

Chapter 2

SOME SURPRISING INVARIANCES

2.1 Introduction

The colour of the experimenter's hair does not generally influence the outcome of his experiment, any more than would the positions of the planets or the value of some stock market indicator. So why, if there are so many of them, should we point out factors which have no influence over the phenomenon being studied?²² When teaching an idea within the text of a standard exercise, it is normal practice to mention only that which is strictly “relevant”, and, in most cases, identified with the variables whose symbols appear in the algebraic expressions used. This results in a draconian selective filtering process. The history of ideas is often one of abandoning parameters hitherto thought to be relevant. The typical wording of exercises reflects this acceptance and adds a layer of simplification for other parameters which should, in principle, be taken into account: how many times have we seen “ignoring friction....”? It's already complicated enough, we hear them say.

Temporarily leaving aside the question of a sometimes over-simplified model, it's worth reflecting on what we don't say on what doesn't count. Or rather, on what we don't emphasise, as such. The value of an invariance (or a non-dependence) lies in its surprising character.

2.2 The speed of light *in vacuo*

It is fairly well known that light in a vacuum, or in air, which is pretty much the same, travels very fast. Its speed, denoted c , is very close to 300,000 km/s ($c = 2.99792458 \cdot 10^8 \text{ m.s}^{-1}$). A common symbol and a value: but have all aspects of

22 S. ROZIER explored this question in 1983 (*L'implicite en physique : les étudiants et les fonctions de plusieurs variables*, mémoire de tutorat (year 5 at university) DEA de didactique, Université Paris Diderot). VIENNOT L. (1982) L'implicite en physique : les étudiants et les constantes, *European Journal of Physics*, **3**, 174-180; and VIENNOT L. (2001) *Reasoning in Physics - The part of common sense*, Dordrecht: Kluwer Ac. Pub., Chapter 9.

this question been considered? If we go into it a bit more deeply, we end up with considerations which are a lot more interesting than that.

The speed of light? What kind of radiation are we talking about? For visible light, everyone knows that this involves an infinite number of wavelengths (or frequencies, if you prefer) each corresponding to a colour. The same remains true outside the visible spectrum (radio waves for example), and we no longer simply talk about colour (though often keeping the adjective “monochromatic” for a given wavelength). Do all these waves travel at the same speed?

The second thing to bear in mind is that when referring to speed, we usually have to state the reference frame in which it is measured.

In fact, if we don't mention it, it is because it doesn't count. For all wavelengths (*i.e.* for all frequencies), electromagnetic radiation, visible or otherwise, has the same speed (“phase velocity” in scientific terms) in a vacuum and in all Galilean reference frames.²³ This last point is particularly startling, and it took some time to convince the world of it. Associated with this astonishing observation are, from 1881 onwards, the celebrated names of MICHELSON and MORLEY, but it was not until 1905 that EINSTEIN in his famous paper on *Special Relativity*²⁴ finally settled the question.

At the start of university, students are, for the most part, able to say that the speed of light is constant, and can give a value for c . However, many do not know, or at least cannot express, the invariances we have just reviewed,²⁵ and that is why it is worth emphasising them.

2.3 Propagation of mechanical signals

A mechanical signal such as a disturbance propagating on a string or a sound in the air can be described by a wave whose speed depends only on the medium.²⁶ Let us now

23 An infinite number of reference frames in mutually rectilinear uniform translation in which NEWTON's laws apply for objects moving much slower than the speed of light.

24 EINSTEIN A. (1905) Zur Elektrodynamik bewegter Körper, *Ann. d.Ph.*, **17**, 892-921 (translation by SOLOVINE, GAUTHIER-VILLARS, 1955, **5**). See also: EINSTEIN A. (1907) Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen, *Jahrbuch der Radioaktivität*, **4**, 411-462 & **5**, 98-99.

25 VIENNOT L. (2001) *Reasoning in Physics - The part of common sense*, Dordrecht: Kluwer Ac. Pub., 155-156.

26 This is in a “non-dispersive” medium in which waves of varying frequency propagate at the same speed, the form of the “bump” being preserved. The expression for the phase velocity (or celerity) of sound is $c = \left[\frac{RT\gamma}{M} \right]^{1/2}$, where R is the ideal gas constant, M is the molar mass of the gas, T is the absolute temperature and γ is a coefficient which depends on the number of atoms in the gas molecule and on T . The speed of propagation of a disturbance on a string is $c = \left[\frac{T}{\mu} \right]^{1/2}$ where T is the tension and μ the mass per unit length of the string.

ask what happens if the source of the signal is more powerful—if the string is struck “harder” or someone shouts louder: a good proportion of people questioned²⁷ predict that the disturbance will propagate faster. And this, despite the fact that the commonly taught formulation (*i.e.* the speed depends only on the medium) is well known to them. What is certain is that they just have not fully realised what this means *i.e.* a surprising thing: that propagation speed is independent of the power of the source.

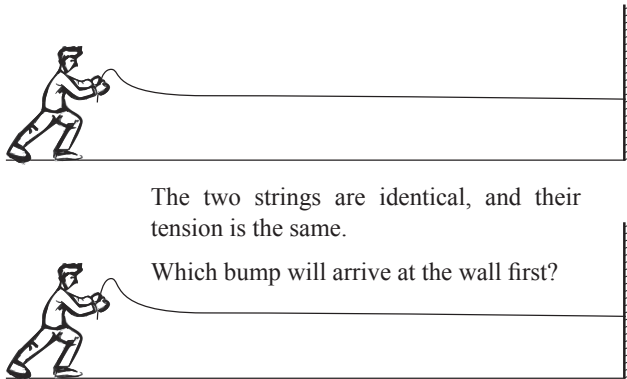


Figure 2.1 - A situation which underlines the meaning of a well-known statement: “for a stretched string, the speed of propagation of a disturbance depends only on the mass per unit length and the tension”. In this model, the race between the disturbances is a foregone conclusion: it’s a dead heat!

There is some benefit to be gained from such knowledge, *i.e.* the ritual statement that propagation speed depends only on the medium, since it implies that this surprising invariance will be questioned by some, or emphasised for others. It is surprises and unexpected phenomena of this kind which illustrate how physical theories are not just a familiar collection of analyses of situations we know how to handle; they have great unifying power for specific cases that we might have thought distinct, but which in fact, from certain points of view, prove to be otherwise.

Sometimes, the merest hint of an explanation is sufficient to induce these welcome surprises, these new and unexpected insights.

2.4 Coefficients of friction

The value of a normal component of contact force F_N and that of its tangential component F_T are coupled by static or dynamic coefficients of friction for two interac-

²⁷ MAURINES L. Spontaneous reasoning on the propagation of visible mechanical signals, *International Journal of Science Education*, 14 (3) 279-293; VIENNOT L. (2001) *Reasoning in Physics - The part of common sense*, Dordrecht: Kluwer Ac. Pub., 143-144. See also WITTMANN M.C. (1998) *Making Sense of How Students Come to an Understanding of Physics: An Example from Mechanical Waves*, Unpublished Ph.D. dissertation, University of Maryland; WITTMANN M.C., REDISH E.F. & STEINBERG R.N. (2003) Understanding and Addressing Student Reasoning about Sound, *International Journal of Science Education*, 25: 8, 991-1013.

ting surfaces. In standard elementary courses on friction,²⁸ the coefficient μ_s enables the maximum allowable value of the tangential component without slipping to be calculated: $F_T \leq \mu_s F_N$ and the coefficient μ_d enables the value of the tangential component, once slipping has started, to be found: $F_T = \mu_d F_N$. For a rectangular block of mass m sliding on an inclined plane (at an angle θ to the horizontal), the written notation of the balance of forces and the fundamental principle of dynamics (air friction being “negligible”) gives the value for tangential acceleration (axis downwards): $a = g (\sin\theta - \mu_d \cos\theta)$.

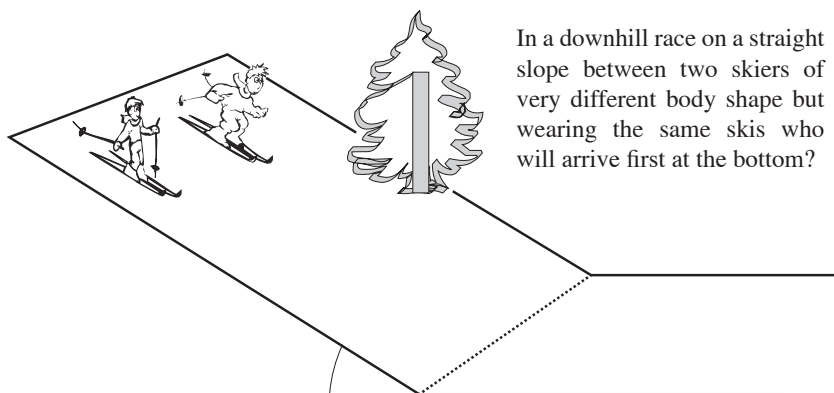


Figure 2.2 - A situation which underlines the meaning of the standard solution to an exercise involving friction: “the acceleration of a skier along the path of steepest descent is: $a = g(\sin\theta - \mu_d \cos\theta)$ ”, where μ_d is the coefficient of sliding friction and g is the acceleration due to gravity. According to this model, the race between the skiers is a foregone conclusion: dead heat. Why is this so improbable?

Let us imagine (this might be a first year university exam question) that this block represents a model of a skier on the line of steepest descent. There is the solution, presented to a tutorial group.²⁹ What can we usefully add?

Here’s a question: if two skiers, otherwise identical in every respect, have skis of different widths, does the solution above predict anything about their performance? This ski width does not appear in the expression for acceleration, a , but is it not relevant? Or else where is it hiding? In the coefficient μ_d perhaps? Otherwise we simply have to resign ourselves to the fact that the solution to this exercise predicts the simultaneous arrival of these two skiers on skis of different widths.

So we come to the question we could so easily have ignored: what exactly **do** these coefficients depend on, and what **don’t** they depend on? It’s surprising that μ_d and μ_s

²⁸ For more information on the history and limitations of this simple model, read BESSON U., BORGHINI L., DE AMBROSIO A. & MASCHERETTI P. (2007) How to teach friction: Experiments and models, *American Journal of Physics*, **75** (12) 1106-1113.

²⁹ See Chapter 4, exercise 4.4.

do not depend on the area of contact. We can discuss the value of this model, but at least we should realise what it means.

The model can be completed with the idea that, although in the case of a greater contact area there are more asperities, they are less crushed and hence play a less active role in friction. With the one cancelling out the other, the result could be the same. In any event, this invariance is more or less explicitly what is being taught today. While it may be surprising, the mere fact of emphasising it draws the students into intellectual activity far more gratifying than simply assigning a value to a coefficient to be mechanically plugged into a formula for calculating some other quantity.

2.5 When mass doesn't count

While the symbol for mass is found right from the outset in course books and exercises books in elementary mechanics, it is, astonishingly, among the variables which don't seem to count for very much. This is also puzzling in the light of common experience.

Quite simply, if the forces involved are proportional to mass, the fundamental principle of dynamics (classically associated with the formula $\vec{F} = m\vec{a}$ ³⁰) leads to an equation containing two terms proportional to this variable which then cancel out. Consider our skier again, but represented as a rectangular parallelepiped. Let's take another skier of the same shape, but much lighter (less dense). There again, do we imagine that these two skiers, who set off together, are going to arrive together at the bottom of the slope? The result's independence of mass is here merely the result of deliberately ignoring frictional forces, of magnitude F_V , due to the viscosity of air. These do not depend on mass, thus the motion does: dividing both sides by m to get the acceleration, one term remains (the one associated with air friction) which includes this parameter in the denominator: $a = g(\sin\theta - \mu_d \cos\theta) - F_V / m$. The bigger the mass, the smaller this retarding term, $-F_V / m$, there is therefore some advantage in being a very dense champion!

Even though downhill skiing may not be our main interest in life, these thoughts allow us to preserve this general formulation: in elementary mechanics,³¹ if all the forces exerted on the object whose motion is being analysed are proportional to its mass, then it is an irrelevant parameter; on the other hand, when some of these forces do not depend on it, the object's motion does depend on it. In *Explorers on the Moon* (the comic in the *Tintin* series by HERGÉ) why is the rocket, with its engines shut

30 Classical notation: total vector force \vec{F} exerted on a "particle" of mass m ; \vec{a} is the acceleration of the particle.

31 The case considered here is the Galilean reference frame. In a system of two or more bodies, where the centre of mass is not necessarily identified with the most massive body, the situation gets complicated.

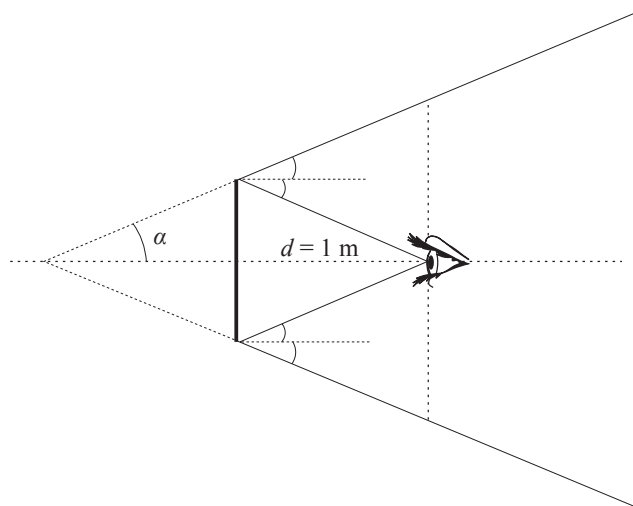
down, not drawn towards the asteroid Adonis, while Captain HADDOCK, on his ill-advised and inebriated space walk, is on a dangerous approach to it?³² Certainly not because the poor captain is less massive than the rocket (with its engines silent and hence just as passive as him).

As for so-called “mass” spectrographs, these make use of forces due to an electric field which is not itself aware of the mass of the object being deflected, whence the influence of mass in the observed deflection. For the same reason, the path of a charged particle in a magnetic field depends very much on its mass.

2.6 The mirror

The field of a mirror is a classic exercise in the first years of university or in secondary school. All you need is DESCARTES’ law in four strokes of a pencil (Fig. 2.3) and there you are, the problem is solved.

Solved maybe, but not finished, at least not in a very interesting way. If we decide to focus our attention on the part of the plane parallel to the mirror and visible to an eye in this plane,³³ this is the same thing as wondering what part of himself somebody can actually see in the mirror.



We can imagine that the answer expected by the teacher is as shown in this figure, based on the equality of the angles of incidence and reflection. For clarity, the scale is not the same in both dimensions.

Figure 2.3 - A model answer for the question of a mirror’s field. Let there be a plane circular mirror 10 cm in diameter. If you place your eye on the axis of the mirror and 1 m from it, draw a diagram to show the region of space you can see in this mirror.

³² See HERGÉ, *On a marché sur la Lune* [Explorers on the moon], Dargaud, Paris 8.

³³ Chapter 4, exercise 4.2. Concerning students’ common ideas about mirrors, see GOLDBERG F. & McDERMOTT L.C. (1986) Student Difficulties in Understanding Image Formation by a Plane Mirror, *The Physics Teacher*, **24** (8) 472-480; GALILI I., GOLDBERG F. & BENDALL S. (1991) Some reflections on plane mirrors and images, *The Physics Teacher*, **29** (7) 471-477.

We can do a miniature simulation of this situation by distributing small, 1 cm square mirrors in class,³⁴ together with small 4 cm cardboard squares with a hole in the centre and gridded in the same format as the mirrors. Looking through the central mirror, with the eye against the box and with the squares facing the mirror, we can count how many we see; this is the same as counting how many square centimetres of one's own face can be seen in the mirror. Typically, our first-year student replies: "four", and "me too" chime in the others. You don't hear immediately: "it doesn't depend on the distance!", that in other words the collection of individual results signifies an invariance. Little by little, one by one, you can see them experimenting, bringing their mirrors closer in and then further away. It is very striking to be able to literally read from people's movements, the sequence of intellectual operations taking place: measuring (as seen by the intensity of their concentration), then testing some possible invariance by changing the position of their arms.

Interesting, precisely because it is unexpected, this independence with respect to distance means that there is no point in going backwards to see your belt in the mirror if it has not already appeared in a first rough inspection.

As is often the case, the surprising invariance involves two factors which cancel each other out and for which only one was spontaneously taken into account.

As far as friction is concerned, one might think that the contact surface area is a determining factor, without necessarily realising that the crushing of asperities diminishes with increasing area. With the small mirror, we imagine that in stepping back more of one's own surface is fitted in a sort of cone of visibility subtended by the mirror, and whose apex would be at the location of the image of the eye in the mirror. However, this is to forget that the included angle of this cone (α in Fig. 2.3) decreases. THALES' theorem provides an exact compensation: the image of the eye is always twice as far from the apex of the cone as the mirror. Hence, the area of the plane of the eye which is still visible is always four times bigger than that of the mirror.

In terms of interest, is there anything in common between the presentation of the four lines in Figure 2.3 and what (with its rich additions) has just been described: collective experimentation,³⁵ invariance, discussion, a fresh look at a demonstration?

³⁴ Like those on a disco (glitter) ball. Experiment suggested by W. KAMINSKI, private communication.

³⁵ There is currently a consensus that the teacher should ask the students what they expect the outcome to be before performing any experiment. See for instance: WHITE R. & GUNSTONE R. (1992) *Probing Understanding*, London: Falmer Press. And, very recently: "The so-called clicker questions (the name comes from the electronic device that students use to record their answers) usually focus on common students misconceptions about the concepts (...)": "A transformed course typically begins not with a lecture but with a clicker question. Students gather in small groups to discuss it, and a fellow assigned to the course circulates through the classroom to guide the inquiry process. Once the students have punched in their answers, the faculty member might

To be sure, the familiar features of interest are also there: the “mini-demo”, the discussion, and the link with everyday life. Why would we ignore them? Nevertheless, this does not invalidate the idea that highlighting an unexpected invariance is in itself a source of satisfaction, even though we cannot always analyse both the common initial expectation and the reason for the surprising result.³⁶

2.7 The power and equivocalness of invariance

When we come across an unexpected invariance, how can we do otherwise but be intrigued? There, we have a vast array of situations for which, after dealing with one of them, we no longer have to invest our energy other than specifying precisely the field in question. Excessive generalisation leads to disappointment, but to do so insufficiently is to pay short shrift to the power of the solution. Although it occurs extremely frequently in physics, the function $y = \text{Constant}$, needs to be specified: constant with respect to which interesting variables and sensitive to variations in what others?³⁷ The two sides to this analysis will be looked at later from various different angles; repetition should not seem too annoying in the light of current practice. In particular, we shall see that what holds true for the value of a variable also applies to more qualitative statements. For instance, the following, very much in favour for the ideal gas law ($pV = nRT$, in the usual notation), to which we shall return in Chapter 3: “All gases which may be assumed to be perfect gases behave in the same manner”. Obviously, it all rather depends on what is meant by “behave”, and the risk of misunderstanding is ever present. Thermodynamics is not the only field in which such preoccupations are relevant.

A variable with a surprising lack of influence is noteworthy, but a pleasure to observe nonetheless.

offer a microlecture aimed at correcting their mistakes and filling in gaps in their knowledge. Once the concept is clear, the class moves on to the next clicker question.”; MERVIS J. (2013) Transformation is Possible if University Really Cares, *Science*, **340** (6130) 292-296. The proposed handling of this situation is deliberately different: the aim here is the gradual emergence of the question of invariance, which might otherwise be short-circuited by the method usually suggested.

³⁶ Note that it's a lot less easy in the case of the phase velocity of a wave.

³⁷ See VIENNOT L. (2001) *Reasoning in Physics - The part of common sense*, Dordrecht: Kluwer Ac. Pub., Chapter 6.

Thinking in Physics

The pleasure of reasoning and understanding

Viennot, L.

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