Preface

Nowadays, mathematical and numerical modeling has become an essential component of the general scientific process. Ever since the 1960s, numerical analysis and scientific computation have made up the most rapidly growing part of mathematics. One of the challenging problems in this area is the creation of fully reliable computer simulation methods, which could become an adequate complement to experimental sciences. This book aims to give an overview of mathematical methods and computer technologies focused on reliable verification of computed solutions and present recently developed methods. We hope that it will be useful for an audience much larger than just advanced specialists in numerical analysis and computer simulation methods. In actuality, the book can be used in three different ways.

For engineers and specialists in natural sciences interested in quantitative analysis of mathematical models, it is best to concentrate on algorithms and prescriptions, which explain how to measure the accuracy of a numerical solution. In Chap. 2, we discuss various error indicators, which are used in mesh adaptive numerical algorithms in order to achieve proper restructuring (refinement) of the computational mesh (or changing the set of trial functions). We suggest a unified approach to this question and discuss different error indicators. Chapter 3 is concerned with the question: "how can guaranteed and computable bounds of errors associated with approximations of differential equations be derived?". We tried to explain this in simple terms without a deep excursion into the mathematical background. In other words, the reader whose main purpose is to use the results (estimates) will find the corresponding detailed recommendations. Certainly, they are given for a limited amount of typical problems. Other cases can be found in the literature cited or require additional analysis (in the latter case, a good understanding of the mathematical theory is necessary).

For advanced specialists interested in the development of new error estimation methods, Chaps. 3–5 are the most interesting. Here, we discuss mathematical technologies that provide guaranteed error control and applications to analysis of problems with uncertain data. These chapters essentially use materials exposed in the books P. Neittaanmäki and S. Repin [NR04] and S. Repin [Rep08] (in [NR04] the reader can find a complete set of a posteriori error estimation theory generated by the variational duality approach and [Rep08] is mainly devoted to the method using transformations of integral identities, which define generalized solutions of boundary value problems). We recommend them for further study of the mathematical theory of a posteriori error estimation. However, in this book (unlike the above-mentioned publications) we pay more attention to computational aspects and try to supply the reader with practical prescriptions. Chapter 5 is devoted to a special but important topic: analysis of effects caused by indeterminacy (incomplete knowledge) of problem data. It contains many new results. We show that studying problems with incompletely known data leads to conceptions and methods, which differ from those used in "classical" error analysis. In particular, they lead to the notion of an *a priori limited accuracy*, which leads to a new perspective on quantitative analysis of mathematical models. Chapter 5 and Sects. 4.1.2 and 4.1.3 (related to beams) use materials of the Ph.D. thesis of O. Mali [Mal11]. The material exposed in Chaps. 4 and 5 may be especially interesting for specialists in computational mechanics interested in finding bounds of the accuracy generated by approximation errors and data indeterminacy.

The entire book (maybe with the exception of Chaps. 4–6) can also be considered as a *textbook* for undergraduate and postgraduate students studying applied mathematics and mathematics of computations. For these reasons, we append three chapters (Appendices A, B, and C), in which basic mathematical knowledge is summarized. These chapters present a concise lecture course "Numerical analysis of differential equations" (which has been developed by the authors for graduate and undergraduate students of the University of Jyväskylä). It discusses the main methods used for quantitative analysis of partial differential equations. Chapters 2 and 3 are also written in the textbook style. Here, we have used materials from lecture courses on a posteriori error estimation methods that have been delivered to undergraduate and postgraduate students by S. Repin in Jyväskylä, Radon Institute of Computational and Applied Mathematics in Linz, Helsinki University of Technology, and University of Saarbrucken.

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Many materials related to theoretical justification and practical implementation of new a posteriori error estimation methods are results of joint research exposed in joint publications with our colleagues, which are referred to in the respective parts of the book. We express sincere gratitude to all of them for the cooperation and interesting discussions. Finally, we would like to thank Springer-Verlag publishing group for the friendly cooperation.

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Olli Mali
Pekka Neittaanmäki
Sergey Repin

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