Chapter 2
Suboptimal Recursive Methodology for Takagi-Sugeno Fuzzy Models Identification

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2.1 Introduction

When there is no one mathematical model of the system it is necessary to use modeling techniques based on input–output data (López-Baldán et al. 2002). This process is critical in control systems, since both, system analysis (Al-Hadithi et al. 2007; Andújar and Barragán 2010; Andújar et al. 2006; Aroba et al. 2007; García-Cerezo et al. 1994; Gordillo et al. 1997; Jiménez et al. 2009) and controller design (Andújar and Bravo 2005; Andújar et al. 2009; Wang et al. 1996), require to obtain a model as accurate as possible.

Fuzzy modeling, especially modeling based on Takagi-Sugeno (TS) fuzzy systems, allows to obtain highly accurate models from a small number of rules (Wang 1997). As it is known, TS models are universal approximators, and they can achieve high accuracy with a small number of rules (Kosko 1994; Wang 1992; Ying 1998; Zeng et al. 2000), therefore, is an ideal tool for this purpose. However, it is also known that the number of rules in TS models is increased as a lower approximation error is desired (Kóczy and Hirota 1997), so the quality of the modeling algorithm

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can be critical. On the other hand, is relatively easy to convert them into nonlinear state models (Andújar and Barragán 2005; Andújar et al. 2009), which support formal analysis to use in control engineering. Up to date, many fuzzy modeling algorithms based on input–output data have been proposed (Babuška 1995; Denaï et al. 2007; Horikawa et al. 1992; Jang 1993). Moreover, in many cases it is required that modeling algorithm works online with the system, and do it properly in the presence of noise.

In order to design a TS fuzzy modeling algorithm based on input–output data, which can work online with the system, properly in presence of noise and can be very efficient computationally, the extended Kalman filter (EKF) (Kalman 1960; Maybeck 1979) is used. The Kalman filter is the minimum-variance state estimator for linear dynamic systems with white noise with zero-mean value. This is an efficient recursive filter that estimates the internal states of a linear dynamic system from a series of noisy measurement. It is used in a wide range of engineering and econometric applications, from radar and computer vision to estimation of structural macroeconomic models, and it is an important topic in control theory and control systems engineering. The EKF (Maybeck 1979) is a modification of the Kalman filter that can be used to estimate the state in nonlinear systems. EKF linearizes the system around the current parameters. These algorithms update the parameters been consistent with previous data, and usually converges in a few iterations.

The Kalman filter has been used with fuzzy logic in various applications, such as the extraction of rules from a given rule base (Liang and John 1999), parameters optimization of defuzzification mechanisms that are based on both Gaussian and polynomial distributions (Jiang and Li 1996) or in optimization of consequents of Takagi-Sugeno models (Ramaswamy et al. 1993). In 2002, Simon introduced the use of Kalman filter for adjusting the parameters of a TS fuzzy model (Simon 2002, 2003), assuming that antecedents were membership functions of triangular type, and using its center of gravity to perform the adaptation process. Later, other proposals have been made (Al-Hadithi et al. 2012; Chafaa et al. 2007).

Motivated by the successful use of Kalman filter in the works presented above, and considering that this algorithm can be applied in real time (Jiménez et al. 2008), a general methodology for use EKF to estimate the adaptive parameters of a general TS fuzzy model, was presented in (Barragán et al. 2011a, b, 2013). This methodology uses the excellent features of Kalman filter to obtain fuzzy models of unknown systems from input–output data. This chapter focuses on this latter approach, because it adjusts both the antecedents such as the consequents, it does not limit the size of input/output vectors, neither the type or distribution of the membership functions used in the definition of fuzzy sets of the model. For this, firstly presents the formulation of the problem and the EKF. Subsequently, the problem of using the EKF to fuzzy modeling is treated, and several algorithms for EKF fuzzy modeling are presented. Finally, to illustrate the procedure, several examples are performed and compared the results with several methodologies accepted by the scientific community.

2.1.1 Problem Formulation

Let n be the number of input variables and m the number of output variables of a completely general system to model; a discrete Multiple Input Multiple Output—MIMO—TS fuzzy model can be represented by the following set of rules (Babuška 1995; Babuška et al. 1996; Nguyen et al. 1995; Takagi and Sugeno 1985):

$$\begin{aligned} R^{(l,i)}: &\text{ If } x_1(k) \text{ is } A^l_{1i} \text{ and } \ldots \text{ and } x_n(k) \text{ is } A^l_{ni} \\ &\text{Then } y^l_i(k) = a^l_{0i} + \sum_{j=1}^{n} a^l_{ji} x_j(k), \end{aligned} \tag{2.1}$$

where $l = 1 \ldots M_i$ is the index of the rule and M_i the number of rules that model the evolution of the ith system output, $y_i(k)$. a^l_{ij}, $j = 0 \ldots n$, represents the set of adaptive parameters of the consequents of the rules, thus they must be determined by the modeling process. k indicates the current sampling time.

Note that in using the above representation, there are multiple inputs and outputs, and each output can be modeled by a different number of rules. This representation facilitates the reduction of the total number of rules needed to model correctly a complex system, and, therefore, facilitates the modeling process by reducing the number of model parameters.

If input vector is extended in a coordinate (Andújar and Barragán 2005; Andújar et al. 2009) by $\tilde{x}_0 = 1$, extended vector $\tilde{\mathbf{x}}$ takes the form:

$$\tilde{\mathbf{x}} = (\tilde{x}_0, \tilde{x}_1, \ldots, \tilde{x}_n)^{\mathrm{T}} = (1, x_1, \ldots, x_n)^{\mathrm{T}} \tag{2.2}$$

Using the weighted average as a method of aggregation and the extension of the state vector given in (2.2) , the output y_i generated by the set of rules $R^{(l,i)}$, can be calculated by (Wang 1994, 1997):

$$y_i(k) = h_i(\mathbf{x}(k)) = \sum_{j=0}^{n} a_{ji}(\mathbf{x}) \tilde{x}_j(k), \tag{2.3}$$

being $a_{ji}(\mathbf{x})$ variables coefficients (Wong et al. 1997) defined by

$$a_{ji}(\mathbf{x}) = \frac{\sum_{l=1}^{M_i} w^l_i(\mathbf{x}) a^l_{ji}}{\sum_{l=1}^{M_i} w^l_i(\mathbf{x})}, \tag{2.4}$$

where $w^l_i(\mathbf{x})$ is calculated by (2.5) and represents the degree of activation of the rules of the fuzzy model:

$$w^l_i(\mathbf{x}) = \prod_{j=1}^{n} \mu^l_{ji}(x_j(k), \boldsymbol{\sigma}^l_{ji}). \tag{2.5}$$

$\mu^l_{ji}(x_j(k), \boldsymbol{\sigma}^l_{ji})$ represents the jth membership function of the l rule for the ith model output, which determines the fuzzy set A^l_{ji}. $\boldsymbol{\sigma}^l_{ji}$ represents the set of adaptive parameters of this membership function, so these values, with the adaptive parameters of the consequents of the rules, a^l_{ji}, shall be determined according to estimation algorithm to achieve an appropriate system model. Then, the problem to be solved is to determine the values of the adaptive parameters of both, antecedents, $\boldsymbol{\sigma}^l_{ji}$, and consequents, a^l_{ji}, of the rules of (2.1).

2.1.2 Extended Kalman Filter

Kalman filter was developed by Kalman (1960, 1963) and allows to construct an optimal observer for linear systems in presence of white noise both in model and in measures. The Kalman filter is a recursive estimator, this means that only the estimated state from the previous time step and the current measurement are needed to compute the estimate for the current state. In contrast to batch estimation techniques, no history of observations and/or estimates is required. Complex systems are often nonlinear, but the basic Kalman filter is limited to a linear assumption. The nonlinearity can be associated either with the process model or with the observation model or with both. To cover nonlinear systems, the Kalman filter was adapted via EKF (Maybeck 1979), if the system supports linearized models around any working point. Although the EKF is not optimal, since it is based on a linear approximation of a model and its accuracy depends heavily on the goodness of such approximations, is a powerful tool for estimation in environments with noise.

If a general nonlinear discrete system as follows is considered:

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{f}\,(\mathbf{x}(k), \mathbf{u}(k)) + \mathbf{v}(k)\\ \mathbf{y}(k) &= \mathbf{g}\,(\mathbf{x}(k)) + \mathbf{e}(k)\end{aligned} \tag{2.6}$$

where $\mathbf{v}(k)$ and $\mathbf{e}(k)$ are white noises that represent uncertainty both in the model of equation of state and in output, respectively. And being the Jacobian matrices of the system:

$$\boldsymbol{\Phi}(k) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}(k),\mathbf{u}=\mathbf{u}(k)} \tag{2.7}$$

$$\boldsymbol{\Gamma}(k) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right|_{\mathbf{x}=\mathbf{x}(k),\mathbf{u}=\mathbf{u}(k)} \tag{2.8}$$

and

$$\mathbf{C}(k) = \left.\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}(k)}. \tag{2.9}$$

The Kalman filter can be conceptualized as two distinct phases: predict and update. The predict phase uses the estimated state from the previous time-step to produce an estimation of the state at the current time-step. This predicted state is also known as the a priori estimated state because, although it is an estimate of the state at the current time-step, it does not include observation information from the current time-step. In the update phase, the current a priori prediction is combined with current observation information to refine the estimate state. This improved estimate is termed the a posteriori estimate state.

Following the previous, the EKF can be solved by iterative application of the following set of equations (Grewal and Andrews 2001):

Predict:

$$\hat{\mathbf{x}}(k|k-1) = \mathbf{\Phi}(k)\hat{\mathbf{x}}(k-1|k-1) + \mathbf{\Gamma}(k)\mathbf{u}(k) \tag{2.10}$$

$$\mathbf{P}(k|k-1) = \mathbf{\Phi}(k)\mathbf{P}(k-1|k-1)\mathbf{\Phi}^{\mathrm{T}}(k) + \mathbf{R}_v \tag{2.11}$$

Update:

$$\mathbf{K}(k) = \left(\mathbf{\Phi}(k)\mathbf{P}(k|k-1)\mathbf{C}^{\mathrm{T}}(k) + \mathbf{R}_{ve}\right)\left(\mathbf{C}(k)\mathbf{P}(k|k-1)\mathbf{C}^{\mathrm{T}}(k) + \mathbf{R}_e\right)^{-1} \tag{2.12}$$

$$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)\left(\mathbf{y}(k) - \hat{\mathbf{y}}(k)\right) \tag{2.13}$$

$$\mathbf{P}(k|k) = \mathbf{\Phi}(k)\mathbf{P}(k|k-1)\mathbf{\Phi}^{\mathrm{T}}(k) + \mathbf{R}_v - \mathbf{K}(k)\left(\mathbf{C}(k)\mathbf{P}(k|k-1)\mathbf{\Phi}^{\mathrm{T}}(k) + \mathbf{R}_{ve}^{\mathrm{T}}\right), \tag{2.14}$$

where $\hat{\mathbf{x}}(k|k-1)$ and $\hat{\mathbf{x}}(k|k)$ represent respectively the estimate of $\mathbf{x}$ predicted (a priori), and updated estimation (a posteriori) given observations up to, and including at time k. $\mathbf{P}(k|k-1)$ is the predicted (a priori) estimate covariance, and $\mathbf{P}(k|k)$ is the updated (a posteriori) estimate covariance matrix. $\mathbf{K}(k)$ is the Kalman gain, $\hat{\mathbf{y}}(k)$ are the estimated outputs, and $\mathbf{R}_v$, $\mathbf{R}_{ve}$ and $\mathbf{R}_e$ are the noise covariance matrices, estimated from the hope operator, $E(\cdot)$:

$$\mathbf{R}_v = E\left(\mathbf{v}(k)\mathbf{v}^{\mathrm{T}}(k)\right) \tag{2.15}$$

$$\mathbf{R}_{ve} = E\left(\mathbf{v}(k)\mathbf{e}^{\mathrm{T}}(k)\right) \tag{2.16}$$

$$\mathbf{R}_e = E\left(\mathbf{e}(k)\mathbf{e}^{\mathrm{T}}(k)\right). \tag{2.17}$$

The iterative process starts with an initial estimate of state vector $\hat{\mathbf{x}}(0) = \mathbf{m}_0 = E\,(\mathbf{x}(0))$ and an initial value of the covariance matrix $\mathbf{P}(0) = E((\mathbf{x}(0) - \mathbf{m}_0)(\mathbf{x}(0) - \mathbf{m}_0)^T)$, being known $\mathbf{x}(0|-1)$, $\mathbf{u}(0)$ and $\mathbf{y}(0)$. Then it is evolving online with respect the system, obtaining a solution that minimizes both estimation error and its covariance matrix for the linearization obtained at each instant.

2.2 Application of the EKF to Fuzzy Modeling

A so interesting application of EKF is the adaptive identification of parameters in nonlinear systems, which allows the online obtaining of the adaptive parameters set of a discrete nonlinear model with noise presence and in a pseudo-optimal way (is optimal in linear systems). The identification of a TS fuzzy model can be seen as the obtaining of parameters of a nonlinear model, so the Kalman filter can be applied using the extended algorithm for estimating these parameters.

First it is necessary to raise the problem of estimation by EKF. For this, a system whose states depend directly on the parameters to be estimated (Simon 2002) must be build, and then apply recursively from (2.10) to (2.14).

Let $\mathbf{p}(k)$ be the set of adaptive parameters of a fuzzy system, and $\mathbf{y}(k)$ the set of outputs of this fuzzy system, the system represented in (2.18) and the diagram shown in Fig. 2.1, allows to obtain these parameters using the EKF.

$$\begin{aligned}\mathbf{p}(k+1) &= \boldsymbol{\Psi}\mathbf{p}(k)\\ \mathbf{y}(k) &= \mathbf{h}(\mathbf{x}(k), \mathbf{p}(k)) + \mathbf{e}(k).\end{aligned} \tag{2.18}$$

$\boldsymbol{\Psi}$ is a constant matrix that relates the parameters between themselves, for example, to meet the requirements of the Standard Fuzzy Partition (SFP) (Xiu and Ren 2005) frequently used in control applications (Al-Hadithi et al. 2007; Xiu and Wang 2007). If all parameters can be adjusted freely, $\boldsymbol{\Psi}$ will be the identity matrix. $\mathbf{e}(k)$ is the uncertainty of the measurement of the output system and is represented by a white noise, whose covariance is $\mathbf{R}_e$.

Thus, the first thing to do is the calculation of Jacobian matrices of the system using (2.7), (2.8) and (2.9). Applying these expressions on (2.18):

$$\boldsymbol{\Phi}\left(\mathbf{p}(k)\right) = \boldsymbol{\Psi} \tag{2.19}$$

$$\boldsymbol{\Gamma}\left(\mathbf{p}(k)\right) = \mathbf{0} \tag{2.20}$$

and

$$\mathbf{C}\left(\mathbf{p}(k)\right) = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{p}}\right|_{\mathbf{p}=\hat{\mathbf{p}}(k)} \tag{2.21}$$

being $\hat{\mathbf{p}}(k)$ the current estimation of the parameters vector of the TS fuzzy model.

Note that, given formulation exposed in Sect. 2.1.1, the estimation problem of the TS fuzzy model (2.1) is to determine the values of the adaptive parameters of both

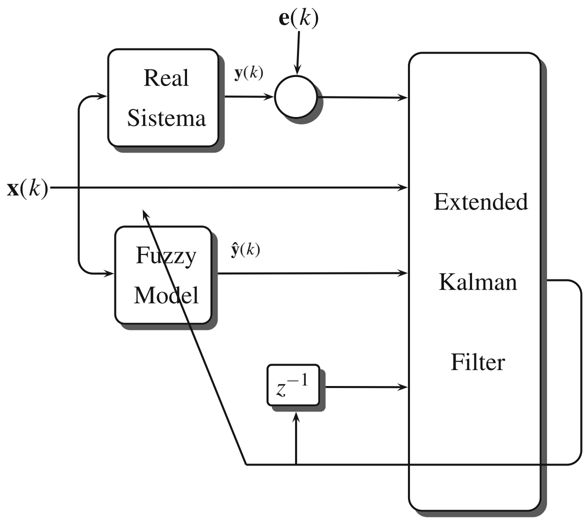

Fig. 2.1 TS Fuzzy modeling using the EKF

antecedents, $\boldsymbol{\sigma}^{l}_{ji}$, and consequents, a^{l}_{ji}, of rules. Therefore, for a TS fuzzy model, the expression $\mathbf{h}(\mathbf{x}(k), \mathbf{p}(k))$ corresponds to (2.3), and (2.21) must be obtained from the derivative of this expression with respect to each of adaptive parameters of the TS fuzzy model.

As can be seen in (2.3) and (2.4), function $\mathbf{h}(\cdot)$ is linear with respect the set of adaptive parameter of consequents, a^{l}_{ji}, so:

$$\frac{\partial h_i}{\partial a^{L}_{JI}} = \begin{cases} \dfrac{w^{L}_{I}\tilde{x}_J}{\sum\limits_{l=1}^{M_I} w^{l}_{I}} & \text{if } i = I \\ 0 & \text{if } i \neq I, \end{cases} \tag{2.22}$$

where L, J and I determine the particular parameter a^{L}_{JI} of the possible set of consequent parameters. Since $\mathbf{h}(\cdot)$ is linear with respect to the parameters of the resulting adaptive, the adjustment of these parameters is optimal in the sense of Kalman filter (it is not necessary to use the extended Kalman filter).

Moreover, for each parameters set of the membership function of antecedent, is obtained, if exist:

$$\frac{\partial h_i}{\partial \boldsymbol{\sigma}^{L}_{JI}} = \sum_{j=0}^{n} \frac{\partial \left(\dfrac{\sum_{l=1}^{M_i} w^{l}_{i} a^{l}_{ji}}{\sum_{l=1}^{M_i} w^{l}_{i}} \right)}{\partial \boldsymbol{\sigma}^{L}_{JI}} \tilde{x}_j. \tag{2.23}$$

Only the Ith output depends of the $\boldsymbol{\sigma}_{JI}^{L}$ parameter, thus,

$$\frac{\partial h_i}{\partial \boldsymbol{\sigma}_{JI}^{L}} = 0 \quad \text{if} \quad i \neq I. \tag{2.24}$$

Developing the partial derivative of (2.23), and considering (2.24):

$$\frac{\partial \left(\frac{\sum_{l=1}^{M_I} w_I^l a_{jI}^l}{\sum_{l=1}^{M_I} w_I^l} \right)}{\partial \boldsymbol{\sigma}_{JI}^{L}} = \frac{\frac{\partial \left(\sum_{l=1}^{M_I} w_I^l a_{jI}^l \right)}{\partial \boldsymbol{\sigma}_{JI}^{L}} \sum_{l=1}^{M_I} w_I^l - \frac{\partial \left(\sum_{l=1}^{M_I} w_I^l \right)}{\partial \boldsymbol{\sigma}_{JI}^{L}} \sum_{l=1}^{M_I} w_I^l a_{jI}^l}{\left(\sum_{l=1}^{M_I} w_I^l \right)^2}. \tag{2.25}$$

Given that $\boldsymbol{\sigma}_{JI}^{L}$ is only present in L rule of Ith output, is easy to deduce that

$$\frac{\partial \left(\sum_{l=1}^{M_I} w_I^l a_{jI}^l \right)}{\partial \boldsymbol{\sigma}_{JI}^{L}} = \frac{\partial w_I^L}{\partial \boldsymbol{\sigma}_{JI}^{L}} a_{jI}^L \tag{2.26}$$

and working in a similar way,

$$\frac{\partial \left(\sum_{l=1}^{M_I} w_I^l \right)}{\partial \boldsymbol{\sigma}_{JI}^{L}} = \frac{\partial w_I^L}{\partial \boldsymbol{\sigma}_{JI}^{L}}. \tag{2.27}$$

Replacing (2.26) and (2.27) in (2.25):

$$\begin{aligned} \frac{\partial \left(\frac{\sum_{l=1}^{M_I} w_I^l a_{jI}^l}{\sum_{l=1}^{M_I} w_I^l} \right)}{\partial \boldsymbol{\sigma}_{JI}^{L}} &= \frac{\partial w_I^L}{\partial \boldsymbol{\sigma}_{JI}^{L}} \left(\frac{a_{jI}^L \sum_{l=1}^{M_I} w_I^l - \sum_{l=1}^{M_I} w_I^l a_{jI}^l}{\left(\sum_{l=1}^{M_I} w_I^l \right)^2} \right) \\ &= \frac{\partial w_I^L}{\partial \boldsymbol{\sigma}_{JI}^{L}} \left(\frac{\sum_{l=1}^{M_I} w_I^l (a_{jI}^L - a_{jI}^l)}{\left(\sum_{l=1}^{M_I} w_I^l \right)^2} \right). \end{aligned} \tag{2.28}$$

Replacing the above expression in (2.23), the final expression is obtained:

$$\frac{\partial h_i}{\partial \boldsymbol{\sigma}_{JI}^{L}} = \begin{cases} \frac{\partial w_I^L}{\partial \boldsymbol{\sigma}_{JI}^{L}} \sum_{j=0}^{n} \left(\frac{\sum_{l=1}^{M_I} w_I^l (a_{jI}^L - a_{jI}^l)}{\left(\sum_{l=1}^{M_I} w_I^l \right)^2} \right) \tilde{x}_j & \text{if } i = I \\ 0 & \text{if } i \neq I \end{cases}. \tag{2.29}$$

Finally, to conclude the calculation of (2.29) is necessary to determine the derivative of the degree of activation from the rules of the TS fuzzy model, w_i^l, with respect to each of the parameters of the antecedents. Obviously, this calculation is dependent on the type of membership function that is used for each antecedent, however, is possible to express in a general form:

$$\frac{\partial w_I^L}{\partial \boldsymbol{\sigma}_{JI}^L} = \frac{\partial}{\partial \boldsymbol{\sigma}_{JI}^L}\left(\prod_{q=1}^{n} \mu_{qI}^L(x_q(k), \boldsymbol{\sigma}_{qI}^L)\right) \tag{2.30}$$

or a more developed form:

$$\frac{\partial w_I^L}{\partial \boldsymbol{\sigma}_{JI}^L} = \frac{\partial \mu_{JI}^L(x_J(k), \boldsymbol{\sigma}_{JI}^L)}{\partial \boldsymbol{\sigma}_{JI}^L} \prod_{q=1, q\neq J}^{n} \mu_{qI}^L(x_q(k), \boldsymbol{\sigma}_{qI}^L). \tag{2.31}$$

Note that $\frac{\partial \mu_{JI}^L(x_J(k), \boldsymbol{\sigma}_{JI}^L)}{\partial \boldsymbol{\sigma}_{JI}^L}$ represents the derivative of the membership function that is defined by the parameters set $\boldsymbol{\sigma}_{JI}^L$. Thus, the calculation of this derivative depends on the type of membership function used and it can be performed from the expression that defines it.

Note it is not necessary that the membership functions are differentiable, but it is enough to be piecewise differentiable. Piecewise membership functions could provide a jump discontinuity in its derivative, however, since the set of singular points is a null set, in numerical implementations this is not a real problem. For example, for a triangular membership function:

$$\mu_{Tri}[a, b, c](x) = \begin{cases} \dfrac{x-a}{b-a} & \text{if } a < x \leq b \\ \dfrac{c-x}{c-b} & \text{if } b \leq x < c \\ 0 & \text{in other case} \end{cases} \tag{2.32}$$

$\boldsymbol{\sigma}$ represents the vector $[a, b, c]$, and the derivatives are:

$$\frac{\partial \mu_{Tri}[a, b, c](x)}{\partial a} = \begin{cases} \dfrac{x-b}{(b-a)^2} & \text{if } a < x < b \\ 0 & \text{in other case} \end{cases} \tag{2.33}$$

$$\frac{\partial \mu_{Tri}[a, b, c](x)}{\partial b} = \begin{cases} \dfrac{a-x}{(b-a)^2} & \text{if } a < x < b \\ \dfrac{c-x}{(c-b)^2} & \text{if } b < x < c \\ 0 & \text{in other case} \end{cases} \tag{2.34}$$

and

Fig. 2.2 EKF(c) algorithm. Adaptation of consequents of a TS fuzzy model

$\tilde{\mathbf{p}}_c(0|-1) = \mathbf{0}$
$\mathbf{P}_c(0|-1) = \mathbf{I}\alpha$
for $k = 0..k_{end}$ **do**
  Calculate $\mathbf{P}_c(k|k-1)$ by (2.11)
  Estimate $\tilde{\mathbf{y}}(k)$ using the fuzzy model
  Calculate $\mathbf{C}_c(k)$ by (2.22)
  Get $\mathbf{K}_c(k)$ by (2.12)
  Update $\tilde{\mathbf{p}}_c(k|k)$ by (2.10) and (2.13)
  Update $\mathbf{P}_c(k|k)$ by (2.14)
end for k

$$\frac{\partial \mu_{Tri}[a,b,c](x)}{\partial c} = \begin{cases} \dfrac{x-b}{(c-b)^2} & \text{if } b < x < c \\ 0 & \text{in other case} \end{cases}. \tag{2.35}$$

It is possible to suppose that the derivatives in a, b or c are the same as the derivative in a point infinitesimally close to the right, to the left, or the average of these derivatives.

2.3 Algorithms for the Parametric Adaptation of a TS Fuzzy Model Based on EKF

In this section, two algorithms that allow to adjust the antecedents and consequents of a TS fuzzy model based on the Kalman filter are presented. Before executing adjustment algorithms, the fuzzy model must be initialized. If there is no prior information about the plant, all consequents can be initialized to 0, and the antecedents can be initialized using uniform partitioning, or it could be initialized by other procedures Benmakrouha (1997). However, if data are available for the system to be modeled, these data can be used to obtain a better initial model, for example, by applying a clustering algorithm (Bezdek and Dunn 1975; Bezdek 1981; Chiu 1994; Dunn 1973; Gustafson and Kessel 1979; Kim et al. 1997; Wang and Mendel 1992b), even an offline modeling algorithm (Andújar et al. 2006; Jang 1993; Jang and Sun 1995; Wang and Mendel 1992a).

Once it has an initial model, the EKF algorithm can be used for online fuzzy modeling of the system. The algorithm shown in Fig. 2.2 can be employed to adjust the consequents of a TS fuzzy model. k is the discrete time, $\tilde{\mathbf{p}}_c$ is the adaptable set of parameters of the consequents, $\mathbf{P}_c$, $\mathbf{C}_p$ and $\mathbf{K}_c$ are matrices of the EKF, $\mathbf{I}$ is the identity matrix and α is a positive integer which represents the certainty that the algorithm should give to the initial parameters. For example, α will be higher if the consequents

Fig. 2.3 EKF(a) algorithm. Adaptation of antecedents of a TS fuzzy model

$\mathbf{p}_a =$ initial antecedents parameters
$\mathbf{P}_a(0|-1) = \mathbf{I}\beta$
for $k = 0..k_{end}$ **do**
    Calculate $\mathbf{P}_a(k|k-1)$ by (2.11)
    Estimate $\tilde{\mathbf{y}}(k)$ using the fuzzy model
    Calculate $\partial\mathbf{w}/\partial\sigma$ by (2.31)
    Calculate $\mathbf{C}_a(k)$ by (2.29)
    Get $\mathbf{K}_a(k)$ by (2.12)
    Update $\tilde{\mathbf{p}}_a(k|k)$ by (2.10) and (2.13)
    Update $\mathbf{P}_a(k|k)$ by (2.14)
end for k

of the fuzzy model are initialized to 0 that if they do so by a clustering algorithm. Choosing a too high value of α would lead the algorithm to make very abrupt initial changes of the consequents, implying a high volatility modeling algorithm. It is also important to note that if the system has multiple outputs may be interesting to use a different α value for each.

In a similar way as has been done for consequents, the algorithm shown in Fig. 2.3 can be employed to adjust the antecedents of a TS fuzzy model, where $\tilde{\mathbf{p}}_a$ is the adaptable set of parameters of the antecedents. The function of β in this algorithm is identical to that of α in the above algorithm, except that normal will always be that the antecedents are best initialized that the consequents, so it is logical that β is less than α if both algorithms are used simultaneously.

If you want to adjust both antecedents and consequents of a fuzzy model, it is advisable to first make the consequents adjustment and subsequently that of the antecedents, because otherwise you will be getting worse results Barragán et al. (2011a, b). A C++ implementation of the algorithms presented in this section can be found in Barragán and Andújar (2012).

2.4 Examples

The following will show some examples of application of the Kalman filter modeling TS fuzzy systems. The first example is carried out 10 times with a different noise signal (tenfold cross validation Kohavi (1995)), while the second example does not incorporate noise. Both, the online modeling capability and the quality of the final models are studied (the final models are the models obtained at the end of the training phase. This model is not changed during the validation phase).

Table 2.1 Average values and standard deviation of 10 runs

	EKF(c)	EKF(c + a)
Online training error	3.115126 ± 0.185	3.084547 ± 0.167
Final models error (training)	5.283666 ± 0.312	5.049607 ± 0.300
Final models error (validation)	3.081644 ± 0.139	3.030364 ± 0.144
RMSE (validation)	4.676209 ± 0.506	4.630756 ± 0.575

2.4.1 Example 1. Nonlinear Static System

Be the nonlinear system:

$$f(x) = e^{-0.03x} \sin(0.1x) \tag{2.36}$$

with $x \in [-150, 150]$, to which is added a noise signal whose covariance is $R_e = 0.5$. During the validation process the noise is removed to check if the model fit to the real system without its interference.

The function (2.36) will be modeled using two different initial configurations. In the first case, an initial model composed only of membership functions of Gaussian type is used, while the second case use a mixture of different membership functions and uses the $\mathbf{\Psi}$ matrix to include a restriction on the antecedents of the fuzzy model. Each case is carried out 10 times, with a 60–40 % random split between the training and validation data subsets. Two fitting algorithms are to be used, the Algorithm 2.2 to set only consequents, EKF(c), and the Algorithm 2.2 followed by the Algorithm 2.3, EKF(c + a), to adjust both consequents and antecedents of the TS fuzzy model. The covariance matrices for each of the algorithms will be initialized with $\alpha = 10^5$ and $\beta = 0.1$ for EKF(ac) algorithm, and $\alpha = 10^5$ and $\beta = 1$ for EKF(c + a).

2.4.1.1 Case I: Gaussian Membership Functions

In this case, the antecedents of the initial model starts with Gaussian membership functions uniformly distributed, with all consequents set to zero, and $\mathbf{\Psi} = \mathbf{I}$. After run ten times, the average errors are shown in Table 2.1. Note that the validation results are better than training since the algorithm performs online modeling.

For one of the runs, the absolute errors of the final models are shown for training and validation data in Figs. 2.4 and 2.5. The evolution of online modeling outputs for each algorithm shown in Fig. 2.6, the final response from validation data in Fig. 2.7, and the online evolution of absolute errors are shown in Fig. 2.8. Finally, Fig. 2.9 shows the resulting antecedents of each model, where the changes made by the EKF(c + a) algorithm from the original, EKF(c), antecedents can be seen.

Based on the results obtained can be checked that the algorithms modeled properly online, and obtain final models that adequately represent the original function.

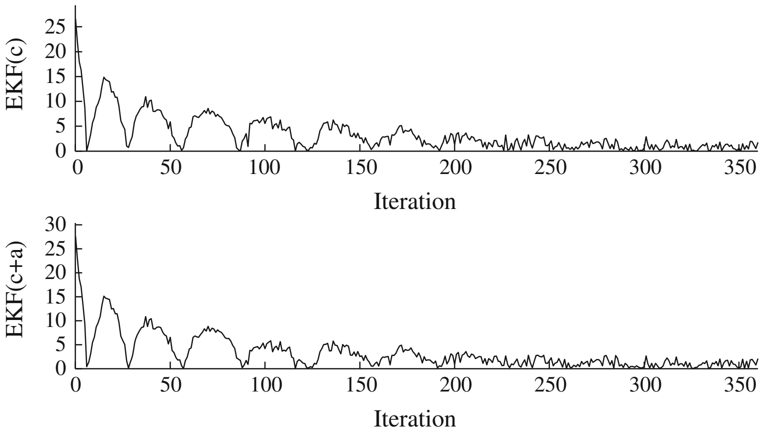

Fig. 2.4 Final models errors from training data

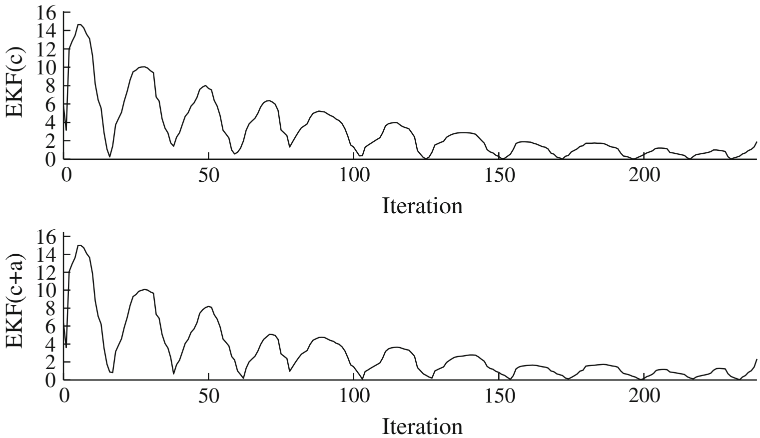

Fig. 2.5 Final models errors from validation data

Algorithm EKF(c + a) perform better because it can adjust the antecedents and this gives it more flexibility. Analyzing the standard deviations in Table 2.1, is possible to see that the algorithms are fairly consistent, since they get very similar results to changes in input data and noise that affects the equation.

2.4.1.2 Case II: Membership Functions of Different Type and Antecedents Restriction

In this case, the antecedents of the initial TS fuzzy model are defined by a S membership function, a Z, two trapezoidal and two triangular. These antecedents have been

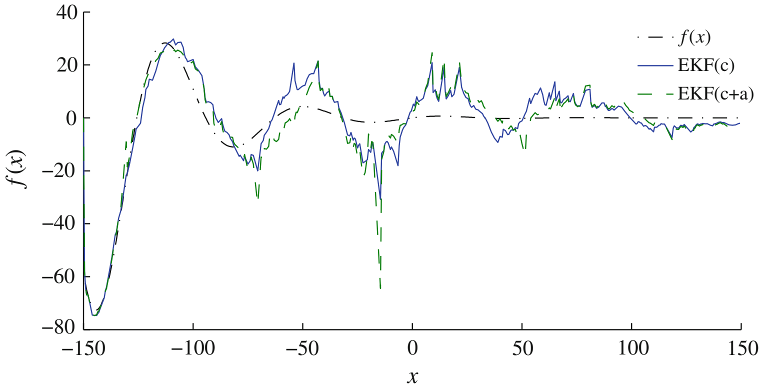

Fig. 2.6 On-line evolution of the outputs of the models

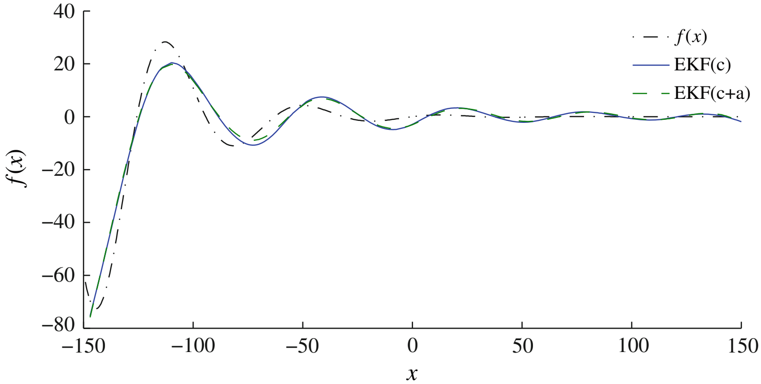

Fig. 2.7 Outputs of the final models from validation data

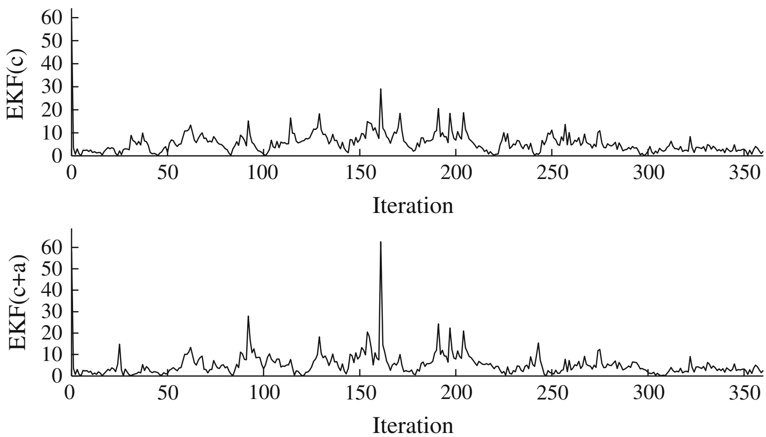

Fig. 2.8 On-line evolution of absolute errors

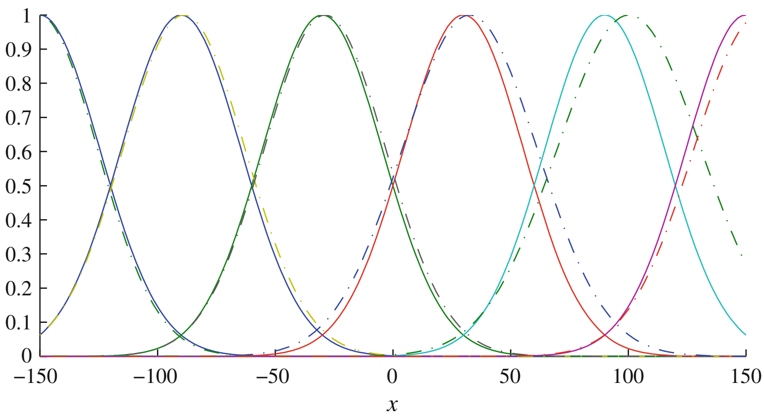

Fig. 2.9 Final antecedents: — EKF(c), -·- EKF(c + a)

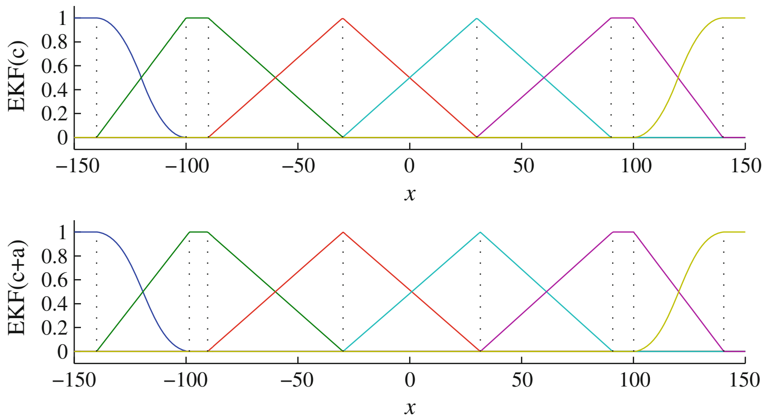

Fig. 2.10 Final antecedents: — EKF(c), -·- EKF(c + a)

distributed according to the EKF(c) graph of Fig. 2.10. In order to demonstrate the operation of $\mathbf{\Psi}$ in (2.18), a restriction on the antecedents is defined, so that each of them depends on the previous antecedent, and they are never overlap in the regions of full membership. If the membership are straight lines overlapping by pairs, the adaptation problem does not have an unique solution which can lead to convergence problem of the algorithm. This fact and a solution has been discussed in Al-Hadithi et al. (2012).

In the EKF(c + a) algorithm, $\mathbf{\Psi}$ is used both to define the relationship between the parameters itself of the antecedents and as the consequent. To define the relationship between consequents, $\mathbf{\Psi}$ matrix is always the identity, whereas the relationship between the parameters of the antecedents is determined by (2.37). Note that bold numbers in (2.37) marks changes from the identity matrix.

Table 2.2 Average values and standard deviation of 10 runs

	EKF(c)	EKF(c + a)
Online training error	2.759294 ± 0.128	3.007364 ± 0.124
Final models error (training)	3.304343 ± 0.153	3.277433 ± 0.121
Final models error (validation)	2.731736 ± 0.130	2.965235 ± 0.164
RMSE (validation)	3.569301 ± 0.025	3.852766 ± 0.032

$$\boldsymbol{\Psi}_a = \left(\begin{array}{c|cc|cccc|ccc|ccc|cccc|cc}
 & \multicolumn{2}{c|}{Zmf} & \multicolumn{4}{c|}{Trapmf1} & \multicolumn{3}{c|}{Trimf1} & \multicolumn{3}{c|}{Trimf2} & \multicolumn{4}{c|}{Trapmf2} & \multicolumn{2}{c}{Smf} \\
\hline
Zmf & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
Trapmf1 & \mathbf{1} & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & \mathbf{1} & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
Trimf1 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
Trimf2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
Trapmf2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & \mathbf{0} & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & \mathbf{0} & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\hline
Smf & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & \mathbf{0} & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & \mathbf{0}
\end{array}\right) \tag{2.37}$$

After run ten times, the average errors are shown in Table 2.2. As in the previous case, is taken one of the executions of the algorithm, where the absolute errors of the final models are shown in Figs. 2.11 and 2.12, the modeling outputs in Fig. 2.13, the final response from validation data in Fig. 2.14, and the online evolution of absolute errors are shown in Fig. 2.15. Figure 2.10 shows the resulting antecedents, where can be seen that EKF(c + a) has complied with the antecedents relationship from (2.37).

Based on the results obtained, it is possible to draw the same conclusions as in the previous case, but can be seen that the use of the matrix $\boldsymbol{\Psi}$ can impose restrictions on the adjust of antecedents.

2.4.2 Example 2. Mackey-Glass Chaotic Time Series

In this case, the EKF algorithms will be used to predict 6, 12 and 85 steps ahead of Mackey-Glass chaotic time series based on the values of the current signal, 6, 12 and 18 steps back ($x = [v(t-18), v(t-12), v(t-6)v(t)]^{\mathrm{T}}$). This series is a well-known

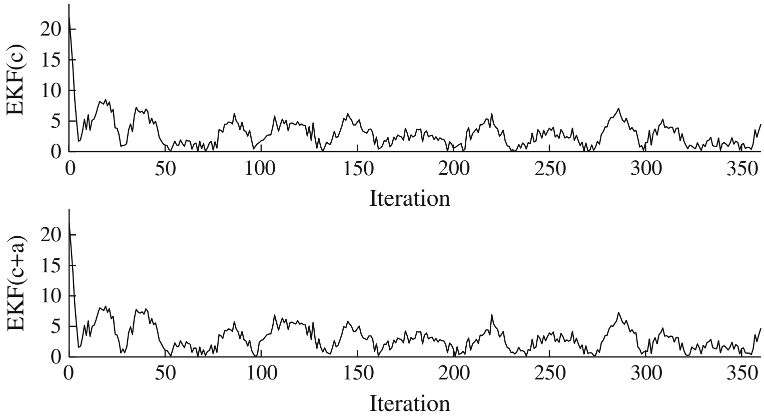

Fig. 2.11 Final models errors from training data

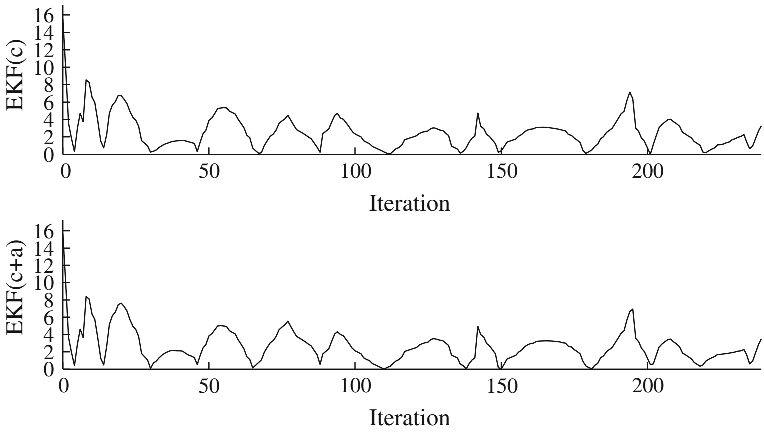

Fig. 2.12 Final models errors from validation data

benchmark function, and it is generated from the differential delay equation defined by Angelov and Buswell (2002), Angelov and Filev (2004), Chiu (1994):

$$\dot{v}(t) = \frac{\gamma v(t-\tau)}{1 + v(t-\tau)^{10}} - \delta v(t) \tag{2.38}$$

with $\gamma = 0.2$, $\delta = 0.1$, $\tau = 17$ and $v(0) = 1.2$.

The initial fuzzy models start with 8 rules for each output (24 rules in total), with 2 Gaussians uniformly distributed for $v(t-12)$, $v(t-6)$ and $v(t)$. In this case a noise signal will not be incorporated in the system. The first 300 values of the series are used as training data, and the next 500 values as validation data. The initial

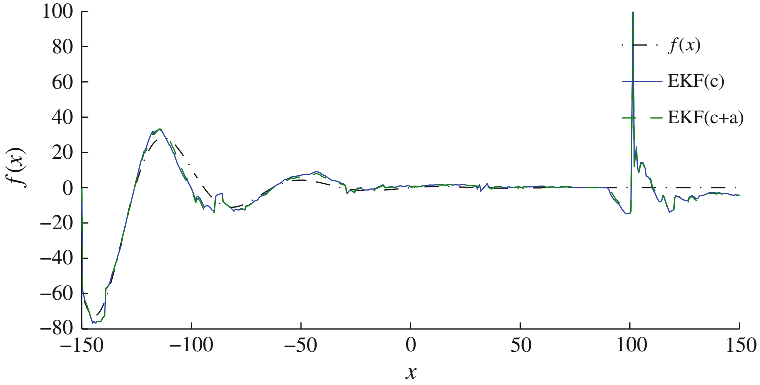

Fig. 2.13 On-line evolution of the outputs of the models

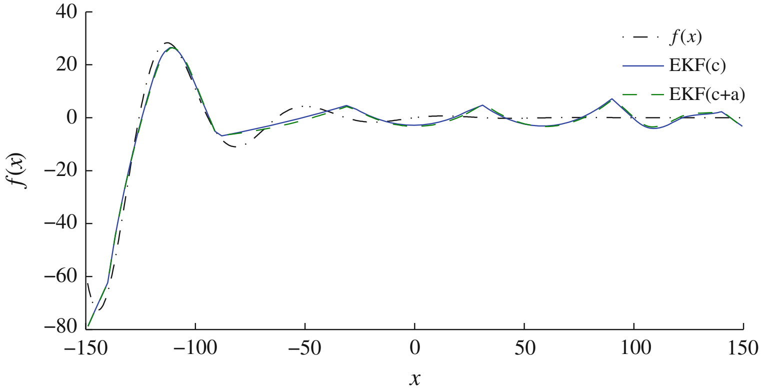

Fig. 2.14 Outputs of the final models from validation data

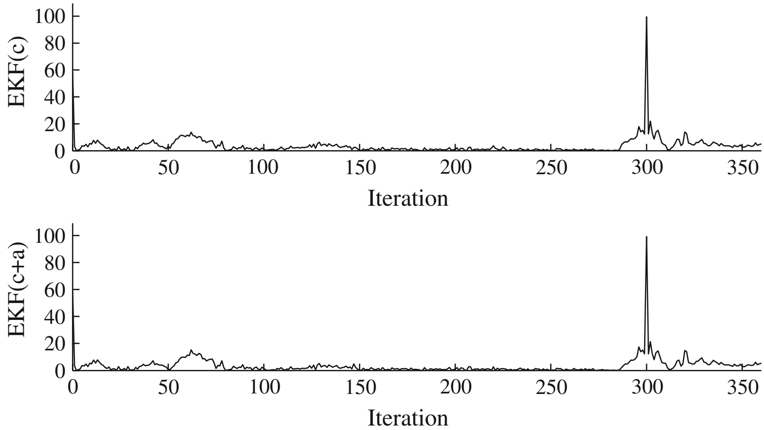

Fig. 2.15 On-line evolution of absolute errors

Table 2.3 Comparison of the prediction RMSE for validation data

	EKF(c)	EKF(c + a)
v(t + 6)	0.0094	0.0091
v(t + 12)	0.0058	0.0045
v(t + 85)	0.0810	0.0767

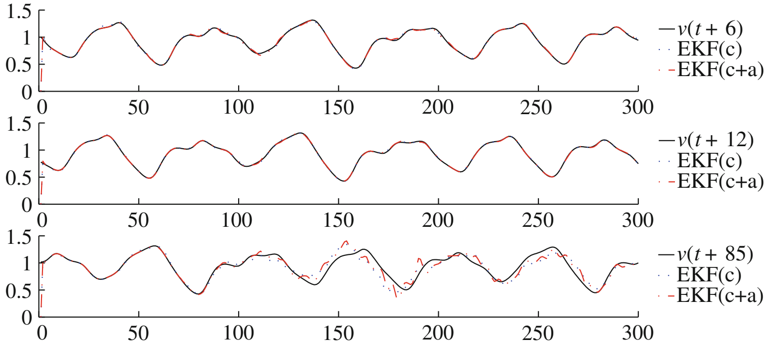

Fig. 2.16 Model training in prediction 6, 12 and 85 steps ahead of Mackey Glass chaotic time series

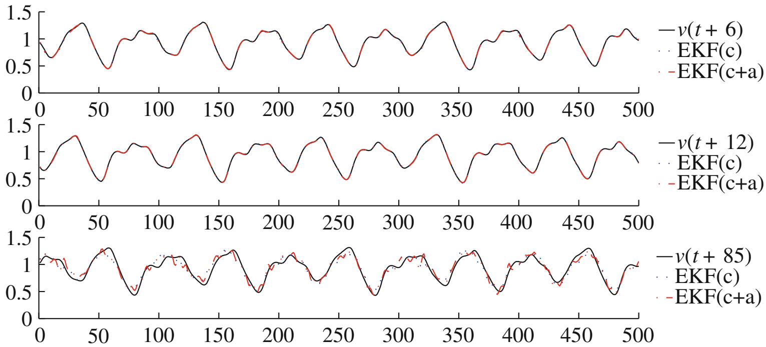

Fig. 2.17 Model validation in prediction 6, 12 and 85 steps ahead

covariance matrix is initialized as $\alpha = 1{,}000$ for EKF(c) algorithm, and $\alpha = 9 \times 10^{12}$ and $\beta = 10^{-6}$ for EKF(c + a) algorithm.

The results of the execution of the two algorithm are shown in Table 2.3. Figures 2.16 and 2.17 show the Mackey Glass chaotic series and the predictions for training and validation data respectively.

2.5 Conclusions

In this chapter the application of extended Kalman filter (EKF) for the parametric adaptation of a TS fuzzy model is presented, which allows obtaining accurate models without renounce the computational efficiency that characterizes the Kalman filter, and allows its implementation online with the process. It is also proposed two algorithms to adjust the antecedents and consequents of a TS fuzzy model, and have been presented several examples.

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