, we need better protocols for classroom observation and analysis.

In the absence of better evaluation tools and methods, studies have fallen back on inadequate measures that are widely accepted for quite different purposes, usually accountability. The evaluation picture for the NSF-funded curricula had to be pieced

[17] Japan, where a substantial part of the teacher's week is spent in lesson planning and lesson study, has larger classes. In the US and UK teachers and their unions are profoundly skeptical that the trade-off would be sustained. "They'll cut the PD again after a year without reducing the class sizes". This exemplifies a whole set of other system issues.

[18] I estimate that a thorough formative evaluation of some NSF-funded curricula and some traditional comparators would require funding comparable to the original development program, roughly $100 million.

together (Senk and Thompson 2003) from a large number of separate studies. Together they gave a result that was fairly unambiguous, but not clear enough to command the acceptance it deserved. The results on the widely accepted narrow tests were comparable with those from other curricula, but these tests did not assess the broader performance goals that were the raison d'etre for these curricula. The studies that showed substantial gains on broad spectrum tests did not receive the same attention, probably because they were fewer and the tests were "non-standard".

We need to go beyond this, to look behind the outcomes in depth at the range of what happens throughout the process, in classrooms, and in the associated professional development. We need to know how the outcomes depend on the processes and the variables: students, teachers, school and district environments, and system structures. This information will provide a sound basis for future development.

Design and "Engineering"

Realizing a planned curriculum change is an unsolved problem in most school systems; nonetheless, a lot is known about what to do and what *not* to do. The smooth implementation of a substantial change in the curriculum requires a pathway of change for all the key groups along which they can move. Ideally, all should feel that the change is, in a broad sense, in their interest; this limits pushback to outsiders—often formidable enough. A change program like this requires a well-engineered mixture of pressure and support on each of the groups involved, with the tools and processes that will enable all those involved to succeed. This is clearly a major design and development challenge; it is rarely recognized as such.

In a rational outcome-focused world, pressure and support should be developed with policy, with the goals matched with the resources available. However, this is a constraint that, in education, politicians are so far unwilling to contemplate. As a result of the political sense of urgency, policy decisions on innovation are usually developed with some "consultation" but without either exploratory design or careful development. Viewed strategically like this, it is not surprising that few changes work out as intended.

The last decade has seen the growth of a more organized community of professional designers in mathematics and science education, supported through the *International Society for Design and Development in Education* and its on-line journal *Educational Designer*. However, as we have seen, much more remains to be done to raise standards—above all, policy makers' awareness of the contribution that high-quality engineering can make to realizing their goals.

In Summary

This chapter has argued that we know enough, and have the tools, to enable typical teachers with reasonable support to deliver a mathematics education for their students that is vastly better than most of them get currently. That is good news. Less encouraging is the evidence that the major problems in the way of implementation are at system level, involving the factors just described.

Since design and development at system level is inevitably larger in scale than, for example, classroom studies, progress will require substantial commitment, probably at a political level. History in other fields suggests (Burkhardt 2006; Burkhardt and Schoenfeld 2003) that, while persuasion is important, large scale research funding will follow only from unmistakable examples of successful impact—like antibiotics in medicine or radar and operational research in military science. Breaking out of this "chicken and egg" situation will require the creation, identification and trumpeting of successful examples like some of those mentioned above.

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