

# Chapter 2

## Uncertainty and Fuzzy Decisions

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**Abstract** Uncertainty is the main reason that makes human free to choose. Many actions, strategies are designed to handle or reduce the uncertainty to make decision makers life easier. But there is no common accepted theory in the academia. Researchers still struggling to create a common understanding. There are theories that we may follow to make decisions under uncertainty. Among them, probability theory, fuzzy theory and evidence theory can be given. Decision problem is constructed in with the help of these theories. Fuzzy Logic and Fuzzy theory may be considered as the recent advancement and has been applied in many fields for different type of decision problems.

**Keywords** Uncertainty • Taxonomy • Chaos and complexity • Fuzzy sets and logic • Computing with words • Meta-Linguistic expressions

### 2.1 Introduction

There is no uncertainty theory that is commonly accepted in academia. A Google search with keywords “uncertainty theory” yields tens of thousands of results. Since the knowledge is limited and measurements are imprecise, future events can only be

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predicted with some confidence under strong assumptions. Naturally decisions have to be made in uncertain environments in the real world.

Like everyone else, leaders must make decisions under uncertainty and, in many cases, uncertainty itself becomes the real problem for leaders. As Simon [38] put forward, decisions are made with criteria that satisfy our needs. We make decisions even though there may not be an optimal action that can be found with an application of scientific methodologies or there may not be any action that satisfies our needs.

Although it is a subject of several research fields, there are some attempts to construct theories of uncertainty in mathematics [see for example, [30, 31]]. Uncertainty theory might be a subject of a specific branch of mathematics that deals with human uncertainty that is usually not sufficient. It is generally a subject of several theories such as fuzzy logic (FL), probability theory (PT), complexity theory and philosophy, etc. There are different views and thoughts about uncertainty. These views and thoughts are all related with the nature of problems in different fields. According to Weber [55, 56] these views are epistemologically biased. He states;

[...] the concept of uncertainty is epistemologically biased, in that uncertainty is viewed as an attribute of how we know what we know. This epistemological bias has led to the development of four branches of uncertainty literature based on an actor's (individual, group, or organization): (1) ability to gather and process information; (2) ability to predict consequences of actions; (3) use of intuition; or (4) perception of the environment. (Weber [55]: 455)

Since, humans are the creators of the concept of uncertainty, it all makes sense that, our abilities, capabilities, intuitions and perceptions are the real ingredient in this subject. As it is put forward by Zadeh [63, 64], Fuzzy logic is create an approximate reasoning mechanism to handle the uncertainty associated with human perception.

The emphasis of this chapter is mainly on uncertainty and fuzzy decision making under uncertainty. After the brief introduction of both uncertainty and Fuzzy Logic, the concept of fuzzy decision making and as a particular extension, perception based decision making is introduced.

## 2.2 Uncertainty

Uncertainty has been studied in many fields during the last few centuries. Discussions on uncertainty are frequently encountered in the following fields, but not limited to them; decision sciences, artificial Intelligence, legal fact-finding, economics, medical science, organizational open system theory, psychology, physics, etc. In this section we would like to summarize some important facts and studies that attempt to discuss uncertainty.

More often authors start with dictionary definition of uncertainty. We all use this term in our day to day real life. In order to keep our explanation compact, we avoid giving these definitions. However, some words that are related to uncertainty, such as, not precise, fuzzy, vague, etc., are important for this chapter. Frequently

information is usually presented with some words that have no precise definitions. “About”, “low”, “high”, “big”, “fast”, “slow” are some examples that are used for approximate reasoning in human decision making. Humans do understand them and communicate effectively using them. These words are central research focus for a specific research field, *Computing With Words*, CWW, [61]. Their imprecise meaning is also seen as the starting point of Fuzzy Logic. They help us to communicate and decide even though they are imprecise.

Researchers have tried to identify the different types and dimensions of uncertainty. Among them, Smithson [41], Smets [40], Bosc and Prade [7], Klir and Yuan [24], Walker [54] and Parsons [36] can be mentioned.

In Smithson’s [41] taxonomy, uncertainty is a part of *Incompleteness* which is product of *Errors*. For him, *Fuzziness* can be seen as a specific type of *Vagueness*. Smets [40] sees uncertainty as the basic part of ignorance. His taxonomy puts the concept of *fuzzy* under the data without error part of uncertainty. Dubois and Prade [10] distinguish the concept of imprecision and uncertainty in a way that, imprecision relates to the content of *value* of information. According to them, the concept of *fuzzy* is a qualifier for items of information. Uncertainty relates to the truth or the *confidence* of information. Bosc and Prade [7] suggest that uncertainty arises from a “*lack of information*” closely related to the probability theory proponents which assess the probability as lack of knowledge. Klir and Yuan [24] identifies three basic types of uncertainty. These are, *nonspecificity*, *strife* and *fuzziness*. According to Walker [54] theory of uncertainty has three dimensions in legal fact-finding. These are, linguistic, logical and causal dimensions. Walker classified the uncertainties into six types for scientific evidence about generic causation. These are; *concept uncertainty*, *measurement uncertainty*, *calculation uncertainty*, *sampling uncertainty*, *mathematical modeling uncertainty* and *causal uncertainty*.

Uncertainty is a cognitive process. It is appropriate to give some other perspectives from selected fields. For example, Knight [25] and Keynes [23] viewed uncertainty as something “*simply we do not know*”. Uncertainty is viewed as “*lack of knowledge*”, “*bias*” and “*psychological perception*”<sup>1</sup> in at least some of many disciplines [3, 14, 17, 27, 28, 34, 39] as in agreement with both Keynes and Knight. Uncertainty is seen in psychology as; (i) the psychological perception that creates fear and (ii) the motivation of communication [5, 11, 26].

Every decision we make is also part of the source of uncertainty even though it is the result of a series of negotiations in which we try to reduce uncertainty. This is the main ingredient of Game Theory. The discussions on game of chance lead to the idea of expected utility. Von Neumann-Morgenstern [53].<sup>2</sup> Based on Expected Utility Theory (EUT), the “rational” way to make decision is to maximize the expected utility. To understand more about the real life problems, these discussions were held

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<sup>1</sup>Uncertainty is thought to be then converted to fear that motivates to take some action. In this view it is a cognitive process.

<sup>2</sup>Due to the war-time difficulties, it was first published in 1953.

in toy domain. However imperfect knowledge makes life difficult for the proponents of EUT. Friedman and Savage [13] proposed the axioms of subjective expected utility theory. To show the systematic deviation from the EUT, Allais [1] published “*Allais Paradox*”. Thus, Non-Expected Utility Theory (NEUT) came into existence. In late 1960s decision under ignorance revived. Instead of *normative* theories, researchers started to look for more *descriptive* theories of decision. Several seminal contributions have been made for decision under uncertainty (sometimes called as under “*ignorance*”, “*risk*”). Among the contributors, Kahneman and Tversky [20], Tversky and Kahneman [51], Machina [32, 33], Fishburn [12], Kahneman et al. [21], Lichtenstein and Slovic [29] can be given.

Information and knowledge have a complex nature in real world. Quantitative measures such as probability may be found to be insufficient and/or misleading for many cases. It appears that humans try to overcome this difficulty with using heuristics as the first tool for reasoning under uncertainty. It is unaided and it is a quick and dirty way of handling uncertainty [36]. There is a need for a formal system to handle uncertainty to ensure that the information is effectively used. For this purpose, one can find three widely studied systems that are probability, possibility and evidence theories in literature. In this chapter, we focus on Zadeh’s Fuzzy Logic and Fuzzy System Theory. Both probability and evidence theories will be untouched since they are out of the scope of this chapter.<sup>3</sup>

## 2.3 Fuzzy Theory

After Zadeh’s [59] introduction of Fuzzy Logic and Fuzzy Sets, a vast volume of literature appeared about fuzzy logic and fuzzy system modeling (FSM) in particular. Zadeh’s intention was to create a methodology to mimic the human reasoning to handle the real world uncertainty more efficiently.

Briefly in fuzzy theory, every element belongs to a concept class, say  $A$ , to a partial degree, i.e.,  $\mu_A: X \rightarrow [0,1]$ ,  $\mu_A(x) = a \in [0,1]$ ,  $x \in X$ , where  $\mu_A(x)$  is the membership assignment of an element ‘ $x$ ’ to a concept class  $A$  in a proposition. The above representation is generally accepted as Type-1 fuzzy sets which assumes the membership values are certain. Unfortunately most of all concepts in fuzzy theory are assumed to be definable to be true to a degree.

Zadeh [60] introduced Type-2 fuzzy sets as an extension of Type-1 fuzzy set. Zadeh’s Type-2 fuzzy sets are the fuzzy sets, whose membership functions are classified as Type-1 fuzzy sets. Hence the value of membership function becomes fuzzy or these values are true to a degree. In most of the situations, the uncertainty may not be captured by Type-1 fuzzy sets. Karnik and Mendel [22] have proposed to add at least one higher degree to Type-1 fuzzy sets may provide a measure of

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<sup>3</sup>We would like to encourage interested readers to examine both theories and their role in reasoning under uncertainty.

dispersion for totally precise membership values of Type-1. Hence, Type-2 fuzzy sets capture the extensions of the Type-1 fuzzy sets to a higher degree. Type-2 fuzzy sets have grades of membership that are themselves defined by Type-1 membership functions, which are called secondary membership functions.

Another particular example of type-2 fuzzy set is interval valued fuzzy set where one of the author of this chapter contributes the related literature with several seminal papers. Turksen's [46–49] approach is a pioneering representation of interval valued type-2 fuzzy sets. In his pioneering works, he has shown that Disjunctive and Conjunctive Normal Forms (DNF and CNF) are equal in classical set theory but DNF and CNF are not equal in fuzzy set theory. He then showed that, Fuzzy Disjunctive Normal Forms (FDNF) is not equal to Fuzzy Conjunctive Normal Forms (FCNF) in fuzzy set theory which define the interval valued Type-2 fuzzy sets. His recent research [50] opens a new path to assess objective Full Type-2, to Full Type-n fuzzy system research.

Fuzzy system models (FSM) based of Fuzzy Logic define relationships between input and output variables of a system by using linguistic labels in a collection of IF-THEN rules. Zadeh [59] and Takagi-Sugeno [42] are the most commonly used rule based approaches. If we define k-th information vector  $X_k = \{x_{1k}, x_{2k}, \dots, x_{nk}\}$  where n is the number of attributes of information, and the reasoning for this information vector is a rule in general which can be written as:

***IF  $x_1 \in X_k$  is  $A_{1j}$  AND  $x_2 \in X_k$  is  $A_{2j}$  AND ... AND  $x_n \in X_k$  is  $A_{nj}$  THEN  $y \in Y_k$  is  $B_j$***

where,  $A_{ij}$  and  $B_j$  are linguistic assignments for input and output information objects, respectively, for the j-th rule of the whole number of rules in fuzzy rule base. 'isr' (is in relation to) is introduced by Zadeh and it represents that the definitions or assignments are not crisp, they are fuzzy.

There are at least two advantages of FSM that attracts researchers: (i) its power of linguistic explanation with resulting ease of understanding, and (ii) its tolerance to imprecise data which provides flexibility and stability for prediction. But, very few studies, if any, have been devoted to the study of the "Philosophical Grounding of Fuzzy Theory" since then. For a detailed philosophical grounding readers are referred to Turksen's book [49].

## 2.4 Fuzzy Decisions

In modeling human decision process, one may distinguish the descriptive and prescriptive type approaches. In these approaches, descriptive modeling attempts to identify system structure that capture the behavior characteristics as best as it can, whereas the prescriptive modeling attempts to determine the best approximate reasoning schemas that produce the best prediction of system behavior for a given

descriptive model.<sup>4</sup> Human decision processes depend on the perceived world and decision maker faces uncertainties at any stage of a decision process.

According to mainstream theoretical studies, rational individuals use all available information during the expectation formation process and they optimize the expected value of a well-defined objective function under the assumptions of von Neumann and Morgenstern's expected utility theory. The assumptions of von Neumann and Morgenstern's theory may not be fulfilled since most real world probabilities are imprecise or immeasurable. Even if it is a measurable case, when there is a tolerance for imprecision which can be exploited through granulation to achieve tractability, interpretability, robustness and economy of communication. There is generally a rationale which underlie granulation of attributes and use of linguistic variables [62, 64]. Furthermore, as briefly discussed in the above section, uncertainty may appear in different forms such as ambiguity, vagueness, discord, imprecision and fuzziness [24]. It is an attribute of information. Zadeh [64] suggests that information is a generalized constraint on the values which a variable is allowed to take. It becomes necessary to use uncertainty as an additional source of information that may be helpful to reasoning. As Zadeh [62] pointed out, information can be analyzed by perception based theory of approximate reasoning which is a generalization of classical reasoning that contains the capability to operate with perception based information. Fuzzy logic and fuzzy sets lay the ground for this kind of information processing and decision making.

Modelling of decision problem in formal theory requires; (i) courses of action (acts), (ii) states of nature (events), (iii) payoff associated with each actions-states, (iv) the degree of knowledge about states of nature and (v) decision criterion that helps select the course of action. This is often presented as a decision matrix (payoff matrix) for the sake of simplicity as given below.

|          | Event '1'     | Event '2'     | ... | Event 'h'     |
|----------|---------------|---------------|-----|---------------|
| Acts '1' | Payoff (1, 1) | Payoff (1, 2) | ... | Payoff (1, h) |
| Acts '2' | Payoff (2, 1) | Payoff (2, 2) | ... | Payoff (2, h) |
| ...      | ...           | ...           | ... | ...           |
| Acts 'k' | Payoff (k, 1) | Payoff (k, 2) | ... | Payoff (k, h) |

In general every event has associated probabilities that is assigned. These probabilities are assumed to represent the degree of knowledge (or of course lack of knowledge) about states of natures. Time to time, payoffs are shown as utilities instead of some values such as returns, gains, etc. Every Act then has a value that represent expected utilities (or expected returns/gains) for decision makers. The rational decisions are supposed to be the ones that maximize expected utilities. In contrast to this representation, Acts, Events, payoffs and the degree of

<sup>4</sup>See Baron [4] ch. 5 and 6., for a clear exposition of descriptive and prescriptive modeling in decision making.

knowledge are often known approximately in real world. Their values are imprecise, or perhaps best approximated with different representations. Hence, following Zadeh's [59] seminal paper on Fuzzy Sets and Fuzzy Decision Analysis [6], during 1970s and 1980s the principles of fuzzy theory were applied to classical statistical decision theory. These contributions include "fuzzy acts" [2, 43–45], "fuzzy events" [2, 43–45], "fuzzy probabilities" [9, 57], "fuzzy utilities" [19, 37, 57, 58], and "fuzzy information" [2, 43–45], "fuzzy linguistic modeling" [8, 15, 16]. With these contributions classical statistical decision theory is transformed into fuzzy decision theory. The importance of formulations of perceptions in fuzzy decision theory and formulations of perception based probabilistic reasoning with imprecise probabilities are articulated by Zadeh [62].

Human perception process is a flexible function of experiences. Studies have shown that attention can be directed to objects that are defined on the basis of generic grouping principles based on previous experiences [65]. Previous experiences determine the familiarity of the objects. In most experiments, it is demonstrated that object-based attention are stronger for highly familiar objects than for unfamiliar ones [52]. For instance, gestalt perceptual grouping principles which have proximity, similarity, continuity, common movement, and common fate properties are sufficient to define the objects.

Often objective function based approaches uses clustering algorithm which assigns a membership value for each observation. This value represents the degree of belongingness to each clusters. Membership functions that calculate membership values can often be assigned linguistic labels such as "low", "medium" or "high". Such labeling provides linguistic meaning representation for understanding.<sup>5</sup>

Ozkan and Turksen [35] employ perception based inference method where fuzzy clusters are treated as dictionary catalogs that serve for the basis of objects to analyze currency crisis. They successfully show that one may use Fuzzy Decision approach to understand the very important and rare event of currency crisis. According to their approach, any object can be defined as a pattern that is generated by experience. Clustering the similar patterns provide the definition of translation catalogs that are used in approximate reasoning. This approach is a process which has four properties that are; (i) clustering, (ii) similarity, (iii) flexibility, and (iv) resolution of uncertainty. In this manner, they embed gestalt perceptual prototypes by their properties of similarity, grouping, proximity and continuity in their model. This model starts with definition of decision problems given as:

[...] Assume that  $d$  is the decision problem,  $s$ , is the state of nature,  $f$  is the inference function,  $p$  is payoff,  $\bar{X} = (x_1, x_2, \dots, x_n)$  is an information vector (input vector),  $\bar{v}_x = (\bar{v}_{x,1}, \bar{v}_{x,2}, \dots, \bar{v}_{x,c^*})$  is the cluster center matrix and  $\bar{v}_{x,j} = (v_{1,j}, v_{2,j}, \dots, v_{n,j})^T$  is the  $j$ -th cluster center projected to input space. The decision problem,  $d$ , can be presented as:  $d = (\bar{X}, s, \{p = f(\bar{X}, \bar{v}_x)\})$ , and inference function,  $p$ , can be written

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<sup>5</sup>See Hoppner et al. [18], chapter 8., Rule Generation with Clustering.

$$\text{as, } f(\bar{X}, \bar{v}_x) = \sum_{j=1}^{c^*} g_j(\bar{X}) \left( \frac{\mu_j(\bar{X}, \bar{v}_x)}{\sum_{i=1}^{c^*} \mu_i(\bar{X}, \bar{v}_x)} \right) \text{ where } g_j \text{ is } j\text{th cluster's fuzzy regression}$$

function and  $\mu_j$  is membership values to  $j$ th cluster for information vector  $\bar{X}$  and the normalization term is equal to one.  $f(\bar{X}, \bar{v}_x)$  is simply a smooth interpolation (where this is the local regression models in their exposition) of models and the weights on each local model is the value of the membership function. (Ozkan and Turksen [35])

## 2.5 Conclusion

Leaders, policy makers, authorities and all parties in society are making decisions everyday under uncertainty, possibly, with some complexities. With complexities, we mean that the knowledge about both the environment and the consequences of decisions is not perfect. There all ill-defined concepts that may be changing with unpredictable patterns. It is helpful to understand the uncertainty and the tools that help us to make decision under uncertainty. In order to do so, a brief summary about uncertainty is given. Then fuzzy logic is introduced and fuzzy decisions is given as a tool to make decision under uncertainty.

Uncertainty is a phenomena that has a deep root in daily life. It makes us free to choose. Our brain converts uncertainty into fear in order to create a motivation to do something. But, as Keynes noted, still we do not know much about uncertainty. One may find that several research field has been spending good efforts to understand and if possible to handle the uncertainty. We do understand that it has several dimensions and types. It has characteristics that may be modelled with classical probability theory, evidence theory or fuzzy logic. Starting from heuristics to the modern tools of handling uncertainty is a vast area of research. Fuzzy logic is an approach that has been used effectively to decide under uncertainty.

After Zadeh's [59] seminal paper about fuzzy logic, researchers were attracted to apply the tools created with fuzzy logic for complex problems in almost every fields. The body of research has been increasing very fast. Generally first discussions are about Type 1 fuzzy logic where the degree of memberships are assumed precise. The new discussions where the degree of membership become fuzzy took place after realizing the certain degree of membership may not be the founding ground for fuzzy approaches. This new approach is called as Type 2 (and of course higher order up to Type n introduced) fuzzy systems. Fuzzy logic and fuzzy system modeling is proved to be a close to approximation of human decision making and perception processes. Therefore as the application of fuzzy system modeling, the advances in fuzzy decisions and as a particular case, perception based decision are introduced in the last section of this chapter.

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Chaos Theory in Politics

Banerjee, S.; Sule Ercetin, S.; Tekin, A. (Eds.)

2014, X, 201 p. 13 illus., 7 illus. in color., Hardcover

ISBN: 978-94-017-8690-4