

# Leonid Assur (1878–1920)

Alexander Evgrafov and Denis Kozlikin

**Abstract** Leonid Assur solved a great challenge. He devised a classification system of planar linkages with lower pairs based on the theory of mechanisms. This system turned out to be remarkably productive, as it described not only all of the hinge mechanisms known at that time, but also showed how to form the new ones. After Assur's death, his ideas were further developed in the works of his fellow Russian and foreign researchers.

## 1 Biographical Notes

Leonid Vladimirovich Assur was born on March 31, 1878 in Rybinsk (Yaroslavl province, Russian Empire). Leonid's father, Vladimir Fyodorovich Assur, worked as a customs officer at the Railway Administration. Leonid had two younger brothers: Andrey (born in 1881) and Vladimir (born in 1883). His mother, Lyudmila Andreevna, died when Leonid was seven years old. After his mother's death, his father sent the boy to live with his aunt in Vezenberg (Estonia). Another aunt, the older sister of Leonid's father, Adel Fyodorovna Assur, also lived in this house. She was a gymnasium teacher and she gave Leonid a primary school education at home. Leonid spent seven years in Vezenberg, after which he entered straightway into the fourth form of the gymnasium in Warsaw in 1892. In 1895 Leonid moved back with his father, who lived in Grodno in those days. Being in Grodno, Leonid (Fig. 1) entered the seventh form of the gymnasium and graduated

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**Fig. 1** L. Assur—in gymnasium (This photograph is from Ivan Assur's family archive. A copy of the photograph was given to one of the article's authors. Ivan Vladimirovich Assur (1929–2006) is the son of Leonid Vladimirovich Assur's younger brother, Vladimir Vladimirovich Assur. The photograph was published for the first time in Scientific Review The theory of mechanisms and machines, 2004, No. 1(3). Vol. 2.)



cum laude in 1897. By that time Leonid had a good command of Latin, Greek, French and German. Later he learned English. Apart from his genius for foreign languages and exact sciences, Leonid inherited a talent for music: he played piano and wrote original compositions (Biographical data was taken from Artobolevski and Bogolyubov (1971).)

In the autumn of 1897 Leonid moved to Moscow where he entered the Department of Mathematics of the Physic-Mathematical faculty of Moscow University. Teaching mathematics at Moscow University was a very highly sought-after professional position. Professor N. E. Zhukovsky (1847–1921) was teaching mechanics at the University at that same time. The vast sphere of Zhukovsky's academic interests also covered the theory of mechanisms, to which he had dedicated several of his works. Zhukovsky had a great influence on Assur. Some of Zhukovsky's ideas had become an impetus for Assur's analysis of topological issues.

In 1901 Leonid Assur graduated from Moscow University. At the recommendation of Zhukovsky, Assur had immediately entered the second course of the Department of Mechanics of Moscow Technical School (nowadays—Moscow State Technical University, named after N. E. Bauman). Theoretical mechanics was taught by N. E. Zhukovsky. The course of applied mechanics was worked out and taught by N. I. Mertsalov (1866–1948). Mertsalov's academic interests included the study of hinge mechanisms. He explained engineering mechanics on

## Нъ вопросу о плавности хода паровыхъ машинъ.

*Изъ № 8 Бюллетеня Московскаго Политехническаго Общества за 1906.*

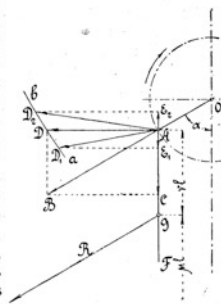
Инж.-мех. Л. Ассуръ.

### Глава I. Соображеніе общаго характера.

Когда автору настоящей статьи пришлось впервые познакомиться съ диаметрально противоположными выводами Radinger'a и Striebeck'a по вопросу о плавности хода паровыхъ машинъ, онъ былъ неприятно пораженъ неполнотой ихъ изслѣдованія. И тотъ, и другой рассматриваетъ ударъ въ кривокопномъ болтѣ и пальцѣ кривошипа, какъ результатъ перемѣны давленія подлѣ угломъ въ  $180^\circ$ . Между тѣмъ въ дѣйствительности явленіе происходитъ совершенно иначе. На палецъ кривошипа и на кривокопный болтъ дѣйствуютъ не одни только параллельныя оси цилиндра усилія, но и перпендикулярныя къ послѣдней. Первымъ изъ нихъ мы присвоимъ въ дальнѣйшемъ названіе *осевыхъ давленій*, вторымъ — *поперечныхъ*, чтобы сохранить одни и тѣ же терминъ какъ для вертикальныхъ, такъ и для горизонтальныхъ машинъ.

Во всѣхъ тѣхъ случаяхъ, гдѣ приходится разсматривать болѣе или менѣе значительныя доли оборота машины (напр. при графическомъ расчетѣ маховиковъ) вліяніе поперечныхъ слагающихся можно считать ничтожнымъ и съ достаточной для практикѣ точностью пренебречь ими. Но если мы разсматриваемъ моментъ перехода осевыхъ слагающихся черезъ нуль и смежныя съ нимъ моменты, когда эти усилія становятся исчезающе малыми, то не странно ли забывать о существованіи поперечныхъ слагающихся, которыя въ теченіе интересующаго насъ промежутка времени пріобрѣтаютъ исключительное, или по крайней мѣрѣ, преобладающее значеніе?

Прежде чѣмъ перейти къ разсмотрѣнію явленія при конечной длинѣ шатуна, разсмотримъ болѣе простой съ теоретической точки зрѣнія случай безконечно длиннаго шатуна, массу котораго назовемъ  $M$ , а длину —  $l$ . Допустимъ, что центръ тяжести  $G$  (фиг. 1) дѣлитъ шатунъ на двѣ части длиною  $\nu l$  и  $\mu l$ . Вращеніе кривошипа происходитъ съ угловой скоростью  $\omega$ . Ось безконечно длиннаго шатуна все время остается параллельной линіи мертвыхъ точекъ; слѣдовательно шатунъ движется поступательно, то есть всѣ его точки описываютъ одинаковыя траекторіи и въ любой моментъ обладаютъ одинаковыми по величинѣ и направленію скоростями и ускореніями. Въ динамикѣ доказывается, что при поступательномъ движеніи твердаго тѣла всѣ внѣшнія силы сводятся къ одной равнодѣйствующей, проходящей черезъ центръ тяжести тѣла, при чемъ послѣдній перемѣщается какъ матеріальная точка, въ которой сосредоточена вся масса тѣла. Сила инерціи  $R$  шатуна должна уравновѣшивать внѣшнія силы, дѣйствующія на шатунъ, слѣдовательно она будетъ приложена въ центръ тяжести, а величина ея опредѣлится, зная массу



Фиг. 1.

**Fig. 2** The first page of L. V. Assur's first published work «about the question of the steam machine's smoothness of movement»

the basis of application of geometrical methods such as analysis and synthesis. He also gave the solutions to several specific problems of the synthesis of hinge mechanisms.

During that time L. V. Assur was interested in the issues of the dynamics of machines and in the problems of kinematics and kinetostatics of hinge mechanisms. In 1906 he had published his first work «About the question of the steam machine's smoothness of movement» (Fig. 2) (The Proceedings of Polytechnical Society that took place in the Empire Technical School, No. 8, pp. 341–352). In the same year L. V. Assur graduated from the Moscow Technical School, entitling him to use the moniker of “mechanical engineer”. Having been unable to find a job in Moscow, he moved to the capital—Saint Petersburg.

Most Russian technical schools were located in Saint Petersburg. Nevertheless, L. V. Assur couldn't get a job as a teacher at first, and he subsequently became employed as an assistant to the head of the urban public bridge-construction workshops. He was engaged in construction planning and material support for bridge construction, working on the Alarchin, Panteleymonovsky, and Mikhailovsky Bridges.

In 1907 the Board of Academics of the Saint Petersburg State Institute had announced admission for studies at the Department of Mechanics for the first time. A job opportunity having opened up, L. V. Assur soon received an offer of employment at the Institute as a teacher of mechanical drawing in the Department of Mechanics.

In 1908 L. V. Assur was charged with conducting exercises in theoretical mechanics and applied mechanics. Lectures on applied mechanics were read by Professor V. L. Kirpichev (1845–1913), and lectures on theoretical mechanics were read by Professor I. V. Meshchersky (1859–1935). The lectures by I. V. Meshchersky were closely related to the sub-disciplines of applied mechanics, which students studied later on. Since 1907 I. V. Meshchersky had been composing tasks on theoretical mechanics that had concrete technical content. Colleagues of I. V. Meshchersky were also involved in this work, including L. V. Assur. The resulting book featuring different tasks met with exceptional success: it was eventually republished 50 times (the 50th edition in 2010).

V. L. Kirpichev had organized a scientific-technical circle at the Polytechnical University that brought young lecturers together. L. V. Assur had become a regular member of that scientific circle (Assur 1909a). In 1908 at one of the circle meetings Assur delivered his paper «Analogues of accelerations and their application to dynamic analysis of the planar linkages» (Assur 1909b). In 1909 Assur presented his second paper, a logical sequel to the first one—«Basic attributes of analogues of accelerations in analytical presentation». Full texts of these papers were published in the «Proceedings of Saint Petersburg Polytechnical Institute».

In these works, L. V. Assur outlined his theory of the analogues of accelerations. He came to a conclusion that «acceleration is a particular case of a more general concept—the analogue of acceleration. Therefore, every theorem derived for analogues of accelerations implies a corresponding theorem for accelerations». Assur had pointed out the possibilities of application of his theory. Among other factors, he had devised the foundations of the graphical analysis of mechanisms with several degrees of freedom. This problem has not only theoretical but also practical importance, especially in our day, and it was Assur who first raised this problem as well as finding the solution to it.

In 1911 Assur took a break from his work on the theory of analogues of accelerations. He had published two text-books for students, one right after the other: «Velocity and acceleration vector diagrams of planar mechanisms» and «Graphical methods for determination of the moment of inertia of a flywheel» (Assur and Roerich 1911). The Assur family picture shown above was likely taken in the same year (Fig. 3).

At the same time he passed examinations and began to work on a doctoral dissertation that had become his life's work: «Research of planar linkages with lower pairs on the basis of their structure and classification».

In 1914 in the «Proceedings of Saint Petersburg Polytechnical Institute» the first part of his work was published—«Teaching about normal multi-arm chains and their role in the formation of mechanisms» (Assur 1914a). In 1915 the second part was issued: «Application of the teaching on normal chains to the general theory of mechanisms» (Assur 1914b). The same year Assur went to Moscow



**Fig. 3** Family picture (sitting: at *left*—V. F. Assur, at *right*—E. M. Assur with son Vsevolod; standing at *right*—L. V. Assur) from the family archive of S. V. Assur—L. V. Assur's grandson. Published for the first time

where he showed his dissertation to N. I. Mertsalov and N. E. Zhukovsky. On February 13th, 1916 at the meeting of the Scientific Council of Saint Petersburg Polytechnical Institute L. V. Assur presented his Ph.D. defense. The official opponents were leading scientists who dealt with the different issues of applied mechanics: professor of Moscow University and Moscow Technical School N. E. Zhukovsky, professor of Kazan University D. N. Zeiliger (1864–1936) and professor of applied mechanics and the dean of the department of mechanics A. A. Radzig (1869–1941). The defense was successful and Assur got a degree of adjunct in the Department of Applied Mechanics. The following picture of L. V. Assur was taken at that time (Fig. 4).

**Fig. 4** Photography of L. V. Assur from the family archive of S. V. Assur—L. V. Assur's grandson.  
Published for the first time



After his Ph.D. defense, Assur couldn't be engaged in scientific work with the same intensity as he had been. In 1914 World War I began. Russia's involvement in the war led in May 1916 to Assur being enlisted to work on the Petrograd Military-Industrial Committee for front assistance. Engineers working on this committee had very high qualification. Assur conducted the project planning for different systems of military assets.

After the Revolution of 1917 and with the beginning of the Civil War, food supplies and provision of necessities had deteriorated significantly. Some lecturers left the city. Assur had to teach at two institutes: at the Polytechnical Institute and the School of Forestry. His teaching load had increased greatly, taking up time that could have been spent on scientific work. As time went on, Assur's health was ruined. In June 1919 Assur went to Voronezh where his family lived. He planned to return to Saint Petersburg (called Petrograd in those days) by the beginning of the new semester, but owing to the conditions caused by the civil war, he didn't get a chance to do that. Assur's health never improved. In May 1920 he went into the hospital, where he had two operations. After the second operation, on May 19th he died without regaining consciousness.

L. V. Assur was married to Elena Mikhailovna Mindrina.<sup>1</sup> They had three children: Olga (1907–1909), Vsevolod (1910–1987) and Elena (1913–2001). Olga died tragically at the age of two. Vsevolod and Elena lived long lives and worked as lecturers. The grandchildren and great grandchildren of L. V. Assur live in Moscow, Saint-Petersburg and Paris.

Assur's manuscripts didn't survive. According to family legend, the manuscripts were bought by a nameless scientist, but the authors of this article were unable to find the proof to substantiate that legend.

## 2 List of Works

About the question of the steam machine's smoothness of movement 1906.

Analogues of accelerations and their appliance to dynamic analysis of planar linkages 1908 Finished 1909.

Basic attributes of analogues of accelerations in analytical presentation 1909.

Velocity and acceleration vector diagrams of planar mechanisms 1911.

Graphical methods for determination of the moment of inertia of a flywheel (co-authorship with K. E. Roerich) 1911.

Die Methode der charakteristischen Kurven als Beitrag zur graphischen Auswertung mehrfacher Integrale 1912.

Research of planar linkages with lower pairs on the basis of their structure and classification:

Part 1. Teaching about normal multi-arm chains and their role in the formation of mechanisms 1914

Part 2. Application of the teaching on normal chains to the general theory of mechanisms 1915.

An addition to the second chapter of the first part 1915.

An addition to the first chapter of the second part 1918.

Geometrical construction schemes of some curve lines 1916.

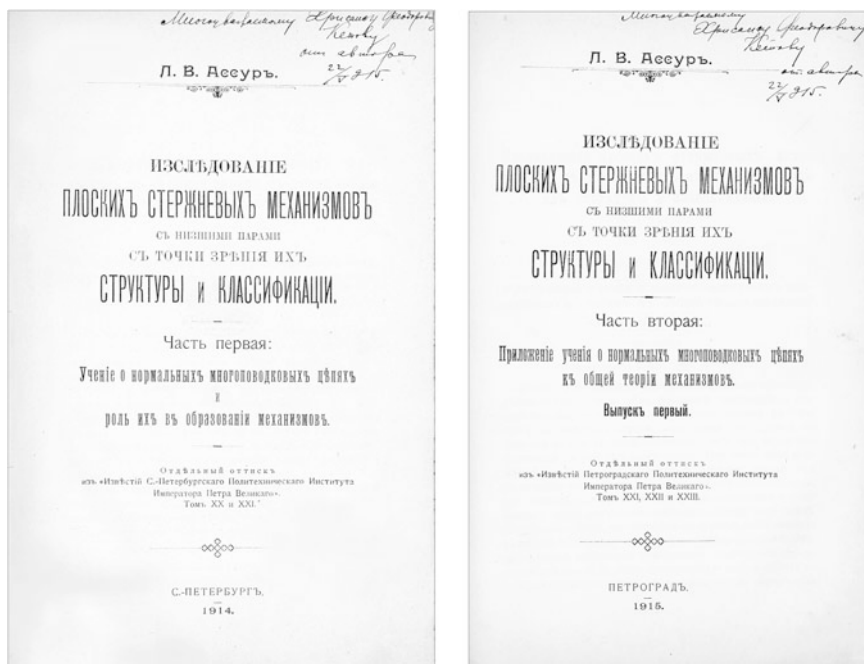
## 3 A Review of the Treatise by Leonid Assur

Undoubtedly, the work most assuring Assur's place in the history of science is «Research of planar linkages with lower pairs on the basis of their structure and classification» (Fig. 5).

The book consists of two parts: Part 1. Teaching about normal multi-arm chains and their role in the formation of mechanisms (published in 1914); Part 2.

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<sup>1</sup> In the monograph by I. I. Artobolevski and A. N. Bogolyubov «L. V. Assur», the surname of Assur's wife is printed as Mindlina by mistake.



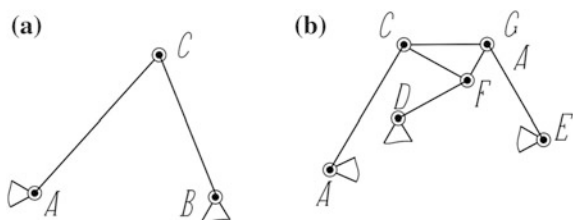
**Fig. 5** Title pages of the book «research of planar linkages with lower pairs on the basis of their structure and classification», parts 1 and 2, autographed by the author. Published for the first time («To dear Chrisanph F. Ketov from the author, 22/V 915». C. F. Ketov (1887–1948) at the time this book was signed in 1915 was a young professor at Leningrad State Polytechnic Institute (now St. Petersburg State Polytechnic University) and would later become Head of the Department of «Theory of Mechanisms and Machines». The book belongs to the Fundamental Library of SpbSPU.)

Application of the teaching on normal chains to the general theory of mechanisms (published in 1915). Later additions were made to chapter 2 of the first part (1915) and to chapter 1 of the second part (1918).

The first part opens with an introduction in which the author gives a short summary of the systems of classification of mechanisms developed by his predecessors: G. Monge (1746–1818), A. Ampere (1775–1836), Ch. Laboulaye (1813–1886), F. Reuleaux (1829–1905) and some smaller contributions by other researchers. Giving preference to the classification of Reuleaux, Assur notes that it nevertheless does not embrace all the existing practices of creation of new combinations and forms of the time. At the end of the introduction, Assur formulates the problem with the research:

«The only theory which may claim scientific value is that which is in a position to specify the ways toward practice: the concern of science is to specify all that is possible, the concern of practice is to choose the practical from the possible. The author hopes that his work will serve as a contribution promoting a scientific

**Fig. 6** Two-arm (a) and three-arm (b) groups



viewing of that area of the theory of mechanisms, which it directly concerns, and that the reader will find in it the aforesaid general scheme».

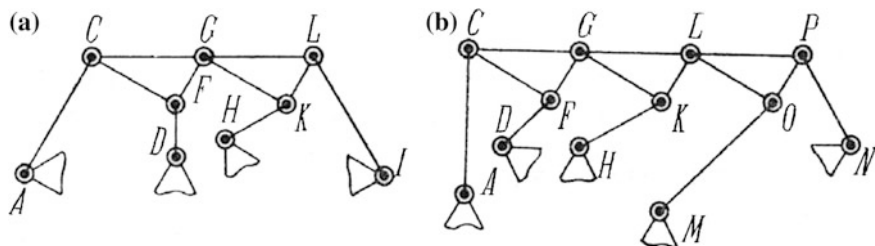
Chapter 1 is called «Survey of the existing methods of formation of new mechanisms». Assur looks for the method (though obviously does not specify it) through which the questions of the synthesis of mechanisms, their structures and classification are organically related. Knowing the place of a mechanism in the classification, it is possible to select the most convenient method for its calculation. The author examines the works of L. Burmester (1840–1927), F. Reuleaux, A. Ampere, M. Grübler (1851–1935) and others. However, Assur did not find a method which would meet his demands. Therefore he puts forward a problem for working out such a method.

In this chapter, Assur sets out the formulation of his classification. He considers the mechanisms with multi-arm open chains of normal types, correlating their structure to the method of their formation. Assur approaches one of the elementary methods of formation of mechanisms: the Silvester dyads are attached to the existing mechanism (Fig. 6a) (Assur calls them two-arm groups). A new mechanism is formed. A dyad consists of two links and three joints, the degree of freedom equal to zero. In the same position of extreme joints, the central joint (and, accordingly, the two links of a dyad) can occupy two positions (we would call it two ways of linkage now). If all three joints (and two links) are placed in one straight line, the dyad gets into a special position (at present they often call them singular). The method also works when any of the joints are replaced with translational pairs (the author excludes the case of three translational pairs).

Further, Assur increases the number of links in a connected chain (group) by means of his method of arm development. The main condition of the formation of such a chain is its degree of freedom being equal to zero. In this case, the chain is statically determinate. Assur referred to this as a normal type chain.

The method of development of the arm consists of the following. To get a new chain, one of the arms (for example, BC in the Silvester dyad) is replaced with a two-arm link (this is a system of three links: DF, EG and CGF). A three-arm chain of normal type is formed (Fig. 6b). Sequentially and repeatedly joining the three-arm groups, one can form new mechanisms.

Applying the method to three-arm chains, Assur builds a four-arm chain, then a five-arm (Fig. 7), etc. Assur comes to an important conclusion: setting an initial configuration of a chain (the way of joining), we define the movement in an



**Fig. 7** Four-arm and five-arm groups

absolutely unique way. Here, Assur shows the way to define special (singular) chain positions.

Thus produced, multi-arm chains (group) possess the following property: being fastened to an unchangeable basis (frame), they form a rigid connection, i.e., such a chain has a zero degree of freedom. Such groups, being joined to the mechanism, always give rise to a new mechanism more complex than the original. Assur formulates the essence of the *method of stratification*: the formation of mechanisms by consecutive joining of a series of normal type groups.

Assur begins the classification with Class I mechanisms. In this classification, Class I mechanisms are all the mechanisms derived from a crank by consecutively adding simple open multi-arm normal type chains. He defines the order of Class I mechanism by the number of arms of the most complex multi-arm chain of the mechanism. Thus, the majority of known mechanisms are the Class I mechanisms of the second order. The mechanisms containing three-arm chains (Stephenson guide, Gutch guide) belong to Class I mechanisms of the third order. The Heisenherr guidance mechanism is the only realized Class I mechanism of the 4th order.

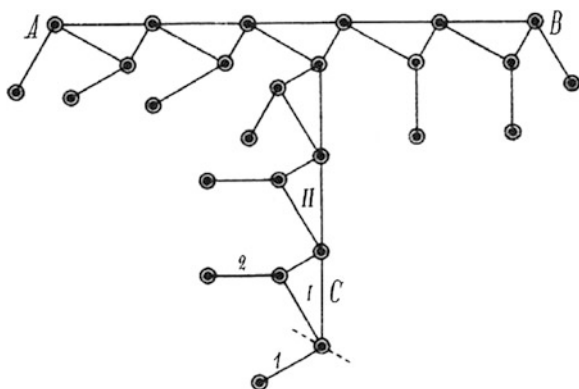
Developing chains of 5 and more arms, Assur discovered the chains with branching (Fig. 8). These are special formations, clearly distinguishable from the other types of chains. Assur called such chains *complex* open multi-arm chains of normal type.

Assur defines a Class II mechanism as any mechanism which can be reduced to a crank by separation of only multi-arm open chains of normal type, just complex or both simple and complex.

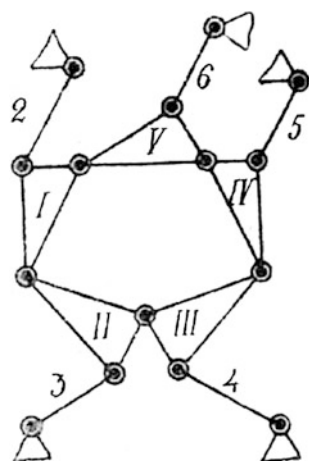
The chains lying in the basis of the formation of Class I mechanisms are what Assur calls Class I chains. The other chains forming Class II mechanisms are Class II chains. The difference between Class I and Class II chains is only that the Class II chains have central links adjacent to more than two neighboring links.

Having considered the possibilities of building Class I and Class II chains, Assur proceeds to the further development of the method. In Chapter 3, he considers simple multi-arm closed chains of normal types with simple and expanded arms. Assur detached the extreme links of an open chain from the frame and connected them between themselves, forming a closed chain (Fig. 9).

**Fig. 8** Complex open multi-arm chain of normal type



**Fig. 9** A simple closed chain



Simple closed chains with expanded arms, as well as those containing them as overlays, are attributed by Assur to Class III. The resulting chains may be distinguished by the number of links of a simple closed chain with simple arms and by complexity of the arms' development. The number of extreme links of a simple closed chain with developed arms defines a chain *category*, and the number of rigid links with simple arms, a chain *order*.

To represent the structure of complex mechanisms graphically, Assur makes a schematic drawing of a structure, consisting of circles and connecting lines (Fig. 10). A circle designates an armless link, a strikethrough circle a single arm link, and two concentric circles a two-arm link. Lines designate the chains connecting the links. Near the line, he puts a figure specifying the number of chain links (when it is important). Sometimes, designations are complemented and changed. The arm coupling chains are designated by straight lines, one-arm links and their groups adjacent to armless links by curves. A dot line designates *locks*—a set of two three-joint links connected with a common joint. The links which are a

**Fig. 10** Scheme of a class III chain of 6th category, 28th order



part of the lock are called lock links. When three curvilinear segments are linked to such a circle, they form a *nodal point*. In other words, a nodal point is an armless link to which three one-arm links are connected.

To study these formations, one has to set a possibility of traversing all the links once, returning to a starting link. For this purpose, assuming flexibility and extensibility of the connecting segments, we may deform the scheme without breaking the mutual arrangement of its components so that all the nodal points (armless links) are placed in a round formed by the segments connecting adjacent links.

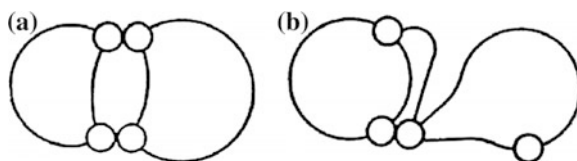
The schematic drawing of the chains led Assur, as he writes, «to the formations treated in a not very popular section of geometry called *Geometria* or *Analysis situs*, a part scattered in separate journal papers, not having any systematic textbook in any languages till now». At present this field of mathematics is called topology. It is quite remarkable that Assur independently arrived at the statement of topological problems.

In Chapter 4, Assur converts Class III chains and gets the complex closed chains which cannot be attributed to Class III. If a complex closed multi-arm chain allows full traversal of all nodal points, then Assur attributes it to Class IV. The problem of structural analysis of mechanisms becomes topological and rather complicated at that. Assur writes: «All the chains investigated in this chapter we will attribute to Class IV. It does not include all the types in this class and the problem as a whole ... the further we analyze, the more complex the issues in point become, the more complexities are piled up ...». Listing the unresolved questions, Assur concludes: «These are the questions for which the answers may be given shortly, although it is possible that their solution is far from being simple».

All the chains which belong to the said four classes Assur includes in one big group, which he calls the first family. Assur addresses the further development of the chains in Chapter 5.

Assur builds the chains of higher families of links, which he calls the links of the second genus. A *link of the second genus* is an independent group of the groups connected between themselves by external coupling chains, provided that it is impossible to connect a portion of the existing groups into one, also allowing

**Fig. 11** A chain of the first (a) and second (b) family



traversal of all armless links. When such a chain forms a rigid, statically determinate joint with a frame and does not disintegrate into the elementary normal ones, it becomes one of the types of the chains of a higher family (Fig. 11).

The chains of the second family possess a great variety of forms in comparison with the chains of the first family. In the second family, the links of second genus can be connected not only with joints (as in the first family), but also with complex open chains.

The first class of the second family forms open chains consisting of 2, 3 and  $n$  links of the second genus. The second class is formed by the chains consisting of the links of the second genus and built in the manner of complex open chains. The third class consists of the chains formed by a closure of simple chains, differing in that there is only one traversal, and there is no third way between the two central links, apart from the two formed by traversal links.

In the case of a Class II chain of the second family, a replacement of such extreme chains with an arm will result in a complex closed chain not containing the latter, but will not eliminate all nodal links; the latter should allow traversal.

If a complex closed chain, not having extreme chains, does not allow traversal of all nodal links, then the nodal links of the second genus break into groups, each of which allow such a traversal. This consideration points to the presence of chains of the third family formed by the links of the third genus where each link is formed from a closed chain of the second family. As it is for the second family, four classes are specified; in the third family, we specify four and, using a traversability criterion, we come to the chains of the fourfold family whereof each link is formed from a closed chain of the third family, etc., indefinitely.

This is, in general, the classification of kinematic chains developed by Assur. It is investigated in detail for the first three classes of normal chains of the first family. The studies of Class IV chains are incomplete; there is no separation of the class into categories and orders. The study of chains of higher families was planned in general.

All existing mechanisms fall into a rather small group of structural formations and only occasionally get to the formations of other types. Assur demonstrated a possible rich range of mechanisms which can be derived by complicating their structure following a definite rule.

When a normal chain of any family and class is connected by joints with a frame, then a more complicated structure—a girder—is formed. Among the girders, one may find the instances of chains of very high classes. Assur thinks that this may have an application for the statics of constructions.

The second part of the treatise is dedicated to the design of a system of kinematic and static analysis of mechanisms, generalized enough to be applied to the studies of any mechanical structure. This problem had not been articulated prior to Assur.

In the introduction to the 2nd part of his study, Assur justifies the necessity of having a classification of mechanisms for conducting kinematic analysis. He gives the examples of when the generally specified methods of kinematic analysis appear suitable only for special cases, for example, for the Class II mechanisms with two-arm links. Assur writes: «Here, the 1st part of our work of classification should be a guideline, on the one hand, allowing us to split the mechanisms into families, classes, categories, orders, establishing certain features of various groups; on the other hand, these are precisely the structural differences which are accepted as the basis for the discrimination.» Setting forward the problem of definition of the speeds of points of a mechanism, Assur chooses the method of defining speeds and accelerations developed by Ch. Mohr (1835–1918). This method, in certain respects, was enhanced and supplemented by Assur.

Chapter 1 of the second part is devoted to the drawing of a picture of the speeds of points of a mechanism through graphical methods. First, a construction of a picture of speeds two-arm, three-arm and four-arm groups is presented. Special (singular) positions are specified, especially for the three arm group. Furthermore, Assur expands the obtained technique to the chains of the first class of all the orders.

After that, Assur consistently considers drawing of the picture of speeds of mechanisms of the second, third and the fourth classes of different types. In the latter case, the author notes the extreme complexity of the question. Nevertheless, he successfully solves this problem, though not completely; this speaks to the restricted possibilities of the graphical methods of research. In effect, all these are the variants of two methods—the method of singular points developed by Assur, and the method of special positions which he borrowed from Mohr, but further developed and complemented.

In this chapter, Assur considers statics of planar linkages. He studies the statics, pursuant to the two main principles: a principle of possible displacements (here he uses the technique of Mohr), and the principle of mutual polyhedrons of J. Maxwell (1831–1879). Assur writes:

Before applying the great general principles, one has to try to divide the mechanism into basic elements for the first time specified in our work, into the normal multi-arm chains the mechanisms are composed of. Only after it has been found out does it become clear how to apply the specified general principles to the mechanism, and also to a rigid connection formed from an abutment and one or a series of stratifications of normal chains. It distinctly points to the organic relation existing between our classification ... and the properties of mechanisms, in particular the methods of kinematic, static and also, in connection with what was laid out in the introduction, dynamic studies of mechanisms.

Thus, Assur finds the counterbalancing force by means of the theory of the auxiliary lever (later called the Zhukovsky lever). Geometrical interpretation of this method foresees the use of a picture of speeds considered in Chapter 1.

Further, Assur addresses a problem of definition of interacting forces between the links (constraint forces). He consistently considers the chains and mechanisms of the 1st, 2nd, 3rd and 4th classes of the types studied in Chapter 1. Apart from a few instances, Assur prefers the graphical methods almost every time. Often they appear to be very complex; they lose accuracy and clarity, although clarity is the basic advantage of graphical methods in comparison with analytical ones.

Assur perfectly understands the immensity of the problem. He writes:

It may seem almost improbable that in the branch of science quite attended in the XX-th century, there was an area which was approached closely but which nevertheless remained unknown as a sealed book. Having found a key to this area in the quite simple domain of development of an arm, the author found himself confronted by an enormous problem. Like a person who has entered a virgin forest, he had to act in an absolutely autocratic and independent manner; he could not find any laid roads or footpaths here; they brought him only to the borderline of this domain. This area is rather broad though, and for its successful study in full one needs something more than the key idea of development of an arm, which opened this area to the eyes of the observer, allowing for the definition of its essence and making it possible to break it into the sections subject to study. The latter were many, an enormous number, the material for a lifetime's research. There are probably other, more convenient approaches to the separate sections, the keys to which have not been found yet. Perhaps it is destined that other generations will find them.

## 4 On the Circulation of the Treatise

Assur's «Research of planar linkages with lower pairs on the basis of their structure and classification» was published in limited circulation. The treatise turned out to be too complicated, and besides, the author had died, meaning Assur's ideas not only didn't receive much development, but were also completely forgotten for several years. Scholars mentioned Assur's classification in their work, but didn't use it to solve different tasks of analysis and synthesis of mechanisms. In fact, even in Assur's motherland of Russia, universities taught the structures of mechanisms according to F. Reuleaux. For the classification of mechanisms, different types of the R. Willis classifications were used. Mechanisms were calculated in an old manner: for every task, its own genuine way was found. Even those scholars who knew Assur's treatise well didn't use its ideas. N. I. Mertsalov, Assur's teacher, to whom Assur had shown his dissertation, considered Assur's method to be ineffective, both for the theory and for engineering practice. A. P. Malyshev (1879–1962) maintained that Assur's work was written in a complicated manner and that the developed methods were also too sophisticated. A. A. Ratsig (Assur's opponent at the defense of the thesis) and A. P. Ivanov (Assur's student, who studied applied mechanics) did mention Assur in their textbooks on applied mechanics (in 1930 and 1934 respectively), but didn't use any of his ideas. In 1925 in Leningrad the posthumous edition of a text-book of D. S. Zernov «Applied mechanics» was issued, edited by Ch. F. Ketov. In this book, Ketov, as editor, had confined himself to only one reference to Assur's treatise.

The status of Assur's ideas began to change, owing largely to I. I. Artobolevski (1905–1977), N. G. Bruevich (1896–1987) and V. V. Dobrovolsky (1880–1956). It should be mentioned that the three of them worked at the same department—the Department of Mechanics at the Air Force Academy in Moscow.

I. I. Artobolevski had read A. P. Malyshev's article from 1928, which had introduced him to the works of Assur. Artobolevski had found Assur's treatise with great difficulty, and that was a very rare exemplar. After he had acquainted himself with Assur's work, Artobolevski realized that Assur's theory opens up great possibilities for the development of a general theory of mechanisms. In the same year at the conference at the Moscow Textile Institute Artobolevski read a paper about Assur's work. Assur's structure and classification was enunciated by Artobolevski for the first time at the Moscow Institute of Chemical Machinery and then soon after at Moscow University.

The other scholar, N. G. Bruevich, was also one of the first to appraise the potential of Assur's ideas. In 1935 his work «Application of vector equations in the kinematics of planar mechanisms» was published. Using Assur's methods, Bruevich elaborated the method for solving the problems of kinematics using vector equations.

In 1936 V. V. Dobrovolsky, at the meeting of the theory of mechanisms team affiliated with the Technical Committee of the Academy of Sciences, presented a paper «Basic principles of the classification and structure of mechanisms», in which he set out Assur's main ideas.

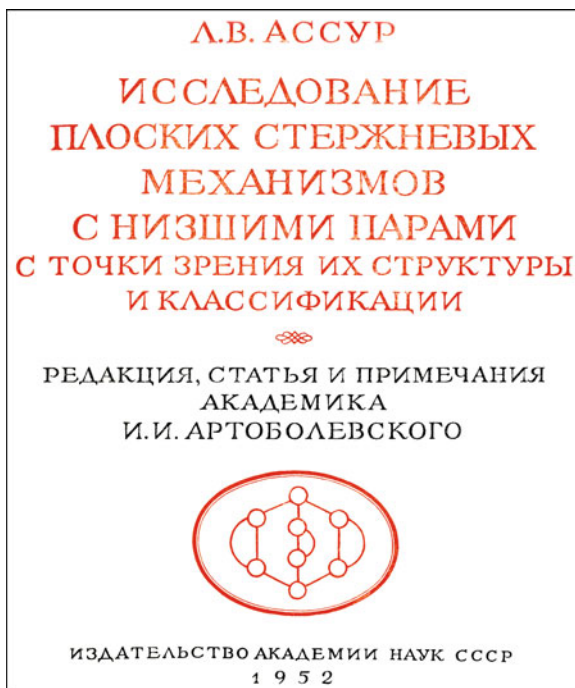
In 1937 and 1938 on the basis of the lectures held at the Moscow Institute of Chemical Machinery and at Moscow University, Artobolevski had published a book, «Theory of machines and mechanisms». In this book, as well as in his lectures, Artobolevski described classifications and structures of mechanisms using Assur's ideas.

In 1938 the educational program for students «Theory of mechanisms and machines» with regard to Assur's study was elaborated and approved. This created a demand for the compilation of new text-books.

In 1939 the first text-book «Theory of mechanisms and machines», written in accordance with the new program, was issued. The authors of the text-book were Ch. F. Ketov (Assur presented him the copy of the treatise) and N. I. Kolchin (1894–1975). The first section of the text-book was dedicated to the analysis of the structure and classification of mechanisms according to Assur.

In 1940 the first edition of I. I. Artobolevski's text-book «Theory of mechanisms and machines» was issued. The author wrote about the classification and structure of mechanisms: «During the period from 1914 to 1918 an outstanding work by the Russian scholar Assur on the classification of planar mechanisms with a very profound analysis appeared. This work sets forth the classification of planar mechanisms, which is closely connected with the methods of kinematic and kinetostatic analyses of these mechanisms. The classification elaborated by Assur comprises almost all of the existing planar mechanisms in engineering. This treatise should be recognized as a work of exceptional value that has provided a wealth of material for a number of further studies by Russian researchers. The fact

**Fig. 12** Title page of Assur's treatise's second edition (1952)



that the classification of Linen is used in only certain places abroad and occasionally in the USSR may be explained by the lack of awareness of Assur's works, as the classification of Linen may be fully inscribed in Assur's classification».

In 1952 the second edition of «Research of planar linkages with lower pairs on the basis of their structure and classification» was issued by the USSR Academy of Sciences printing house under the editorship of I. I. Artobolevski (within the series «Science classic scholars») (Fig. 12). The book comprised the title treatise, two additions written by Assur in 1915 and 1918, and the annex—a review by N.E. Zhukovsky about the treatise, an article by Artobolevski «L. V. Assur and his works on the theory of mechanisms», annotations and a list of scholarly works by Assur. The book had a fairly large circulation (2,500 copies) for a scientific monograph. Assur was acknowledged as a classic scholar of the theory of mechanisms.

This edition of the book had sparked a rising tide of interest in Assur's ideas. Assur's classification became an initial system of the structural classification for further studies within the Russian school of theory of mechanisms and machines. It was used in almost every textbook on the theory of mechanisms: S. N. Kozhevnikov («Theory of mechanisms and machines», first edition published in 1947, was translated into Polish, Czech, Hungarian and Chinese), N. I. Kolchin («Machine engineering» in five parts, first edition published in 1948–1957), Vl. A. Zinoviev («Theory of mechanisms and machines», first edition published in 1955, translated into English in 1963) and many others.

The works of Assur weren't translated into foreign languages, but his theory had become well known outside Russia. Foreign researchers started to use Assur's classification in their own works. At the same time, they made no references to the first or second editions of Assur's treatise. Recently, there had appeared some publications referring to Assur's classification as one that was commonly known, without any reference to the original source. This is a kind of an acknowledgement and recognition of the achievements of Assur in the field of the theory of mechanisms.

## 5 Modern Interpretation of Main Contributions to Mechanism Design

Actually, Assur's work has never been finished. It is known that Assur continued working on the classification, but his papers were lost. However, his ideas were taken over by other researchers.

In the 1930s I. I. Artobolevski developed a theory of three-dimensional mechanisms. In 1937 his monograph «The theory of three-dimensional mechanisms» was published. Artobolevski came to the conclusion that there was much in common in the structure of spherical and planar mechanisms. By analogy with Assur's groups, he developed a theory of multi-axial spatial groups. He noted the possibility of the reverse influence of classification of three-dimensional mechanisms on the classification of the planar ones.

In 1939 Artobolevski published a monograph «Structure, kinematics and kinetostatics of multi-link planar mechanisms» (Artobolevski and Dobrovolsky 1939). In this work, he ingeniously develops Assur's theory. Apart from the studies of the structures of mechanisms, the book contains a solution to the problem of definition of the groups' positions. The kinematics of mechanisms includes the questions of defining the accelerations, in addition to the speeds. The graphical methods applied and developed by Assur were then supplemented by an analytical method for vector equations. The developed graphical/analytical method appeared to be more clear, convenient and accurate than the graphical method. Assur's classification was expanded to the mechanisms which included translatory and higher kinematic pairs. The analysis of kinetostatics embraced the groups up to the third class of the higher orders.

Assur's principle of building a classification was used by V. V. Dobrovolsky. He stated his ideas in the works «The main principles of rational classification of mechanisms» and «The theory of mechanisms». Dobrovolsky (1953) suggested putting several layers of kinematic chains on an initial mechanism which can change the mechanism's degree of freedom. For such chains, he used the term «non-Assur chains» for the first time. Depending on whether the connected chain increases or reduces the degree of freedom, it is attributed to a group of positive or negative order. In this respect, the Assur chain not changing the degree of freedom of an initial mechanism belongs to the chains of zero order. The Assur groups, as they are kinematically and statically determinate, are of the highest

spread. Dobrovolsky sees an expediency of introduction in non-Assur chains in that «they can join several mechanisms with independent movements into one, and thus, serve as a transmission medium between them». Dobrovolsky's work is also remarkable for his having tried to make a structural analysis of a mechanism not with one, but with several degrees of freedom.

I. I. Artobolevski also continued to develop Assur's ideas. In his work «Experience of the structural analysis of mechanisms» and in other works, he defined a structural group. *The Group* is a kinematic chain which, after its joining by extreme free pair elements to a frame, acquires zero degree of movability and which cannot be separated into independent kinematic chains of zero degree of movability. Assur assumed no difference between the notions of group and chain, and used both equally.

According to the classification of Artobolevski, the structural Assur groups are characterized by class and order. A group order corresponds to the number of elements of the kinematic pairs by which the group is connected to the links of previous groups and to the frame. A group class is defined by the number of kinematic pairs which include the links forming the highest contour of the group.

G. G. Baranov (1899–1968) in his study «Classification, structure, kinematics and kinetostatics of planar mechanisms with the pairs of the first kind» (published in 1952) offered another system of classification of the planar Assur groups. According to that work, a group class  $k$  equals one half of the number of its links, thus a two-link group (dyad) is the group of the first class, a four-link group is the group of the second class, and so on. All the groups of class  $k$  are divided into orders, as was suggested by I. I. Artobolevski.

I. I. Artobolevski's classification became the most widespread in Russia. It was not without its drawbacks, as noted by V. V. Dobrovolsky, G. G. Baranov, E. E. Peisach and others. The scientists mentioned that this classification did not fully characterize each separate group and did not embrace all possible groups of few links.

E. E. Peisach (1936–2008) offered an improved definition of an Assur group, more convenient for the structural analysis of mechanisms. On the basis of this definition, he offered a new classification of structural Assur groups. Its two basic structural criteria are the *class* and the *category*. Besides, there are four more additional structural categories—the *number of links*, the *number of kinematic couples*, the *order*, and the *number of changeable closed contours*. When the class and the group category are known, then all four additional structural criteria are defined univocally, i.e., they are dependent on two basic criteria. It is possible to present each mechanism in the form of a certain symbol code. Considering the symbol codes of the structures and casting out the repeated ones, it becomes possible to make electronic catalogues of the planar structural groups of various classes with rotational pairs. By means of the algorithm offered in the «Classification of planar groups of Assur» (2007) and the corresponding program, E. E. Peisach performed a structural synthesis of the Assur groups up to the sixth class inclusive, i.e., for  $k = 1, 2, 3, 4, 5, 6$ , and made an electronic catalogue of the Assur groups for  $k = 2, 3, 4, 5$ .

In 2000 the Springer publishing house published a book by a group of authors (M. Z. Kolovsky, A. N. Evgrafov, Yu. A. Semenov, A. V. Slousch), «Advanced Theory of Mechanisms and Machines». In this book, the following definition of a structural group is offered.

«A kinematic chain with given inputs is referred to as a *normal structural group of movability  $n$*  or simply a *structural group*, if the number of independent chain inputs  $n_c$  coincides with the number of degrees of movability  $w_c$ . A *simple* structural group is one that cannot be split into several structural groups with smaller numbers of links. A simple structural group may have zero degrees of movability (and, therefore, zero inputs as well), i.e.,  $n_c = w_c = 0$ . Such a structural group is called an *Assur group*».

Such a structural group, as well as an Assur group, is kinematically and statically determinate, and an Assur group is a special case out of this group. Layering such groups (as well as an Assur group) on an initial mechanism or on a frame, it is possible to gain new, more complex mechanisms which will always be kinematically and statically determinate. Numerous mechanisms of modern cars, including spatial, multimotor, open-circuited ones, can be presented in the form of a chain of such structural groups. In the article «Program of the structural analysis of mechanisms» by Karlovskiy et al. (2005), a corresponding computer program is described. This program automatically separates Assur and non-Assur structural groups in a mechanism.

Assur's ideas came to the notice of foreign researchers thanks in a large part to I. I. Artobolevski. The notion of an «Assur group» became a recognized term. Many authors use the concept of an Assur group and Assur's ideas: Bouzakis et al. (2004, 2008), Calle et al. (2010, 2011), Ceccarelli (2004), Ceresole et al. (1996), Cheng (2005), Crossley and Seshachar (1971), Zhang and Xu (2011), Galletti (1979), Servatius et al. (2010a, 2010b), Shai (2010), Tereshin (2003, 2004), Dvornikov (2011), Ionescu (2003) and others.

In their article, Sun and Tang (2009) acknowledge the significant role of the methods of Assur groups in engineering planar mechanisms, and they also point out the moderate popularity of this method. Galletti and Giannotti (2009) present in their article an instrument for modeling the kinematics of planar groups in the system SIMULINK, which is based on the method of Assur groups, and for using it in teaching practice. Romaniak (2007) in his article offers the methodology of developing the Assur groups, dividing them into simple, complex and multiple ones. Calle et al. (2011) examine in their article the solution to the problem of kinematic analysis of Assur groups of class four (4). Wohlhart (2008) article is dedicated to the analysis of the positions of open Assur groups and the formation of planar open mechanisms. A short list of these and some other works of foreign researchers is given below in References.

To sum up, it can be said that Assur as a scholar was twice lucky. First, he discovered and developed a very viable and fruitful idea. Secondly, this idea wasn't forgotten after Assur's death, and was treated to further development by other researchers.

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