Chapter 2
Optical Instruments (Paraxial Approximation)

We give only a short description of the most important optical instruments. For more details see the textbooks, e.g. Hecht [1] or Longhurst [2].

2.1 Camera

The simplest camera has a lens, shutter, an iris diaphragm and a box in which the back wall contains the film plane. Axial movement of the lens makes focusing on the object plane possible, see Fig. 2.1.

The F-number, defined as $F\# = \frac{f'}{2a}$, where f' is the focal length of the lens and 2a the diameter of the iris, controls exposure time and the depth of focus. A $\sqrt{2}$ times smaller F-number gives a 2 times smaller exposure time.

The depth of focus Δz is given by

$$\Delta z = \pm \Delta y \; F\#,$$

where Δy is the acceptable unsharpness, for instance the smallest detail discussed above.

With a diffraction limited lens the depth of focus is given by $\pm 4\lambda (F\#)^2$. This is called the Rayleigh DOF and constitutes a minimum of Δz.

The distance from lens to film plane s' is given by the imaging equation

$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f}$$

where s is the object distance.

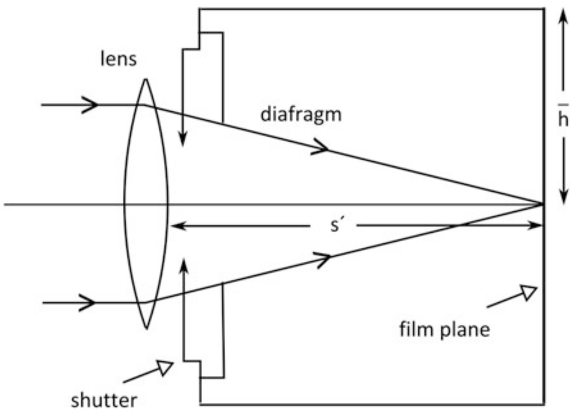
With a height $\bar{h}$ of the film plane, the field angle w is given by $\tan w = \bar{h}/s'$.

The resolution of lenses used in cameras depends on the application of the camera.

The human eye can resolve details of 0.1 mm at a distance of 250 mm. When a negative with a format of 24×36 mm is enlarged by a factor 4, its smallest details should have a size of 0.025 mm.

C. Velzel, *A Course in Lens Design*, Springer Series in Optical Sciences 183,
DOI: 10.1007/978-94-017-8685-0_2,

Fig. 2.1 A simple camera



The lens of a film camera (and also a film projection lens) should have a better resolution over a smaller field (8 or 16 mm).

With a CCD as target in a camera, the pixel size is related to the required resolution. With pixels of 5 μm and arguing that 4 pixels per period are necessary for a good contrast, a resolution of 0.010 mm should be sufficient.

In our review of camera lenses in Sect. 4.1 we will see that a resolution of 30 periods per mm is standard for modern camera lenses.

2.1.1 Camera Obscura

The camera obscura, or pinhole camera, is a predecessor of our modern photo-camera. It was used in the renaissance era to record the contours of landscapes and played an essential role in the discovery of the laws of *perspective*, see Fig. 2.2.

The image on the backside (film or ground glass) is a projection with the entrance diaphragm as a center. It is clear that the size of the image is given by ws', where w is the field angle. We have $w = -\frac{h}{s}$, where h is the object height. With s' constant we see that a more distant object makes a smaller image. This is called *homocentric perspective*.

By imaging the diaphragm with a lens to the left we can make the perspective so that a more distant object gives a larger image. This is called *hypercentric perspective*.

In Sect. 1.5 (Fig. 1.24a) we saw that a telecentric perspective can be obtained when the diaphragm is in the focal plane of the imaging lens.

To make a sharp image the diaphragm must be made small. Diffraction at a small pinhole gives a blur with the size $\frac{\lambda s'}{d}$, where λ is the wavelength; equating this with the diameter d of the pinhole we get as optimum $d = \sqrt{\lambda s'}$. With $\lambda = 0.5$ μm and $s' = 125$ mm we find $d_{opt} = 0.25$ mm. This is also the optimum of resolution.

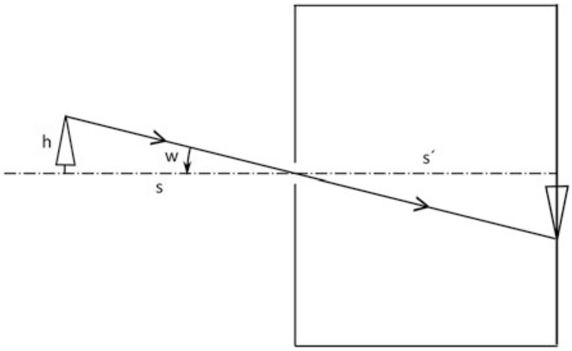


Fig. 2.2 Camera obscura

2.2 Human Eye

In optics the following components of the eye (and their properties) are important. See Fig. 2.3.

- The cornea, the eye's first surface, is covered with a tear layer of about 50 μm thickness. The cornea is transparent and approximately spherical with a radius of curvature of 7.8 mm (average).
- Between the cornea and the eye lens there is a watery fluid with index $n = 1.336$.
- The eye lens is situated about 7 mm behind the cornea and has a refractive index that varies in the axial direction between 1.386 (outside) and 1.406 (center). Its axial thickness and curvature are controlled by the ciliary muscle, so that its power can vary between 17 and 25 dioptres (accommodation) in persons under the age of 45.
- The iris diaphragm is in front of the eye lens and can vary its diameter from 2 to 8 mm to adapt to the light level.
- Between the lens and the retina there is a watery gel with index 1.336.
- The retina is situated at an axial distance of about 25 mm behind the cornea; it is curved with a radius of about 12 mm.

At the backside of the retinal tissue we find the sensitive elements, "cones" and "rods". The cones are concentrated in the macula, the cone density is greatest in the fovea with distances between their centres from 2.5 to 5 μm. The cones are used for seeing with high resolution (about 0.5 mrad) in daylight.

The rods are distributed over the peripheral retina; they are connected by nerve patterns and serve for pattern recognition and movement detection. The resolution in the periphery is considerably lower than in the macula; the sensitivity of the rods is highest at low light levels.

In Fig. 2.4 we give a model of the eye based on work of Gullstrand [2, p. 425].

In this figure, 1 is the cornea, 2 and 3 are surfaces of the eye lens, 4 is the retina. The iris (pupil) is just in front of the eye lens. The refractive indices behind cornea

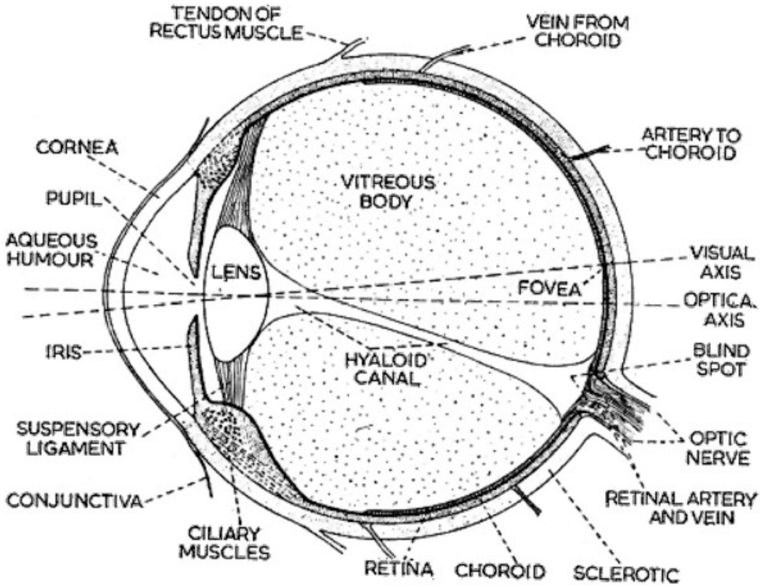
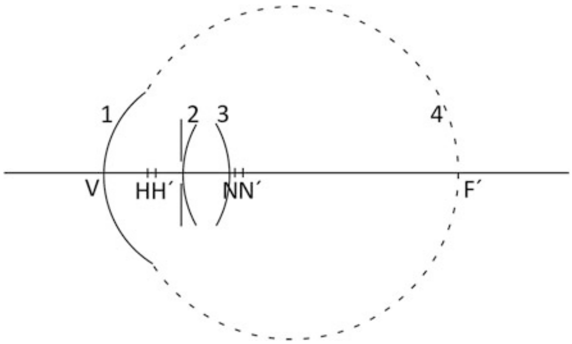


Fig. 2.3 The human eye, horizontal cross-section of the eye. Nose is *below*, so it is the *right* eye

Fig. 2.4 Gullstrand's eye model



and lens are those of aqueous and vitreous bodies. The refractive index of the lens in this model is an average value.

The data of this eye model are

	#	Radius	Distance	Index
OBJ	0	–	∞	1
	1	7.8	7	1.336
	2	10.0	3.6	1.413
	3	−6.0	13.4	1.336
IMA	4	−12.0	–	–

The stop (iris) is in front of the first lens surface. The principal planes lie at distances VH = 1.47 mm and VH′ = 1.50 mm from the cornea. The distance VF′ is 24 mm. The radius of curvature of the image surface is 12 mm. The power is 60 diopters. See [2, Sect. 17.2].

2.2.1 Nodal Points

The *nodal points* of an optical system are defined as axial points with an angular magnification of +1, so that $u' = u$. In Fig. 2.4 N and N′ are the nodal points and H and H′ the principal points.

With $n'u' = nu\text{-}Kh$ (1.19) and $u' = u$ we have

$$HN = H'N' = h/-u = (n' - n)/K = f' + f$$

Note that $HN = H'N' = 0$ when $n' = n$.

2.2.2 Exercise 4, Nodal Points of the Eye

When the eye is rotated around the second nodal point (N′ in Fig. 2.4) the image on the retina of a far object will not move (check this with your own eye).

Calculate the position of the nodal points in Gullstrand's eye model.

Tip: first find the position of the principal planes.

A few words about eye corrections:

- cornea too steep gives *nearsightedness*, to be corrected by a negative glass,
- cornea too shallow *farsightedness*, to be corrected by a positive glass,
- at old age accommodation fails, one needs reading spectacles then,
- when the cornea has different curvature in two perpendicular directions (with arbitrary orientation) we have *astigmatism*, to be corrected by "cylinder power",
- the eye lens can become opaque by cataract; then a plastic "intra-ocular lens" can be inserted.

In this course we do not consider

- color vision
- binocular vision
- eye movements.

Cones can discriminate between red, green and blue accurately, so that many different shades of color can be perceived. The cones are more sensitive for green than for red and blue, with maximum sensitivity of 680 lumen/watt at 555 nm.

By comparing the images of both eyes, the brain can see depth.

The eye can be rotated in two dimensions by two sets of muscles (see Fig. 2.3) to direct the eye axis to objects of interest.

2.3 Magnifier and Microscope

With the unaided eye we can resolve details of the order of 0.1 mm at a distance of 250 mm.

When we use a magnifier lens in front of the eye, with focal length f', we have an *angular magnification*

$$M = \frac{250}{f'} \tag{1}$$

compared to the original situation, when we put the object in the focal plane. See Fig. 2.5.

One could expect that with a magnification M, details of the order of magnitude of 100/M μm could be resolved. This was confirmed in the work of Anthoni van Leeuwenhoek.

Anthoni van Leeuwenhoek (1632–1723) used lenses in the form of small glass spheres, where

$$K = \frac{2(n-1)}{nr}$$

With $r = 4$ mm and $n = 1.6$ we have $f' = 5.3$ mm and $M = 48$. It turned out that the observable details were of the order of 2 μm.

The *compound microscope* consists of an objective lens that makes a magnified image of the object, and an ocular that projects an image at infinity, so that the eye has a focused image on its retina (see Fig. 2.6).

The angular magnification of the compound microscope is given by

$$M = M_{ob}\frac{250}{f_{oc}}, \tag{2}$$

where M_{ob} is the (linear) magnification of the objective and f_{oc} the focal length of the ocular. With $M_{ob} = 20$ and $f_{oc} = 25$ we have a magnification of 200× and we expect to see details of 0.5 μm.

This is of the order of the wavelength of light, so we should consider the effect of diffraction.

According to the theory of diffraction the smallest detail that can be resolved by an objective with aperture angle u is given by [see Sect. 4.7, (4.53)]

$$\Delta y = 0.6\frac{\lambda}{n \sin u},$$

where n is the index in object space, λ is the wavelength in vacuo; $n \sin u$ is called the *numerical aperture*. With $\lambda = 0.5$ μm (green light) and $\sin u = 0.5$ (quite normal for a 20× objective), $n = 1$, we have $\Delta y = 0.6$ μm. With such an objective details of 0.5 μm cannot be observed.

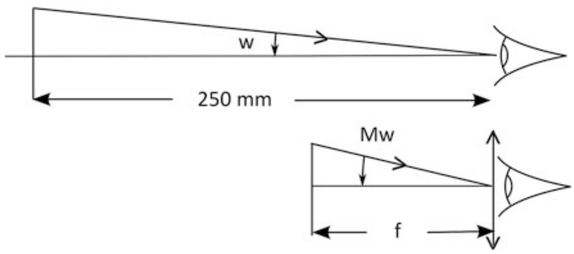


Fig. 2.5 Magnifier

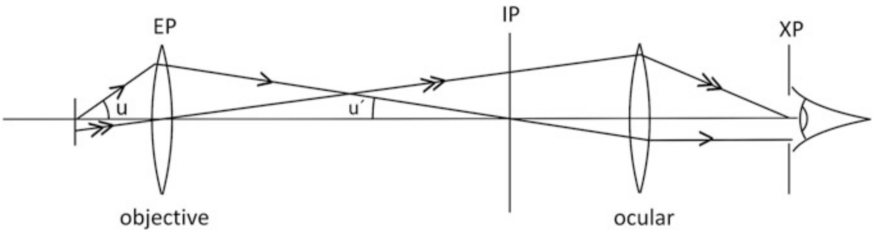


Fig. 2.6 Compound microscope

By making the refractive index higher than 1 in object space we can improve the resolution. Microscopy in a transparent medium, for which water and other fluids are being used in practice, is called *immersion microscopy*.

With a water *immersion* we have n = 1.33, and the diffraction limit becomes, with the same values of u and λ as before, equal to 0.45 μm; smaller than 0.5 μm.

2.3.1 Maximum Magnification

We calculate the maximum magnification as function of the diameter of the eye pupil. This is important for visual microscopy (and also for observation with a telescope).

According to *Abbe's sine rule* we have, for a well corrected objective (see Fig. 2.6)

$$n' \sin u' = \frac{n \sin u}{M_{ob}}$$

Abbe's sine rule and its consequences are explained in Sect. 14-3 of Longhurst [2].

Usually n = 1, $M_{ob} \gg 1$, so that $n' \sin u'$ is small and

$$n' \sin u' = \frac{\Phi}{2 f_{oc}},$$

where Φ is the diameter of the eye pupil, is a good approximation.

The ocular magnification is $M_{oc} = \frac{250}{f_{oc}}$, so that we find for the total magnification

$$M = M_{ob} M_{oc} = 500 \frac{n \sin u}{\Phi}. \tag{3}$$

Because the smallest pupil diameter is $\Phi = 2$, the maximum magnification is

$$M_{max} = 250\, n \sin u \tag{4}$$

when the eye pupil is the limiting diameter.

When we make the exit pupil of the ocular narrower, we can take a somewhat higher value for M; but beware of diffraction (a Φ_{oc} of 1 mm gives a spot of 5 μm on the retina). Van Heel recommends to take the maximum magnification as 1,000 n sin u [3].

2.4 Telescopes

Kepler's (astronomical) telescope consist of an objective and a positive ocular, see Fig. 2.7.

It gives an inverted image; the inversion can be corrected by inserting a prism arrangement (for instance that of Porro) or an inverting system between objective and ocular. Here we will not consider this any further, but in the design of telescopes of this type it is an important issue (see Sect. 6.2).

In this arrangement, with a single lens ocular, we take the aperture stop at the objective. Then the exit pupil is at a distance behind the ocular given by

$$\frac{1}{s_2'} = \frac{-1}{f_1 + f_2} + \frac{1}{f_2}$$

Because $f_1 + f_2 \gg f_2$, we have $s_2' \cong f_2$; s_2' is called the *eye relief*. The *linear magnification*, defined as the ratio between the ray heights in the exit and entrance pupils, is $M_L = -\frac{f_2}{f_1}$.

The *angular magnification*, defined as the ratio $\frac{W'}{W}$ (see Fig. 2.8) is given by $M_A = -\frac{f_1}{f_2}$.

($M_L = \frac{1}{M_A}$ is an example of a more general relationship, known as Lagrange's invariant, see Sect. 2.5.)

Usually we designate a hand-held telescope by a product like 8 × 30, where the first figure denotes M_A and the second the diameter of the entrance pupil.

The diameter of the exit pupil is now $\frac{30}{8} = 3.75\,\text{mm}$.

The system of Fig. 2.9 suffers from *vignetting*: a chief ray from the centre of the entrance stop (at the objective) misses the ocular when its angle w with the axis (the *field angle*) is larger than

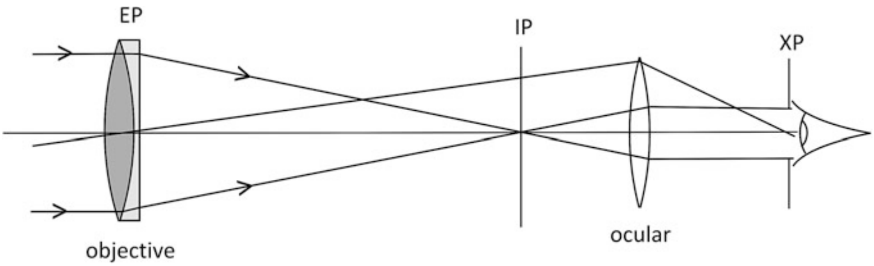


Fig. 2.7 Kepler's telescope

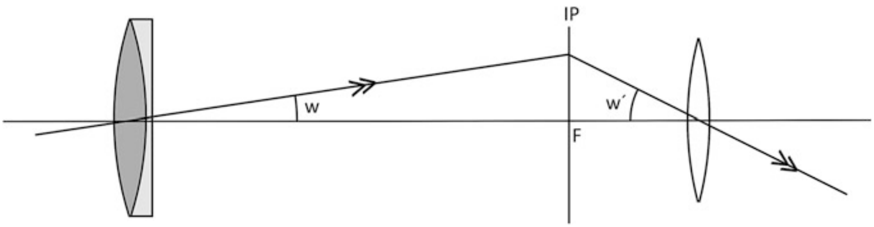


Fig. 2.8 Magnification

$$w = \frac{1}{2}\frac{\Phi_{oc}}{(f_1 + f_2)}$$

where Φ_{oc} is the diameter of the ocular. With our 8×30 telescope and $f_1 = 200$ mm we have $f_1 + f_2 = 225$ mm. With $\Phi_{oc} = 4.5$ mm we could have $w = 0.01$. The full field would be $2w = 0.02$ or 20 mrad.

Van Heel [3] points out that with the exit pupil XP rather far from the eye lens and partial illumination of this pupil due to vignetting, the observer has to move his head to see objects at the edge of the field. This is shown in Fig. 2.9.

An improvement of this situation can be obtained by the use of a *field lens*, see Fig. 2.10.

When we put a lens in (or near) the point F where intermediate image is, and take the power of this lens so that the objective is imaged by it on the eye lens, we can take the diameter of the field lens Fig. 2.16, so that the field angle $\frac{\Phi_F}{f_1'}$ is what we want.

In our example $\Phi_F = 4$ mm would give $2w = 0.02$. Because, as we will see later, a lens near the image plane does not contribute much to the aberrations, we can make its diameter easily larger to obtain a larger field angle.

With telescope and microscope oculars $\Phi_F = 20$ mm is frequently used.

Now the eye lens acts as aperture stop, with a minimum diameter of 3.75 mm, and the field lens acts as a field stop. Over the whole field we have (nearly) no vignetting.

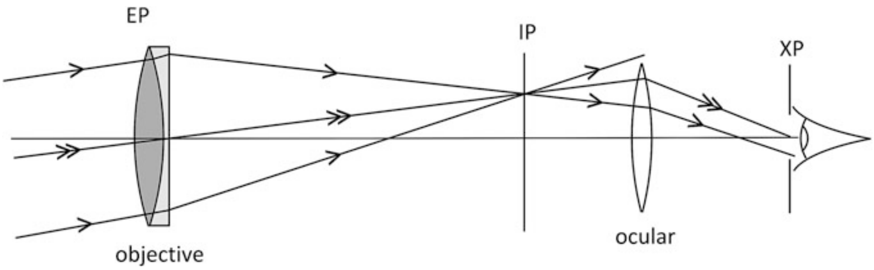
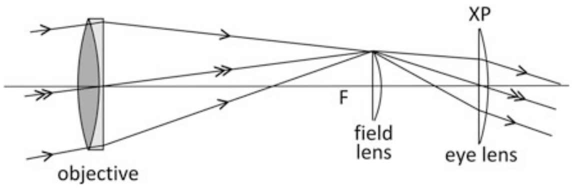


Fig. 2.9 Vignetting in a simple telescope

Fig. 2.10 Telescope with field lens



In this set-up the eye of the observer should be immediately behind the eye lens; in many situations this is not practical, an eye relief of a few centimeters is usually required. This leads to a compromise, by taking a slightly larger Φ_{oc} and increasing the focal length of the field lens, we obtain the final lay-out. See Fig. 2.11.

The parameters of this lay-out are given in the following table:

		Focal length	Distance	Diameter
Objective	1	300	300	30
Field lens	2	81.8	37.5	20
Eye lens	3	37.5	25	17.2
Stop	4			3.75

Galilei's telescope consists of a positive objective and a negative eye lens. With the stop at the objective its exit pupil lies between the lenses, and therefore far from the eye, so that vignetting cannot be avoided.

This telescope has no inversion, so that it can be used directly for terrestrial applications; it is applied (at low magnifications) in the theatre and in sports. See Fig. 2.12.

A modern application of the telescope, in reverted order, as a beam expander for laser beams. Between the lenses (in F) a pinhole filter takes away stray light. See Fig. 2.13.

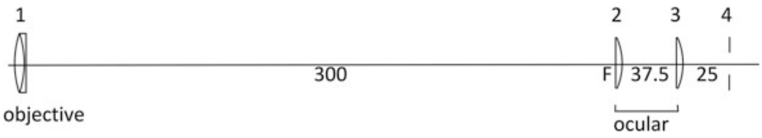


Fig. 2.11 Lay-out of a telescope, scale 2:1

Fig. 2.12 Galilei's telescope, $M_A = 2$

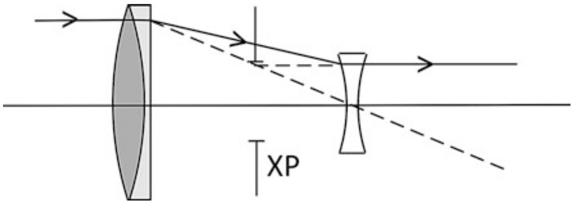
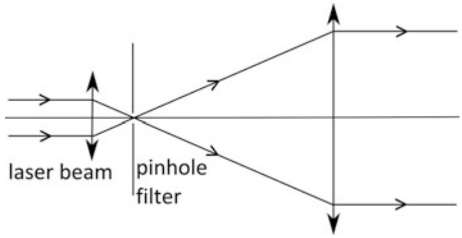


Fig. 2.13 Beam expander



2.5 Illumination

Some instruments, such as the microscope and the projector, need artificial illumination. Others, like the camera and the telescope (and also the human eye) have objects that send out light themselves, or light reflected by light sources already present.

A simplified illumination system for a microscope contains a light source, a condenser lens, a transparent object, the objective and the stop. See Fig. 2.14.

In the figure the object size is greatly exaggerated; the full field of a 10× microscope objective has a diameter of about 2 mm.

The condenser and the objective image the source in the aperture stop. It can be shown that with a flat source, the light flux (Watt/m^2) through the object is given by

$$\Phi = \pi B\, S \sin^2 u_c \tag{5}$$

where S is the area of the source and u_c is the aperture angle of the condenser.

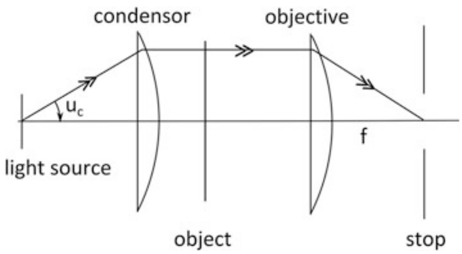


Fig. 2.14 Illumination system

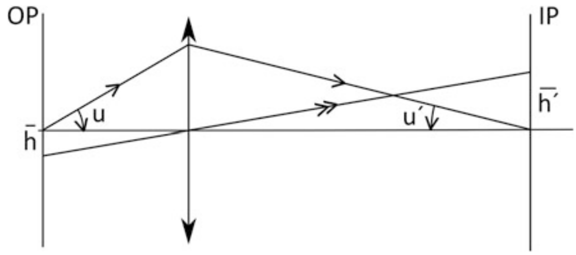


Fig. 2.15 Lagrange invariant

The constant B, called *radiance*, is a property of the source. With a thermal light source it is a function of the temperature of the source.

In paraxial optics we have the invariant

$$S'u'^2 = Su^2. \tag{6}$$

This equation is a special case of the optical invariant.

$$H = n(h\bar{u} - \bar{h}u),$$

where (h, u) and $(\bar{h}, \bar{u})$ are two paraxial rays. See Sect. 1.4, (1.22).

H is equal in all planes of the system. Here we consider object and image plane. See Fig. 2.15.

In the above object plane, when we take $h = 0$, we have $H = -n\bar{h}u$. This is equal to $H' = -n'\bar{h}'u'$ in the image plane, so that, with $n' = n$, we have $Su^2 = S'u'^2$. This is called Lagrange's invariant or Helmholtz-Lagrange invariant in the literature.

Ernst Abbe showed that for well corrected optical systems

$$S' \sin^2 u' = S \sin^2 u \tag{7}$$

Without loss of light (by absorption or scattering) in the system therefore the radiance of the source is equal to that of its image $B = B'$.

This is in agreement with the second law of thermodynamics (Clausius): the temperature in the image cannot be higher than in the source itself.

The quantity $S \sin^2 u$ is called the *throughput* of the system. It is proportional to H^2; we have seen that the throughput determines the transport of energy through the system.

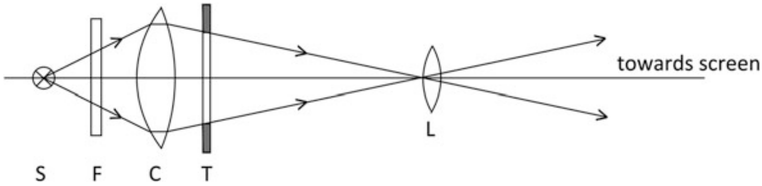


Fig. 2.16 Slide projector

Throughput is also important in the description of the *channel capacity*; according to Shannon [4] this is the number of bits that can be transported per second through the system.

The channel capacity is proportional to the number of degrees of freedom N, defined as

$$N = \frac{S_0}{S_D}$$

where S_0 is the area of the object and S_D is the area of the object-side diffraction spot,

$$S_D = \frac{\lambda^2}{n^2 \sin^2 u} \frac{\pi}{4}$$

so that we have

$$N = S_0 \sin^2 u \frac{4n^2}{\pi \lambda^2}$$

Because throughput is an invariant ($\sim H^2$) also N is invariant (the "pixel theorem").

With a field radius of 1 mm and $\frac{\lambda}{n \sin u} = 1\,\mu m$ we have $N = 4.10^6$. This is typical for a 20× microscope objective. With a stepper lens N can be of the order of 5×10^{10}.

Example

As an example of an illumination system we discuss a simple projection device: the slide projector. Slide projectors are not in frequent use anymore, but their modern successors operate according to the same principle. See Fig. 2.16 for a scheme of a slide projector.

A light source S is imaged by a condenser lens C in the pupil of the projection lens L. Between the source and the condenser there is a filter F that transmits only the visible part of the source spectrum, so as to prevent heating of the transparency (slide) T that follows the condenser. The slide is imaged on a screen (not in the figure) that is far from L, typically a few meters (at home) or some tens of meters (in a lecture room).

Let us assume that the projector source is a 100 W halogen lamp, that will produce about 3,000 lumen.

How much of this light will reach the screen depends on the aperture of the condenser and the transmission of the components between the source and screen.

Assuming that the lamp sends an equal amount of light in all directions (it is a uniform source, not Lambertian) the condenser will receive a portion $u^2/4$, where u is the condenser aperture. With $u = \pi/6$, or 30° this factor is equal to 0.0685. Let the filter F have a transmission coefficient of 0.8 and let us assume that we have reflection losses of 2 % for each optical surface and there are 10 of these; the transmission coefficient without transparency then becomes 0.64.

A blank slide does not contain useful information; let us assume that the average transparency transmits 50 % of the light. Then the luminous flux that hits the screen is given approximately by

$$\Phi = 3000 \times 0.0685 \times 0.64 \times 0.5 \approx 66\,\mathrm{lm}$$

When we assume that the screen is a diffuse scatterer that absorbs 30 % of the light impending on it and the spectator is at a distance of 4 m from the screen in an otherwise dark room, his two eyes will receive a portion of u_e^2 of the light reflected from the screen, where u_e is the aperture angle of the eye measured from the screen. With a pupil diameter of the eye of 8 mm we have $u_e = 0.001$.

The luminous flux received by the eye pupil is therefore given by

$$\Phi = 66 \times 0.7 \times (0.001)^2 = 0.46 \times 10^{-4}\,\mathrm{lm}$$

When we estimate that the screen image on the retina occupies an area of about 100 mm^2, the illuminance on the retina will be of the order of 0.5 lm/m^2 = 0.5 lux.

This is enough for a well-adapted eye to perceive a clear screen image.

More on the subject of illumination is found in Mouroulis and Macdonald [5].

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A Course in Lens Design
Velzel, C.
2014, XV, 334 p. 231 illus., Hardcover
ISBN: 978-94-017-8684-3