Chapter 2
Edge of the World: When Are Manifolds Metrisable?

Abstract This chapter might seem odd in that it lists a huge number of topological properties and connections between them. What it shows is that the requirement that a manifold be metrisable is extremely versatile. We list over 100 conditions each of which is equivalent to metrisability of a manifold. At one extreme, metrisability of a manifold implies that it may be embedded as a closed subset of some Euclidean space while at the other extreme knowing that every open cover of the form $\{U_\alpha \ / \ \alpha < \omega_1\}$ with $U_\alpha \subset U_\beta$ whenever $\alpha < \beta$ has an open refinement which is point countable on a dense subset is sufficient to guarantee that a manifold is metrisable. Space precludes giving full details of the proofs. Instead we give brief ideas of the proofs and refer the interested reader to original sources for complete proofs. The content of this chapter is taken from [21].

2.1 Definitions

Firstly we must list all of the definitions needed for our grand theorem. Throughout this section X is a topological space and $\mathscr{F}$ a family of subsets of X.

- X is *paracompact* (respectively *metacompact*, *paraLindelöf* and *metaLindelöf*) if every open cover $\mathscr{U}$ has a locally finite (respectively point finite, locally countable, and point countable) open refinement, i.e. there is another open cover $\mathscr{V}$ such that each member of $\mathscr{V}$ is a subset of some member of $\mathscr{U}$ and each point of X has a neighbourhood meeting only finitely (respectively lies in only finitely, has a neighbourhood meeting only countably, and lies in only countably) many members of $\mathscr{V}$;
- X is *strongly paracompact* if every open cover $\mathscr{U}$ has a *star-finite open refinement* $\mathscr{V}$, i.e. for any $V \in \mathscr{V}$ the set $\{W \in \mathscr{V} \ / \ V \cap W \neq \varnothing\}$ is finite. If in addition, given $\mathscr{U}$, there is an integer m such that $\{W \in \mathscr{V} \ / \ V \cap W \neq \varnothing\}$ contains at most m members then X is *star finitistic*;
- X is *screenable* (respectively *σ-metacompact* and *σ-paraLindelöf*) if every open cover $\mathscr{U}$ has an open refinement $\mathscr{V}$ which can be decomposed as $\mathscr{V} = \cup_{n\in\omega}\mathscr{V}_n$ such that each $\mathscr{V}_n$ is disjoint (respectively point finite and locally countable);

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- X is *(linearly) [ω_1-]Lindelöf* if every open cover (which is a chain) [which has cardinality ω_1] has a countable subcover;
- X is *(nearly) [linearly ω_1-]metaLindelöf* if every open cover $\mathscr{U}$ of X [for which $|\mathscr{U}| = \omega_1$ and $\mathscr{U}$ is a chain] has an open refinement which is point-countable (on a dense subset);
- X is *almost metaLindelöf* if for every open cover $\mathscr{U}$ there is a collection $\mathscr{V}$ of open subsets of X such that each member of $\mathscr{V}$ lies in some member of $\mathscr{U}$, that each point of X lies in at most countably many members of $\mathscr{V}$, and that $X = \bigcup \left\{\overline{V} \;/\; V \in \mathscr{V}\right\}$;
- X is *(strongly) hereditarily Lindelöf* if every subspace (of the countably infinite power) of X is Lindelöf;
- X is *k-Lindelöf* provided every open *k-cover* (i.e. every compact subset of X lies in some member of the cover, but X itself is not a member of the cover) has a countable k-subcover;
- X is an *$\aleph_0$-space* [29, p. 493] provided that it has a countable *k-network*, i.e. a countable collection $\mathscr{N}$ such that if $K \subset U$ with K compact and U open then $K \subset N \subset U$ for some $N \in \mathscr{N}$;
- X is *cosmic* if there is a countable family $\mathscr{C}$ of closed subsets of X such that for each point $x \in X$ and each open set U containing x there is a set $C \in \mathscr{C}$ such that $x \in C \subset U$;
- X is an *$\aleph$-space* [29, p. 493] provided that it has a σ-locally finite k-network;
- X is *hemicompact* if there is an increasing sequence $\langle K_n \rangle$ of compact subsets of X such that for any compact $K \subset X$ there is n such that $K \subset K_n$;
- X is *Hurewicz* if for each sequence $\langle \mathscr{U}_n \rangle$ of open covers of X there is a sequence $\langle \mathscr{V}_n \rangle$ such that $\mathscr{V}_n$ is a finite subset of $\mathscr{U}_n$ for each $n \in \omega$ and $\cup_{n\in\omega}\mathscr{V}_n$ covers X (note the alternative definition of Hurewicz, [11]: X is *Hurewicz* if for each sequence $\langle \mathscr{U}_n \rangle$ of open covers of X there is a sequence $\langle \mathscr{V}_n \rangle$ such that $\mathscr{V}_n$ is a finite subset of $\mathscr{U}_n$ and for each $x \in X$ we have $x \in \cup\mathscr{V}_n$ for all but finitely many $n \in \omega$. For a manifold these two conditions are equivalent.);
- X is *selectively screenable*, [1], if for each sequence $\langle \mathscr{U}_n \rangle$ of open covers of X there is a sequence $\langle \mathscr{V}_n \rangle$ such that $\mathscr{V}_n$ is a family of pairwise disjoint open sets refining $\mathscr{U}_n$ for each $n \in \omega$ and $\cup_{n\in\omega}\mathscr{V}_n$ covers X;
- X is *Polish* if X is a separable, complete metric space;
- X is *Lašnev* if it is the image of a metrisable space under a closed map;
- X is M_1 if it has a *σ-closure preserving base* (i.e. a base $\mathscr{B}$ such that there is a decomposition $\mathscr{B} = \cup_{n=1}^{\infty}\mathscr{B}_n$ where for each n and each $\mathscr{F} \subset \mathscr{B}_n$ we have $\overline{\cup\mathscr{F}} = \cup\left\{\bar{F} \;/\; F \in \mathscr{F}\right\}$;
- X is *stratifiable* or M_3 if there is a function G which assigns to each $n \in \omega$ and closed set $A \subset X$ an open set $G(n, A)$ containing A such that $A = \cap_n\overline{G(n, A)}$ and if $A \subset B$ then $G(n, A) \subset G(n, B)$;
- X is *finitistic* (respectively *strongly finitistic*) if every open cover of X has an open refinement $\mathscr{V}$ and there is an integer m such that each point of X lies in (respectively has a neighbourhood which meets) at most m members of $\mathscr{V}$ (finitistic spaces have also been called *boundedly metacompact* and strongly finitistic spaces have also been called *boundedly paracompact*);

- X is a *Moore space* if it is regular and has a *development*, i.e. a sequence $\langle \mathscr{U}_n \rangle$ of open covers such that for each $x \in X$ the collection $\{st(x, \mathscr{U}_n) : n \in \omega\}$ forms a neighbourhood basis at x;
- X is *θ-refinable* if every open cover can be refined to an open θ-cover, i.e. a cover $\mathscr{U}$ which can be expressed as $\cup_{n\in\omega}\mathscr{U}_n$ where each $\mathscr{U}_n$ covers X and for each $x \in X$ there is n such that $ord(x, \mathscr{U}_n) < \omega$;
- X is *subparacompact* if every open cover has a σ-discrete closed refinement;
- X is *perfectly normal* if for every pair A, B of disjoint closed subsets of X there is a continuous function $f : X \to \mathbb{R}$ such that $f^{-1}(0) = A$ and $f^{-1}(1) = B$;
- X is *monotonically normal* if for each open $U \subset X$ and each $x \in U$ it is possible to choose an open set $\mu(x, U)$ such that $x \in \mu(x, U) \subset U$ and such that if $\mu(x, U) \cap \mu(y, V) \neq \varnothing$ then either $x \in V$ or $y \in U$;
- X is *extremely normal* if for each open $U \subset X$ and each $x \in U$ it is possible to choose an open set $\nu(x, U)$ such that $x \in \nu(x, U) \subset U$ and such that if $\nu(x, U) \cap \nu(y, V) \neq \varnothing$ and $x \neq y$ then either $\nu(x, U) \subset V$ or $\nu(y, V) \subset U$;
- X is *weakly normal* if for every pair A, B of disjoint closed subsets of X there is a continuous function $f : X \to S$, for some separable metric space S, such that $f(A) \cap f(B) = \varnothing$;
- X has a *regular G_δ-diagonal* if the diagonal Δ is a *regular G_δ-subset* of X^2, i.e. there is a sequence $\langle U_n \rangle$ of open subsets of X^2 such that $\Delta = \cap U_n = \cap \overline{U_n}$;
- X has a *quasi-regular G_δ-diagonal* if there is a sequence $\langle U_n \rangle$ of open subsets of X^2 such that for each $(x, y) \in X^2 - \Delta$ there is n with $(x, x) \in U_n$ but $(x, y) \notin \overline{U_n}$;
- X has a *G^*_δ-diagonal* if there is a sequence $\langle \mathscr{G}_n \rangle$ of open covers of X such that for each $x, y \in X$ with $x \neq y$ there is n with $\overline{\mathrm{st}(x, \mathscr{G}_n)} \subset X - \{y\}$;
- X has a *quasi-G^*_δ-diagonal* if there is a sequence $\langle \mathscr{G}_n \rangle$ of families of open subsets of X such that for each $x, y \in X$ with $x \neq y$ there is n with $x \in \overline{\mathrm{st}(x, \mathscr{G}_n)} \subset X - \{y\}$;
- X is *submetrisable* if there is a metric topology on X which is contained in the given topology;
- X is *jointly metrisable on compacta* or a *JCM-space* [3], provided that there is some metric d on X such that for each compactum $K \subset X$ the restriction of d to K generates the subspace topology inherited from X;
- X has the *Moving Off Property* [31], provided that every family $\mathscr{K}$ of non-empty compact subsets of X large enough to contain for each compact $C \subset X$ a disjoint $K \in \mathscr{K}$ has an infinite subfamily with a *discrete open expansion* (a family $\{S_\alpha \ / \ al \in I\}$ of subsets of a topological space has a discrete open expansion provided there is a family $\{U_\alpha \ / \ al \in I\}$ of open sets such that $S_\alpha \subset U_\alpha$ and $\forall x \in X, \exists U \subset M$ open such that $x \in U$ and U meets at most one of the sets U_α);
- X has *property pp*, [35], provided that each open cover $\mathscr{U}$ of X has an open refinement $\mathscr{V}$ such that for each choice function $f : \mathscr{V} \to X$ with $f(V) \in V$ for each $V \in \mathscr{V}$ the set $f(\mathscr{V})$ is closed and discrete in X;
- X is a *q-space* if each point admits a sequence of neighbourhoods Q_n such that $x_n \in Q_n$ implies that $\langle x_n \rangle$ clusters;
- X is *Fréchet* or *Fréchet-Urysohn* if whenever $x \in \overline{A}$ there is a sequence $\langle x_n \rangle$ in A that converges to x;

- X is *countably tight* if for each $A \subset X$ and each $x \in \bar{A}$ there is a countable $B \subset A$ for which $x \in \bar{B}$;
- X is *countably fan tight* if whenever $x \in \bigcap_{n\in\omega}\overline{A_n}$ there are finite sets $B_n \subset A_n$ such that $x \in \overline{\bigcup_{n\in\omega} B_n}$;
- X is *countably strongly fan tight* if whenever $x \in \bigcap_{n\in\omega}\overline{A_n}$ there is a sequence $\langle a_n \rangle$ such that $a_n \in A_n$ for each n and $x \in \overline{\{a_n \,/\, n \in \omega\}}$;
- X is *analytic* if it is the continuous image of a Polish space (equivalently of the irrational numbers);
- X is *sequential* if for each $A \subset X$, the set A is closed whenever for each sequence of points of A each limit point is also in A;
- X is *radial* [12] provided for any $A \subset X$ and any $x \in \bar{A}$, there is a transfinite sequence $\langle x_\alpha \rangle$ in A which converges to x;
- X is *weakly α-favourable* if there is a winning strategy for player α in the Banach-Mazur game (defined below);
- X is *strongly α-favourable* if there is a stationary winning strategy for player α in the Choquet game (defined below);
- X is *pseudocomplete* provided that it has a sequence $\langle \mathscr{B}_n \rangle$ of π-bases ($\mathscr{B} \subset 2^X - \{\varnothing\}$ is a *π-base* if every non-empty open subset of X contains some member of $\mathscr{B}$) such that if $B_n \in \mathscr{B}_n$ and $\overline{B_{n+1}} \subset B_n$ for each n, then $\bigcap_{n\in\omega} B_n \neq \varnothing$;
- X is *Baire* provided that the intersection of any countable collection of dense G_δ subsets is dense;
- X is *strongly Baire* provided that X is regular and there is a dense subset $D \subset X$ such that β does not have a winning strategy in the game $\mathscr{G}_S(D)$ (defined below) played on X;
- X is *Volterra* [25], provided that the intersection of any two dense G_δ subsets is dense;
- X is a *k-space* if A is closed whenever $A \cap K$ is closed for every compact subset $K \subset X$;
- for each $x \in X$ the *star* of x in $\mathscr{F}$ is $st(x, \mathscr{F}) = \cup\{F \in \mathscr{F} \,/\, x \in F\}$;
- $\mathscr{F}$ is *point-star-open* if for each $x \in X$ the star $st(x, \mathscr{F})$ is open.

Next we introduce some topological games. Usually a *topological game* involves two 'players' playing on a topological space X, alternately choosing subsets, perhaps points, of X and subject to certain rules. Finitely or infinitely many (even uncountably many for some games) moves may be allowed and there is a rule to determine which player wins, if any.

- The *Banach-Mazur game* has two players α and β whose play alternates. Player β begins by choosing a non-empty open subset of X. After that the players choose successive non-empty open subsets of their opponent's previous move. *Player α wins* iff the intersection of the sets is non-empty; otherwise *player β wins*.
- The *Choquet game* has two players α and β whose play alternates. Player β begins by choosing a point in an open subset of X, say $x_0 \in V_0 \subset X$. After that the players alternate with α choosing an open set $U_n \subset X$ with $x_n \in U_n \subset V_n$ then β chooses

a point x_{n+1} and an open set V_{n+1} with $x_{n+1} \in V_{n+1} \subset U_n$. *Player α wins* iff the intersection of the sets is non-empty; otherwise *player β wins*.

- *Gruenhage's game $G^o_{K,L}(X)$* [30], has, at the nth stage, player K choose a compactum $K_n \subset X$ after which player L chooses another compactum $L_n \subset X$ so that $L_n \cap K_i = \varnothing$ for each $i \leq n$. *Player K wins* if $\langle L_n \rangle_{n\in\omega}$ has a discrete open expansion.
- For a dense subset $D \subset X$ the game $\mathscr{G}_S(D)$ has two players α and β whose play alternates. Player β begins by choosing a non-empty open subset V_n of X. After that the players choose successive non-empty open subsets of their opponent's previous move, β choosing sets V_n and α choosing sets U_n. *Player α wins* iff the intersection of the sets is non-empty and each sequence $\langle x_n \rangle$, for which $x_n \in U_n \cap D$, clusters in X; otherwise *player β wins*.
- For an ordinal k and families $\mathfrak{A}$ and $\mathfrak{B}$ of collections of subsets of a space X let $\mathsf{G}^k_c(\mathfrak{A}, \mathfrak{B})$ be the game played as follows [4]: at the lth stage of the game, $l < k$, Player One chooses a member $\mathscr{A}_l \in \mathfrak{A}$ then Player Two chooses a pairwise disjoint family $\mathscr{T}_l$ which refines $\mathscr{A}_l$. The play $\mathscr{A}_0, \mathscr{T}_0, \ldots, \mathscr{A}_l, \mathscr{T}_l, \ldots$ $l < k$ is won by Player Two provided that $\cup_{l<k}\mathscr{T}_l \in \mathfrak{B}$; otherwise Player One wins. The game $\mathsf{G}^\omega_c(\mathfrak{A}, \mathfrak{B})$ is denoted by $\mathsf{G}_c(\mathfrak{A}, \mathfrak{B})$.
- When players α and β play a topological game, a *strategy for α* is a function which tells α what points or sets to select given all the previous points and sets chosen by β. A *stationary strategy for α* is a function which tells α what points or sets to select given only the most recent choice of points and sets chosen by β. A *winning (stationary) strategy* for α is a (stationary) strategy which guarantees that α will win whatever moves β might make.

Closely related to topological games are *selection principles*. Typically these involve two families $\mathfrak{A}$ and $\mathfrak{B}$ of subsets of some set. The challenge is to find for each sequence $\langle A_n \rangle$ of members of $\mathfrak{A}$ another sequence $\langle B_n \rangle$ satisfying certain conditions such that $B_n \subset A_n$ for each n and $\bigcup_{n\in\omega} B_n \in \mathfrak{B}$.

- For families $\mathfrak{A}$ and $\mathfrak{B}$ of subsets of some set consider the *selection principles* (cf [8, 42]):
  - $\mathsf{S}_1(\mathfrak{A}, \mathfrak{B})$: for each sequence $\langle A_n \rangle$ of members of $\mathfrak{A}$ there is a sequence $\langle b_n \rangle$ such that $b_n \in A_n$ for each n and $\{b_n \ / \ n \in \omega\} \in \mathfrak{B}$;
  - $\mathsf{S}_{fin}(\mathfrak{A}, \mathfrak{B})$: for each sequence $\langle A_n \rangle$ of members of $\mathfrak{A}$ there is a sequence $\langle B_n \rangle$ of finite sets such that $B_n \subset A_n$ for each n and $\bigcup_{n\in\omega} B_n \in \mathfrak{B}$.

  For a topological space X consider the following examples of families $\mathfrak{A}$ or $\mathfrak{B}$:

  - $\mathfrak{O}$, the family of open covers of X;
  - Λ, the family of *large* covers of X, i.e. those open covers $\mathscr{U}$ for which $X \notin \mathscr{U}$ and every point of X is contained in infinitely many members of $\mathscr{U}$;
  - Ω, the family of *ω-covers* of X, i.e. those open covers $\mathscr{U}$ for which $X \notin \mathscr{U}$ and every finite subset of X is contained in some member of $\mathscr{U}$;
  - $\mathfrak{K}$, the family of *k-covers* (see page 22);
  - Γ, the family of *γ-covers* of X, i.e. those infinite open covers $\mathscr{U}$ for which $X \notin \mathscr{U}$ and each point of X belongs to all but finitely many members of $\mathscr{U}$.

Denote by $\mathscr{H}(X)$ the space of homeomorphisms of X with the compact-open topology.

We will denote by $C_k(X, Y)$ (respectively $C_p(X, Y)$) the space of all continuous functions from X to Y with the compact-open topology (respectively the topology of pointwise convergence). We drop the subscript k or p when we are not interested in the topology on $C(X, Y)$ and, when $Y = \mathbb{R}$, denote by $C^*(X, \mathbb{R})$ the set of bounded continuous functions on X.

We will denote by \$ the space $\{0, 1\}$ with the *Sierpinski topology* $\{\varnothing, \$, \{0\}\}$. Then for any space X we denote by $[X, \$]$ the space of continuous functions from X to \$ with the *upper Kuratowski topology*, i.e. that in which a subset $\mathscr{F} \subset [X, \$]$ is open if and only if

(i) for each $f \in \mathscr{F}$ and each $g \in [X, \$]$ if $g \leq f$ then $g \in \mathscr{F}$;
(ii) if $\mathscr{G} \subset [X, \$]$ is such that $\inf \mathscr{G} \in \mathscr{F}$ then there is a finite subfamily $\mathscr{G}' \subset \mathscr{G}$ with $\inf \mathscr{G}' \in \mathscr{F}$.

In this definition we are using the usual ordering on $\{0, 1\}$ when discussing $\leq$ and inf. Of course identifying a closed subset of X with its characteristic function gives a bijective correspondence between $[X, \$]$ and the collection of closed subsets of X. This topology is also variously known as the *cocompact topology* and the *upper Fell topology*, especially when looked at as a topology on the set 2^X of non-empty closed subsets of X. Letting $U^+ = \{C \in 2^X \ / \ C \subset U\}$ for $U \subset X$, this topology has as subbasis $\{U^+ \ / \ U \text{ is open in } X \text{ and } X \setminus U \text{ is compact}\}$. The *Fell topology* [14], denoted by τ_{fell}, has as subbasis

$$\{U^+ \ / \ U \text{ is open in} X \text{and } X \setminus U \text{ is compact}\} \cup \{U^- \ / \ U \text{ is open in } X\},$$

where $U^- = \{C \in 2^X \ / \ C \cap U \neq \varnothing\}$.

Following [15] we say that a family $\Phi \subset C(X, \mathbb{R})$ is *generating** for X with respect to a set M of continuous functions mapping subsets of euclidean space to euclidean space if each $f \in C^*(X, \mathbb{R})$ can be written as a composition of functions from Φ, M and $C(\mathbb{R}, \mathbb{R})$. The set M is called a *set of operations*.

The notion of *microbundle* is introduced in [36, p. 54] as a diagram $B \xrightarrow{i} E \xrightarrow{j} B$ where B and E are topological spaces and i and j are continuous functions such that ji is the identity map and for each $b \in B$ there are open neighbourhoods U of b and V of $i(b)$ with $i(U) \subset V$ and $j(V) \subset U$ such that there is a homeomorphism $h_b : V \to U \times \mathbb{R}^n$ for some fixed n such that the diagram

$$\begin{tikzcd}
& V \arrow[dr, "j\restriction V"] \arrow[dd, "h_b"] & \\
U \arrow[ur, "i\restriction U"] \arrow[dr, "\times 0"'] & & U \\
& U \times \mathbb{R}^n \arrow[ur, "p_1"'] &
\end{tikzcd}$$

commutes, where $\times 0$ denotes the map $u \mapsto (u, 0)$ and $p_1(u, x) = u$. [36, Lemma 2.1] shows that for a manifold M the diagram $M \xrightarrow{\Delta} M \times M \xrightarrow{p_1} M$ is a microbundle, called the *tangent microbundle*, where $\Delta(x) = (x, x)$ and $p_1(x, y) = x$. The paper [36] was followed soon after by [34]; apparently as a result *Microbundles Part II* never appeared and the theory of microbundles did not develop far. However, as noted in Theorem 2.1(51) below, Kister's assumption that his base spaces are paracompact is vital: the only manifolds to which his result applies for the tangent microbundle are metrisable.

2.2 Conditions Equivalent to Metrisability

Here we present 119 conditions which are equivalent to metrisability on a manifold along with an indication of the proofs of the equivalence.

Theorem 2.1 *Let M^m be a manifold. Then the following are equivalent:*

1. *M is metrisable;*
2. *M is paracompact;*
3. *M is strongly paracompact;*
4. *M is screenable;*
5. *M is metacompact;*
6. *M is σ-metacompact;*
7. *M is paraLindelöf;*
8. *M is σ-paraLindelöf;*
9. *M is metaLindelöf;*
10. *M is nearly metaLindelöf;*
11. *M is Lindelöf;*
12. *M is linearly Lindelöf;*
13. *M is ω_1-Lindelöf;*
14. *M is ω_1-metaLindelöf;*
15. *M is nearly linearly ω_1-metaLindelöf;*
16. *M is almost metaLindelöf;*
17. *M is hereditarily Lindelöf;*
18. *M is strongly hereditarily Lindelöf;*
19. *M is k-Lindelöf;*
20. *M is an $\aleph_0$-space;*
21. *M is cosmic;*
22. *M is an $\aleph$-space;*
23. *M has a star-countable k-network;*
24. *M has a point-countable k-network;*
25. *M has a k-network which is point-countable on some dense subset of M;*
26. *M is second countable;*
27. *M is hemicompact;*
28. *M is σ-compact;*
29. *M is Hurewicz;*

30. *M satisfies the selection criterion* $\mathsf{S}_1(\mathfrak{K}, \Gamma)$*;*
31. *Player Two has a winning strategy in the game* $\mathsf{G}_c^{n+1}(\mathfrak{O},\mathfrak{O})$ *played on M;*
32. *Player Two has a winning strategy in the game* $\mathsf{G}_c(\mathfrak{O},\mathfrak{O})$ *played on M;*
33. *M is selectively screenable;*
34. *only countably many coordinate charts are needed to cover M;*
35. *M may be embedded in some euclidean space;*
36. *M may be embedded properly in some euclidean space;*
37. *M is Polish;*
38. *there is a continuous discrete map* $f : M \to X$ *where X is Hausdorff and second countable;*
39. *there is a surjective immersion* $f : \mathbb{R}^m \to M$*;*
40. *there is a continuous surjection* $f : \mathbb{R}^n \to M$ *for some n;*
41. *there is a continuous surjection* $f : \mathbb{R}^n \to M$ *for all n;*
42. *M is Lašnev;*
43. *M is an* M_1*-space;*
44. *M is stratifiable;*
45. *M is finitistic;*
46. *M is strongly finitistic;*
47. *M is star finitistic;*
48. *there is an open cover* $\mathscr{U}$ *of M such that for each* $x \in M$ *the set* $st(x, \mathscr{U})$ *is homeomorphic to an open subset of* $\mathbb{R}^m$*;*
49. *there is a point-star-open cover* $\mathscr{U}$ *of M such that for each* $x \in M$ *the set* $st(x, \mathscr{U})$ *is Lindelöf;*
50. *there is a point-star-open cover* $\mathscr{U}$ *of M such that for each* $x \in M$ *the set* $st(x, \mathscr{U})$ *is metrisable;*
51. *the tangent microbundle on M is equivalent to a fibre bundle;*
52. *M is a normal Moore space;*
53. *M is a normal* θ*-refinable space;*
54. *M is a normal subparacompact space;*
55. *M is a normal space which has a* σ*-discrete cover by compact subsets;*
56. $M \times M$ *is perfectly normal;*
57. *M is a normal space which has a sequence* $\langle \mathscr{U}_n \rangle_{n\in\omega}$ *of open covers with* $\bigcap_n \overline{st(x, \mathscr{U}_n)} = \{x\}$ *for each* $x \in M$*;*
58. *M is separable and monotonically normal;*
59. $M \times M$ *is monotonically normal;*
60. *M is monotonically normal and of dimension* ≥ 2 *or* $M \approx \mathbb{S}^1$ *or* $\mathbb{R}$*;*
61. *M is extremely normal;*
62. *M is perfectly normal and there is a sequence* $\langle \mathscr{U}_n \rangle_{n\in\omega}$ *of families of open sets such that* $\bigcap_{n\in C(x)} \overline{st(x, \mathscr{U}_n)} = \{x\}$ *for each* $x \in M$*, where*
$$C(x) = \{n \in \omega \ / \ \exists U \in \mathscr{U}_n \text{ with } x \in U\};$$
63. *M is separable and there is a sequence* $\langle \mathscr{C}_n \rangle_{n\in\omega}$ *of point-star-open covers such that* $\bigcap_n \overline{st(x, \mathscr{C}_n)} = \{x\}$ *for each* $x, y \in M$ *and for each* $x, y \in M$ *and each* $n \in \omega$ *we have* $y \in \overline{st(x, \mathscr{C}_n)}$ *if and only if* $x \in \overline{st(y, \mathscr{C}_n)}$*;*

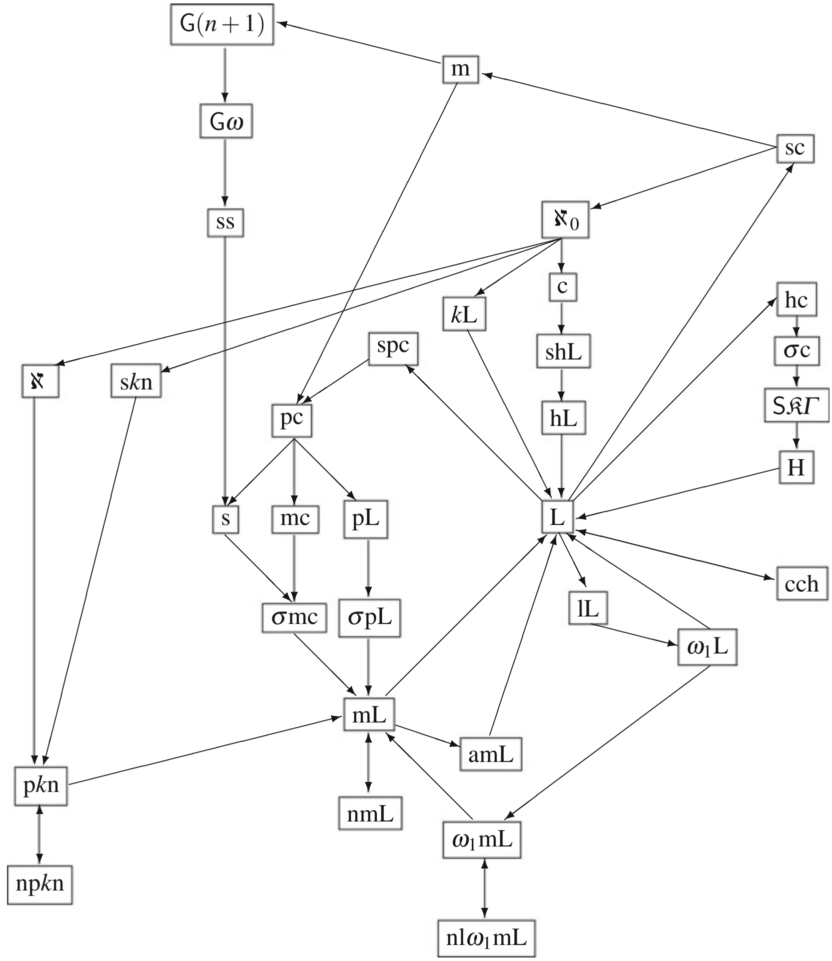
64. *M is separable and there is a sequence $\langle \mathscr{C}_n \rangle_{n\in\omega}$ of point-star-open covers such that $\bigcap_n \overline{st(x, \mathscr{C}_n)} = \{x\}$ for each $x \in M$ and for each $x \in M$ and each $n \in \omega$, $ord(x, \mathscr{C}_n)$ is finite;*
65. *M is separable and hereditarily normal and there is a sequence $\langle \mathscr{C}_n \rangle_{n\in\omega}$ of point-star-open covers such that $\bigcap_n \overline{st(x, \mathscr{C}_n)} = \{x\}$ for each $x \in M$;*
66. *M is separable and there is a sequence $\langle \mathscr{U}_n \rangle_{n\in\omega}$ of families of open sets such that $\bigcap_{n\in C(x)} \overline{st(x, \mathscr{U}_n)} = \{x\}$ for each $x \in M$, and $ord(x, \mathscr{C}_n)$ is countable for each $x \in M$ and each $n \in \omega$;*
67. *$M \times M$ has a countable sequence $\langle U_n : n \in \omega \rangle$ of open subsets, such that for all $(x, y) \in M \times M \setminus \Delta$, there is $n \in \omega$ such that $(x, x) \in U_n$ but $(x, y) \notin \overline{U_n}$;*
68. *For every subset $A \subset M$ there is a continuous injection $f : M \to Y$, where Y is a metrisable space, such that $f(A) \cap f(M \setminus A) = \varnothing$;*
69. *For every subset $A \subset M$ there is a continuous $f : M \to Y$, where Y is a space with a quasi-regular-G_δ-diagonal, such that $f(A) \cap f(M \setminus A) = \varnothing$;*
70. *M is weakly normal with a G^*_δ-diagonal;*
71. *M has a quasi-G^*_δ-diagonal and for every closed subset $A \subset M$ there is a countable family $\mathscr{G}$ of open subsets such that, for every $x \in A$ and $y \in X \setminus A$, there is a $G \in \mathscr{G}$ with $x \in G$, $y \notin \overline{G}$;*
72. *M has a regular G_δ-diagonal;*
73. *M is submetrisable;*
74. *M is a JCM-space;*
75. *M has the Moving Off Property;*
76. *M has property pp;*
77. *every open cover of M has an open refinement $\mathscr{V}$ such that for every choice function $f : \mathscr{V} \to M$ the set $f(\mathscr{V})$ is closed in M;*
78. *every open cover of M has an open refinement $\mathscr{V}$ such that for every choice function $f : \mathscr{V} \to M$ the set $f(\mathscr{V})$ is discrete in M;*
79. *M is a point-countable union of open subspaces each of which is metrisable;*
80. *M has a point-countable basis;*
81. *M is separable and M^ω is a countable union of metrisable subspaces;*
82. *$\mathscr{H}(M)$ is a q-space;*
83. *$\mathscr{H}(M)$ is separable and metrisable;*
84. *selection principle $\mathsf{S}_1(\mathfrak{K}, \mathfrak{K})$ holds on M;*
85. *selection principle $\mathsf{S}_{fin}(\Omega, \Omega)$ holds on M;*
86. *selection principle $\mathsf{S}_{fin}(\Lambda, \Lambda)$ holds on M;*
87. *selection principle $\mathsf{S}_{fin}(\mathfrak{D}, \mathfrak{D})$ holds on M;*
88. *selection principle $\mathsf{S}_{fin}(\mathfrak{K}, \mathfrak{D})$ holds on M;*
89. *$C_k(M, \mathbb{R})$ is Polish;*
90. *$C_k(M, \mathbb{R})$ is completely metrisable;*
91. *$C_k(M, \mathbb{R})$ is first countable;*
92. *$C_k(M, \mathbb{R})$ is second countable;*
93. *$C_k(M, \mathbb{R})$ is a q-space;*
94. *$C_k(M, \mathbb{R})$ is Fréchet;*
95. *$C_k(M, \mathbb{R})$ is countably tight;*
96. *$C_k(M, \mathbb{R})$ has countable strong fan tightness;*

97. *$C_k(M, \mathbb{R})$ is an $\aleph_0$-space;*
98. *$C_k(M, \mathbb{R})$ is cosmic;*
99. *$C_k(M, \mathbb{R})$ is analytic;*
100. *$C(M, \mathbb{R})$ satisfies the selection criterion $\mathsf{S}_1\left(\Omega^k_{\underline{0}}, \Sigma^p_{\underline{0}}\right)$: for each sequence $\langle F_n \rangle$ of subsets of $C(M, \mathbb{R})$ whose compact-open closures contain the constant function $\underline{0}$ there is a sequence $\langle f_n \rangle$, infinitely many members of which are distinct, with $f_n \in F_n$ for all n and $\langle f_n \rangle$ converges pointwise to $\underline{0}$;*
101. *$C_p(M, \mathbb{R})$ has countable tightness;*
102. *$C_p(M, \mathbb{R})$ has countable fan tightness;*
103. *$C_p(M, \mathbb{R})$ is analytic;*
104. *$C_p(M, \mathbb{R})$ is hereditarily separable;*
105. *$C_p(M, \mathbb{R})$ (equivalently $C_k(M, \mathbb{R})$) is separable;*
106. *$[M, \$]$ is first countable;*
107. *$[M, \$]$ is countably tight;*
108. *$[M, \$]$ is sequential;*
109. *$(2^X, \tau_{\text{fell}})$ is metrisable;*
110. *$(2^X, \tau_{\text{fell}})$ is countably tight;*
111. *$(2^X, \tau_{\text{fell}})$ is sequential;*
112. *$(2^X, \tau_{\text{fell}})$ is radial;*
113. *K has a winning strategy in Gruenhage's game $G^o_{K,L}(M)$;*
114. *$C_k(M, \mathbb{R})$ is strongly α-favourable;*
115. *$C_k(M, \mathbb{R})$ is weakly α-favourable;*
116. *$C_k(M, \mathbb{R})$ is pseudocomplete;*
117. *$C_k(M, \mathbb{R})$ is strongly Baire;*
118. *$C_k(M, \mathbb{R})$ is Baire;*
119. *$C_k(M, \mathbb{R})$ is Volterra;*
120. *M has a countable generating* family with respect to a countable set of operations.*

Proof (*Outline*) The diagram below shows how items 1–34 are related. In the diagram we use the following notation.

$\boxed{\text{m}}$ = metrisable; $\boxed{\text{sc}}$ = second countable; $\boxed{\text{shL}}$ = strongly hereditarily Lindelöf; $\boxed{\text{hL}}$ = hereditarily Lindelöf; $\boxed{\sigma\text{c}}$ = σ-compact; $\boxed{\text{hc}}$ = hemicompact; $\boxed{\text{H}}$ = Hurewicz; $\boxed{\mathsf{S}\mathfrak{K}\Gamma}$ = satisfies $\mathsf{S}_1(\mathfrak{K}, \Gamma)$; $\boxed{\text{cch}}$ = countably many charts cover; $\boxed{\aleph_0}$ = $\aleph_0$-space; $\boxed{\aleph}$ = $\aleph$-space; $\boxed{\text{s}k\text{n}}$ = has a star-countable k-network; $\boxed{\text{p}k\text{n}}$ = has a point-countable k-network; $\boxed{\text{np}k\text{n}}$ = has a k-network which is point-countable on a dense subset; $\boxed{k\text{L}}$ = k-Lindelöf; $\boxed{\text{c}}$ = cosmic; $\boxed{\text{L}}$ = Lindelöf; $\boxed{\text{lL}}$ = linearly Lindelöf; $\boxed{\omega_1\text{L}}$ = ω_1-Lindelöf; $\boxed{\text{spc}}$ = strongly paracompact; $\boxed{\text{pc}}$ = paracompact; $\boxed{\text{mc}}$ = metacompact; $\boxed{\text{s}}$ = screenable; $\boxed{\text{ss}}$ = selectively screenable; $\boxed{\text{pL}}$ = paraLindelöf; $\boxed{\sigma\text{mc}}$ = σ-metacompact; $\boxed{\sigma\text{pL}}$ = σ-paraLindelöf; $\boxed{\text{mL}}$ = metaLindelöf; $\boxed{\text{amL}}$ = almost metaLindelöf; $\boxed{\text{nmL}}$ = nearly metaLindelöf; $\boxed{\omega_1\text{mL}}$ = ω_1-metaLindelöf; $\boxed{\text{nl}\omega_1\text{mL}}$ = nearly linearly

ω_1-metaLindelöf; $\boxed{\mathsf{G}(n+1)}$ = Player Two has a winning strategy in the game $\mathsf{G}_c^{n+1}(\mathfrak{O}, \mathfrak{O})$; $\boxed{\mathsf{G}\omega}$ = Player Two has a winning strategy in the game $\mathsf{G}_c(\mathfrak{O}, \mathfrak{O})$.



All arrows denote implications. Downward sloping arrows show an implication which holds in an arbitrary topological space. Upward sloping arrows require one or more properties of manifolds to realise the implication. $\boxed{\mathrm{mL}} \Rightarrow \boxed{\mathrm{L}}$ in every locally separable and connected space. $\boxed{\mathrm{amL}} \Rightarrow \boxed{\mathrm{L}}$ in every regular, locally separable and connected space, [23]. $\boxed{\mathrm{nmL}} \Rightarrow \boxed{\mathrm{mL}}$ in every locally hereditarily separable space. $\boxed{\mathrm{L}} \Rightarrow \boxed{\mathrm{spc}}$ in every T_3 space. $\boxed{\omega_1\mathrm{L}} \Rightarrow \boxed{\mathrm{L}}$ in every locally metrisable space, [2]. $\boxed{\mathrm{L}} \Rightarrow \boxed{\mathrm{sc}}$ in every locally second countable space. $\boxed{\mathrm{L}} \Rightarrow \boxed{\mathrm{hc}}$ in every locally compact space. $\boxed{\mathrm{cch}} \Rightarrow \boxed{\mathrm{L}}$ (Visser) because a countable union of Lindelöf sets is Lindelöf. $\boxed{\mathrm{sc}} \Rightarrow \boxed{\mathrm{m}}$ in every T_3 space (Urysohn's metrisation theorem). $\boxed{\mathrm{m}} \Rightarrow \boxed{\mathsf{G}(n+1)}$ in every space having covering dimension at most n, [4, Theorem 2.4]. $\boxed{\omega_1\mathrm{mL}} \Rightarrow \boxed{\mathrm{mL}}$ in every locally second countable space, [27]. $\boxed{\mathrm{nl}\omega_1\mathrm{mL}} \Rightarrow \boxed{\omega_1\mathrm{mL}}$ in every locally hereditarily separable space, [27]. $\boxed{\mathrm{p}k\mathrm{n}} \Rightarrow \boxed{\mathrm{mL}}$ in every regular Fréchet space. $\boxed{\mathrm{np}k\mathrm{n}} \Rightarrow \boxed{\mathrm{p}k\mathrm{n}}$ in every regular, locally compact, locally hereditarily separable space.

By [39, Proposition 7.3.9] we conclude that a metrisable n-manifold, being separable and of covering dimension n, embeds in $\mathbb{R}^{2n+1}$, so 1 $\Rightarrow$ 35. By choosing a proper continuous real-valued function on M we can add a further coordinate to embed M in $\mathbb{R}^{2n+2}$ so that the image is closed, i.e. the embedding is proper, hence 1 $\Rightarrow$ 36. It is clear that 36 $\Rightarrow$ 37.

Every second countable Hausdorff space satisfies 38 so 26 $\Rightarrow$ 38. Conversely, given the situation of 38, if $\mathscr{B}$ is a countable base for the topology on X then the Poincaré-Volterra Lemma of [16, Lemma 23.2] asserts that

$$\big\{U \subset M /\ U \text{ is second countable and} \\ \text{there is } V \in \mathscr{B} \text{ such that } U \text{ is a component of } f^{-1}(V)\big\}$$

is a countable base for M.

[9, Theorem 1] asserts that 1 $\Rightarrow$ 39 while 39 $\Rightarrow$ 40 is trivial. 40 $\Rightarrow$ 41 because if $\mathbb{R}^p \to M$ is a continuous surjection then we can construct a continuous surjection $\mathbb{R} \to M$ by use of a Peano curve $\mathbb{R} \to \mathbb{R}^p$ and for any other n we may project $\mathbb{R}^n$ onto $\mathbb{R}$ (or go directly to $\mathbb{R}^p$ if $n > p$). The continuous image of a σ-compact space is σ-compact so 41 $\Rightarrow$ 28.

Clearly every metrisable space is Lašnev so 1 $\Rightarrow$ 42. The implication 42 $\Rightarrow$ 43 is [29, Theorem 5.5]. It is easy to show that 43 $\Rightarrow$ 44. The implication 44 $\Rightarrow$ 2 is [29, Theorem 5.7].

The conditions 1, 45, 46 and 47 are shown to be equivalent in [10].

The equivalence of conditions 1 and 48–51 is established as follows: 1 $\Rightarrow$ 48 is reasonably straightforward making use of the fact that metrisable manifolds are σ-compact. Then 48 $\Rightarrow$ 49 is trivial. 49 $\Rightarrow$ 50 requires use of Urysohn's metrisation theorem to deduce that the Lindelöf stars are metrisable. 50 $\Rightarrow$ 11 requires some delicate manoeuvres; see [24]. 51 $\Rightarrow$ 48 is also found in [24] while 1 $\Rightarrow$ 51 is [34, Corollary 2].

The implication 1 $\Rightarrow$ 52 holds in every topological space while its converse holds provided that the space is locally compact and locally connected, [40] or [41, Theorem 3.4]. The equivalence of 52 and 53 comes from [45, Theorem 3], while the equivalence of 52, 53, 54 and 55 is discussed in [38, Theorem 8.2].

The equivalence of conditions 1, 56 and 57 is referred to briefly in [19]. The implications 1 $\Rightarrow$ 56 $\Rightarrow$ 57 hold in any topological space and the implication 57 $\Rightarrow$ 1 uses some properties of a manifold.

Every metric space is monotonically normal and every metrisable manifold is second countable, hence separable, so 1 $\Rightarrow$ 58. To get the converse implication 58 $\Rightarrow$ 2 use is made of the fact that every monotonically normal space is hereditarily collectionwise normal ([32]), and hence no separable monotonically normal space contains a copy of ω_1. On the other hand in [4, Theorem I] it is shown that a monotonically normal space is paracompact if and only if it does not contain a stationary subset of a regular uncountable ordinal.

If M is metrisable, so is $M \times M$, so that $M \times M$ is monotonically normal and hence $1 \Rightarrow 59$. The converse follows from a metrisability result of [32] as manifolds are locally countably compact.

The criterion 60 is [4, corollary 2.3(e)], except that we have listed all of the metrisable 1-manifolds.

Every metrisable space is extremely normal. The implication $61 \Rightarrow 2$ is found in [44].

The equivalence of conditions 1 and 62–65 is discussed in [37].

Proofs of the equivalence of 1 and 66 may be found in [18] and of 1 and 67–71 may be found in [17].

The implication $72 \Rightarrow 1$ holds in every locally compact, locally connected space ([29, Theorem 2.15(b)]) and, as noted in [29, p. 430], every submetrisable space has a regular G_δ-diagonal so $73 \Rightarrow 72$.

Clearly every metrisable space is a JCM-space so $1 \Rightarrow 74$. On the other hand a manifold which is a JCM-space is submetrisable and hence $74 \Rightarrow 73$. We give more details of the proof of this fact because it is not apparently in the literature. Suppose that the manifold M is a JCM-space, say d is a metric on M whose restriction to each compact subset of M induces the subspace topology inherited from M. We claim that d exhibits the submetrisability of M. Suppose that $U \subset M$ is open in the metric space (M, d), and let $x \in U$. Then there is a chart (V, φ) on M such that $\varphi(V) = \mathbb{R}^n$, where n is the dimension of M, and $\varphi(x) = 0$. By the JCM property, $U \cap \varphi^{-1}\left(\mathbb{B}^n\right)$ is open in $\varphi^{-1}\left(\mathbb{B}^n\right)$ and hence is a neighbourhood of x in M. Thus U is a neighbourhood of x in M so U is open in M.

The equivalence of conditions 1, 75 and 118 is discussed in [5].

It is readily shown that every T_1-space which is paracompact has property pp. We now obtain the implication $76 \Rightarrow 5$, again giving more details because the proof does not appear to be in the literature. Suppose that $\mathscr{U}$ is an open cover of M. Use the property pp to find an open refinement $\mathscr{V}$ such that for each choice function $f : \mathscr{V} \to M$ with $f(V) \in V$ for each $V \in \mathscr{V}$ the set $f(\mathscr{V})$ is closed and discrete. We will show that $\mathscr{V}$ is point-finite. Suppose to the contrary that $x \in M$ is such that $\{V \in \mathscr{V} \mid x \in V\}$ is infinite; let $\langle V_n \rangle$ be a sequence of distinct members of $\mathscr{V}$ each of which contains x. Because M is a manifold, hence first countable, we may choose a countable neighbourhood basis $\{W_n \mid n \in \omega\}$ at x. Note that for each n, $V_n \cap W_n \setminus \{x\} \neq \varnothing$ as M has no isolated points. Choose a function $f : \mathscr{V} \to M$ as follows: if $V \in \mathscr{V}$ but $V \neq V_n$ for each n then choose $f(V) \in V - \{x\}$ arbitrarily; if $V = V_n$ choose $f(V_n) \in V_n \cap W_n \setminus \{x\}$. Then $x \in \overline{f(\mathscr{V})} \setminus f(\mathscr{V})$ so that $f(\mathscr{V})$ is not closed, contrary to the choice of $\mathscr{V}$. Thus $\mathscr{V}$ is point-finite so M is metacompact.

It is easy to show that conditions 77 and 78 are equivalent to each other, and hence also to 76; cf [20, Lemma 2.3].

Details for the implication $79 \Rightarrow 9$ appear in [24], while details for the implication $80 \Rightarrow 1$ appear in [13]. Of course $26 \Rightarrow 80$.

The implication 81 $\Rightarrow$ 1 is a consequence of the more general result that if the countable power of a topological space X is a countable union of metrisable subspaces and in X discrete families of open sets are countable then X is metrisable, [28].

The equivalence of 1 and conditions 82 and 83 is shown in [28, Theorem 4.2].

The equivalence of the selection principles 84, 85, 86, 87 and 88 to metrisability of a manifold is the content of [22, Theorems 1.3 and 1.4].

The equivalence of conditions 1 and 89 to 101, excluding 91, 96 and 100, is shown in [26]. A number of properties of manifolds are required, including that every manifold is a q-space and a k-space, and some of the equivalences to metrisability already proved.

Conditions 91 and 96 are shown to be equivalent to condition 11 in [8, Theorem 6] using Hausdorffness, local compactness and first countability of manifolds.

In [8, Theorem 15] there is a proof that in a Tychonoff space 30 and 100 are equivalent.

The equivalence of condition 1 and conditions 109, 110, 111 and 112 is established in [7, Theorem 3.3].

The implication 1 $\Rightarrow$ 114 follows from 89 and [33, Theorem 8.17]. 114 $\Rightarrow$ 115 is trivial. 115 $\Rightarrow$ 113 is [30, Lemma 4.3]. 113$\Rightarrow$ 2 is [30, Theorem 4.1].

Complete metrisability implies pseudocompleteness in any space and in turn pseudocompleteness implies α-favourability in a regular space, so 90 $\Rightarrow$ 116 $\Rightarrow$ 115.

The implications 37 $\Rightarrow$ 117 and 117 $\Rightarrow$ 28 are shown in [5, Theorem 2.2].

The equivalence of 118 was already considered above in the context of 75.

Clearly every Baire space is Volterra and the converse holds in any locally convex topological vector space, [6, Theorem 3.4] so 118 $\Leftrightarrow$ 119.

The equivalence of 1 and 120 is contained in [15, Theorem 4], noting for the implication 1 $\Rightarrow$ 120 that by 1$\Leftrightarrow$ 26 every metrisable manifold is secondcountable, hence separable. $\square$

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