Colorimetric Quantities and Laws

A. Giannini and L. Mercatelli

1 Introduction

The purpose of this chapter is to introduce the fundaments of colorimetry: It illustrates the most important and frequently employed chromatic quantities and laws. These elements are essential to define and describe color concepts, chromatic human eye perception, and color measurement. Colorimetry can be defined as the scientific–technological discipline physically describing and quantifying human color perception.

Colorimetry exploits spectro-photometric measurements and spectral curves to extract the colorimetric quantities, finally expressing the results as chromatic coordinates and classifying the data in color spaces. Basically, this discipline privileges the physical quantities of color perception, frequently expressed in the CIE 1931 XYZ color space, as tristimulus values or using associated parameters.

After a brief description of the colorimetric science, the most popular and useful quantities of colorimetry are reported and concisely explained. Sections 2.1, 2.2, and 2.3 discuss the concepts of trichromatic vision, color matching, and tristimulus. Then, the CIE 1931 color diagram is illustrated in Sect. 3. Color attributes and temperature are examined in Sects. 3.1 and 3.2. To conclude, Sect. 4.1.1 describes CIELAB and Sect. 4.1.2 describes CIELUV 1976 color spaces.

A. Giannini · L. Mercatelli (✉)
CNR-INO National Institute of Optics, Largo E. Fermi, 6, 50125 Florence, Italy
e-mail: luca.mercatelli@ino.it

A. Giannini
e-mail: luca.mercatelli@ino.it

P. Sansoni et al. (eds.), *Sustainable Indoor Lighting*, Green Energy and Technology,
DOI 10.1007/978-1-4471-6633-7_2

2 Colorimetry

In everyday life, everything is characterized by color: The sky is blue, leaves are green, strawberries are reds … but what is *color*?

The word *color* has a lot of different meanings depending on the field of works in which it is used. For example, to the painter, *color* is the pigment that is used to paint; to the physicist, it is a property of the radiant energy; to the psychologist, *color* is a perceptive event; and to the physiologist, it is a response of the nervous system.

In reality, the concept of color is so complex that it can embrace all the previous definitions. That which is sure is the fact that color is not a property of light and objects themselves, as everyday experience could make us think, but it comes from an interaction between light, objects, and the human eye's response.

Since color depends on eye's response, it is a perception, a quality that the human visual system assigns to light and objects, but that is not intrinsic to them. So, an apple is not red but *we see* it red, a leaf is not green, *we see* it green….

Therefore, colorimetry has an important task, which consists in quantifying colors by the creation of standard models based on measurable objective quantities.

Before talking of the *CIE* results, it is important to give a brief description of the process involved in color sensation, a process which begins from the emission of a luminous stimulus (light reaching the eye, also called color stimulus) and finishes with the elaboration of this stimulus by the retinal photoreceptors and finally by the brain.

2.1 Trichromatic Vision

It is common experience that the chromatic vision is typical of photopic conditions, while in night vision, it is not possible to see colors. The reason is that to see colors, it is necessary to have at least two receptors with different sensitivity in different region of the spectrum, but at low levels of illuminations, only rods are active and, since they all are made up of the same pigment, they all have the same spectral sensitivity.

In photopic conditions, the active photoreceptors are cones. As shown in Fig. 1, there are three kinds of cones, each one having a spectral response to light due to a particular pigment: cones L (long) which cover the range of the long visible wavelengths with an absorbance peak at 560 nm; cones M (medium) with an absorbance peak at 530 nm; and cones S (short) with an absorbance peak at 420 nm, in the region of shorter wavelengths.

The color vision is due to cones because of the possibility to give different response to different luminous stimulus. For example, a monochromatic stimulus of 670 nm excites only the L cones; one of 530 nm excites both L and M cones, but in

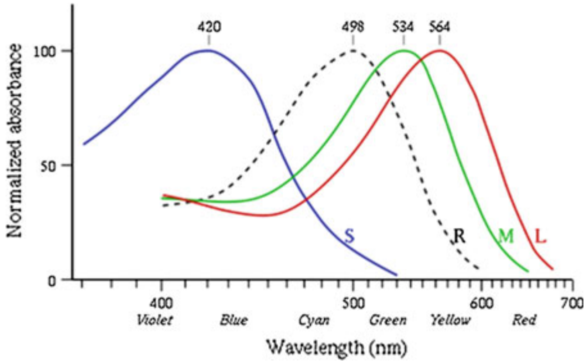


Fig. 1 Spectral absorbance of *cones L*, *M*, and *S* and *rods R* normalized at their maximum

different percentage; finally, a broadband source as sun stimulates each kind of cones at the same time.

The response of each cone is proportional to the fraction of light absorbed within two limits: the inferior limit, *sensitivity threshold*, below which cones are not sensitive anymore, and the superior limit, *saturation threshold*, above which the response is always the same. If the majority of observers belong to one class having similar cone spectral sensitivity, whose color vision is called *normal vision*, there are observers who have a different discrimination of colors because of the missing of a kind a cone (*dichromats*), or two cones (*monochromats*), or because of a different sensitivity of cones respects to the normal observers (*anomalous trichromats*).

Despite these color-deficient observers, the color models are obtained referring to the normal vision of the human standard eye.

2.2 The Laws of Color Matching

Since color is a human sensation, its measure has to be based on a procedure of comparison between a test surface and a reference one, a procedure called "*color matching*" coded in 1931 and revisited in 1961 by *CIE*.

Suppose to have a white reference source (*standard illuminator*), whose wavelengths are selected by a diffraction grating, illuminating one half of a white surface reflecting at 100 % constantly for each wavelength. Suppose then the other half of the surface is illuminated by three monochromatic lights as in Fig. 2.

For each wavelength of the reference luminous stimulus, the observer has to match the additive mixture of the three lights and vary their intensity until he obtains the same sensation of color on the second surface. Additive mixture means that the observer does not see the three lights as independent, but he sees the mixed light reflected by the surface.

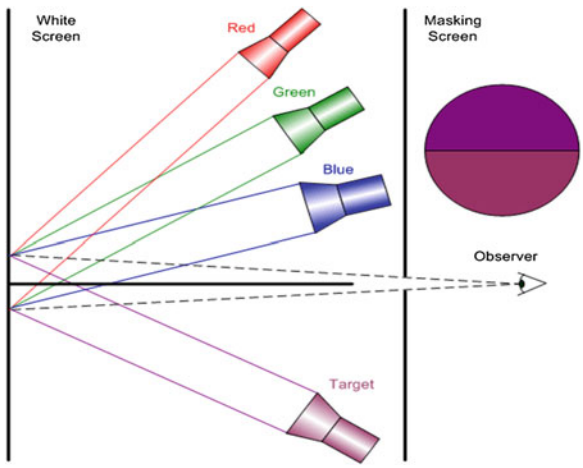


Fig. 2 Color-matching experiment

Mathematically, if $S(\lambda)$ is the reference stimulus and $R(\lambda)$, $G(\lambda)$, and $B(\lambda)$ are the three monochromatic lights, then

$$S(\lambda) \equiv R(\lambda) + G(\lambda) + B(\lambda) \tag{1}$$

where $\equiv$ the symbol of metamerism, i.e., it means that two lights, in this case, the reference stimulus and the additive mixture of red, green, and blue lights, can produce the same color sensation even if they are of different spectral compositions.

Several experiments of color matching demonstrate that the three colors giving the major chromatic scale, and that for this reason are called *fundamental colors*, are red (700 nm), green (546.1 nm), and blue (435.8 nm), but even with this primary colors, it is not possible to match all colors. To do that, a new method of subtractive mixture is introduced. It does not mean subtracting color from the additive mixture, but rather adding this color to the reference stimulus. Mathematically, the subtractive method can be expressed as

$$S(\lambda) + R(\lambda) \equiv G(\lambda) + B(\lambda) \tag{2a}$$

$$S(\lambda) + B(\lambda) \equiv R(\lambda) + G(\lambda) \tag{2b}$$

$$S(\lambda) + G(\lambda) \equiv R(\lambda) + B(\lambda) \tag{2c}$$

Combining additive and subtractive mixtures all colors can be obtained.

Example nr.1. An obvious example of additive mixture is given by the sum of all the colors of the electromagnetic spectrum giving the white light.

Example nr.2. The working principle of a TV color is based on three phosphors sensitive to the primary colors. They are so near each other that observer's eye sees them as a single sum stimulation, able to give colors according to the additive mixture.

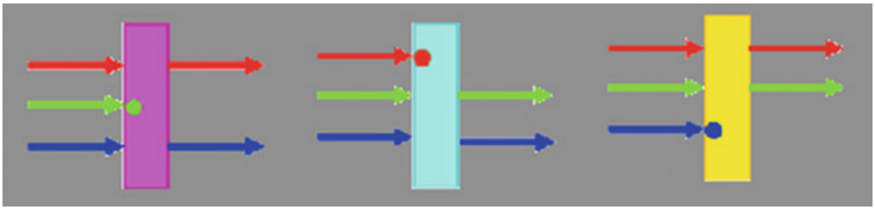


Fig. 3 Subtractive synthesis by three filters which absorb, respectively, the fundamental colors *green*, *red,* and *blue*

Example nr.3. Suppose having a white light formed by the synthesis of the three fundamental colors and three fundamental filters. As shown in Fig. 3, when the filter absorbs blue, it is of yellow color; when the filter absorbs red, it appears turquoise; and finally, when the filter absorbs green, it appears purple color. If we overlap two filters, we obtain one of the primary colors; while if we overlap all these filters, we obtain black, i.e., the absence of light.

In short, the empirical laws of color matching are expressed by the following Grassmann's laws:

1. To specify a color match, only three independent variables are necessary and sufficient;
2. For an additive mixture of color stimuli, only their tristimulus values (see later) are relevant and not their spectral compositions;
3. In additive mixture of color stimuli, if one or more components of the mixture are changed, then the resulting tristimulus values also change.

2.3 Tristimulus Theory

The color-matching experiment allows us to describe a sensation (color) by numerical quantities. In fact, it is possible to record the intensity values of the three primary lights that better match the reference one. Iterating the experiment with several observers, the average of the intensity values obtained by each observer gives the chromatic response of a standard eye. By normalization of the previous results, it is possible to extract the color-matching functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$, which represent the amounts of primaries needed to match the monochromatic test primary. The curves in Fig. 4 are normalized to have constant area beneath them. This area is fixed to a particular value by specifying that

$$\int_{380}^{780} \bar{r}(\lambda)\mathrm{d}\lambda = \int_{380}^{780} \bar{g}(\lambda)\mathrm{d}\lambda = \int_{380}^{780} \bar{b}(\lambda)\mathrm{d}\lambda \tag{3}$$

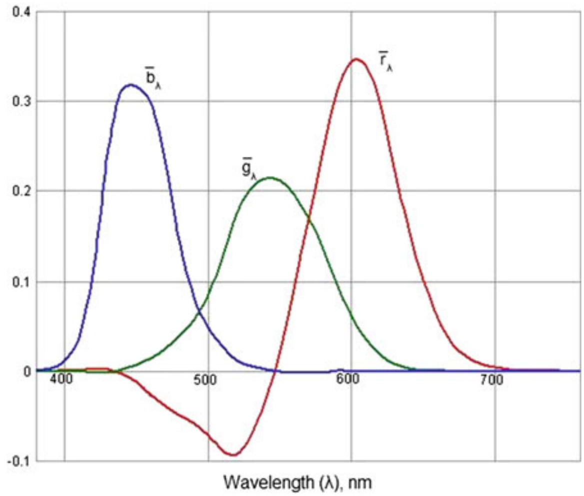


Fig. 4 Color-matching functions of the CIE 1931 standard colorimetric observer

Given a stimulus $S(\lambda)$, the sensation of color (the color stimulus) can be represented and evaluated by an equation similar to those used in photometry to pass from radiometric to photometric quantities:

$$R = k \int_{380\,\mathrm{nm}}^{780\,\mathrm{nm}} S(\lambda)\bar{r}(\lambda)\mathrm{d}\lambda \tag{4a}$$

$$G = k \int_{380\,\mathrm{nm}}^{780\,\mathrm{nm}} S(\lambda)\bar{g}(\lambda)\mathrm{d}\lambda \tag{4b}$$

$$B = k \int_{380\,\mathrm{nm}}^{780\,\mathrm{nm}} S(\lambda)\bar{b}(\lambda)\mathrm{d}\lambda \tag{4c}$$

where $S(\lambda)$ is a radiometric quantity and k is a constant of normalization that permits the passage from radiometric to colorimetric (photometric) quantities.

Because of the subtractive synthesis, color-matching functions have also negative values. To have only positive weights to evaluate the color stimulus as in Fig. 5, it has to be done a vectorial transformation, so that $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$, becomes the $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ color-matching functions, where $\bar{y}(\lambda)$ is evaluated to match the photopic curve $V(\lambda)$.

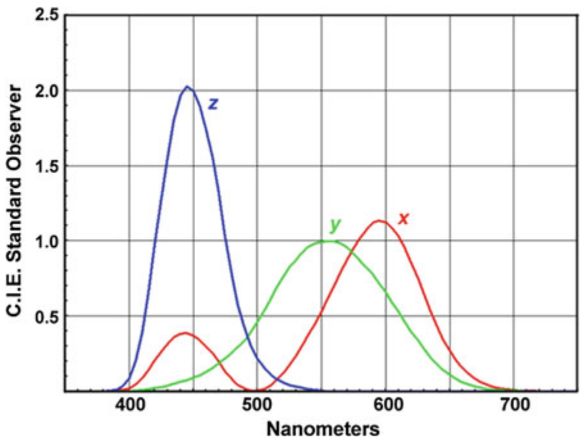


Fig. 5 Color-matching functions of standard colorimetric observer in XYZ CIE

The color stimulus is then represented by the equations:

$$X = k \int_{380\,\mathrm{nm}}^{780\,\mathrm{nm}} S(\lambda)\bar{x}(\lambda)\mathrm{d}\lambda \tag{5a}$$

$$Y = k \int_{380\,\mathrm{nm}}^{780\,\mathrm{nm}} S(\lambda)\bar{y}(\lambda)\mathrm{d}\lambda \tag{5b}$$

$$Z = k \int_{380\,\mathrm{nm}}^{780\,\mathrm{nm}} S(\lambda)\bar{z}(\lambda)\mathrm{d}\lambda \tag{5c}$$

where $S(\lambda)$ generally is spectral radiance and $k = 683$ lm/W so that Y can express the luminance of the luminous stimulus.

We know that the stimulus reaching the eye usually derives from the interaction with objects' surfaces because of their reflectivity or transmissivity properties. Tristimulus values take into account these properties, and the previous equations can be written as:

$$X = k \int_{380\,\mathrm{nm}}^{780\,\mathrm{nm}} S(\lambda)F(\lambda)\bar{x}(\lambda)\mathrm{d}\lambda \tag{6a}$$

$$Y = k \int_{380\,\mathrm{nm}}^{780\,\mathrm{nm}} S(\lambda)F(\lambda)\bar{y}(\lambda)\mathrm{d}\lambda \tag{6b}$$

$$Z = k \int_{380\,\text{nm}}^{780\,\text{nm}} S(\lambda)F(\lambda)\bar{z}(\lambda)\text{d}\lambda \tag{6c}$$

where $F(\lambda)$ can be the reflectivity factor $R(\lambda)$ or the transmissivity one $T(\lambda)$. In this case, the constant k is defined as

$$k = \frac{100}{\int_{380\,\text{nm}}^{780\,\text{nm}} S(\lambda)\bar{y}(\lambda)\text{d}\lambda}. \tag{7}$$

3 CIE 1931 Color Diagram

Since the human vision is based on the response of the three kinds of cones, a representation of color should be a three-dimensional figure. But in 1931, CIE established that the color stimulus could be expressed in terms of luminance and chromaticity, where chromaticity is color regardless of luminance. So, while Y has been chosen to represent the luminance of the stimulus, chromaticity can be graphically represented by x and y, two of the three parameters derived from the normalization of the tristimulus values X, Y, and Z, defined as

$$x = \frac{X}{X+Y+Z} \tag{8a}$$

$$y = \frac{Y}{X+Y+Z} \tag{8b}$$

$$z = \frac{Z}{X+Y+Z} \tag{8c}$$

which are called chromaticity coordinates.

Since $x + y + z = 1$, it follows that x and y are sufficient to characterize every color. The chromaticity coordinate z can be derived simply as the complement to 1 of the sum $x + y$.

Assuming x and y as the axes of a Cartesian diagram such as in Fig. 6 and reporting the chromaticity coordinates obtained from the color-matching test, they place themselves to form a triangle of vertex (0,0), (1,0), (0,1). Inside it, a horse-shoe-shaped curve is plotted which represents the whole visible spectrum, while outside, there are all the imaginary colors used in the subtractive mixture.

Colors that are on the curved line are called *spectral colors* and represent the monochromatic radiation of the visible spectrum, i.e., they are composed by a single wavelength. Instead, colors on the straight line connecting the lowest blue

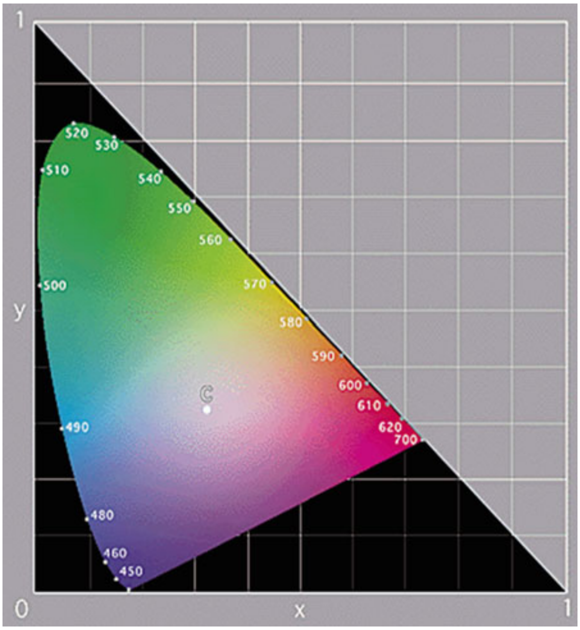


Fig. 6 CIE xyY color diagram 1931

and the highest red are not spectral colors because they derive from a mixture of red and blue lights. This line is called *purple line*.

At the center of the diagram, a point C represents the white point, i.e., the sum of all the wavelengths at the same intensity.

3.1 Color Attributes

Colors can be identified by three attributes: hue, saturation, and brightness.

Hue is the sensation of color determined by a light of a specified wavelength. If light is composed by more wavelengths, hue is determined by the dominant wavelength.

Suppose a color identified by (x_1, y_1) coordinates. If this point P represents a spectral color, its hue is that of the monochromatic wavelength. If it is not so, its hue is determined on the CIE diagram by the interception of a line passing through C and (x_1, y_1) with the spectral line. The intercepting point on this line is the dominant wavelength of the color stimulus.

If we consider a purple color, there does not exist a dominant wavelength for it because the line passing through C and the color point intercepts the base of the color diagram and not the spectral curve. But if the line is extended on the opposite direction respect to the purple line, the color can be defined by its *complementary*.

Given a white source, two colors are complementary if their combination produces a stimulus metameric to the white source. For a given white source, infinite couples of complementary colors exist in the spectrum. If a radiation belongs to the spectrum region of the short wavelengths (blue–violet), its complementary will belong to the long wavelengths (yellow–red) region: For example, a blue stimulus has its complementary in orange one, a violet stimulus has its complementary in yellow, while the middle wavelengths (green region) have the complementary in purple colors.

Consider the line connecting our color point P to the spectral curve, passing through C: On this line, we find all colors obtained by the mixture of white with the dominant wavelength. The dominance of hue respect to white in color is defined as *saturation*: It increases toward the spectral line and it reaches its maximum value in the spectral colors, also called "*pure hues*"; instead, the saturation decreases toward the inner of the diagram, becoming zero at C point. Since white has saturation degree equal to zero, it is called *neutral color*. The saturation degree of every color is then measured by the ratio of the length of the distance of the point from the C point to the distance of the C point from the spectral line.

For example, since in red color, the chromatic component prevails on white, it is more saturated than pink, where white prevails on red.

To completely define color, it is necessary to introduce the third attribute: *brightness*. Brightness is the qualitative equivalent of luminance, and it describes the sensation of luminosity of a stimulus, i.e., if the stimulus appears bright or dark.

For definition, white has the maximum brightness, but if luminance decreases, it also decreases giving all the levels of grays until black, which is absence of light.

Brightness is not represented in the CIE diagram, but it can be considered as its third dimension. In Fig. 7, a good three-dimensional representation of the three attributes of color is given by the Munsell color tree: Each color is represented by a point which defines a direction orthogonal to the central vertical axis according to an orientation which measures the color hue. The distance from the central axis defines the saturation of colors, and the height gives the brightness of the stimulus.

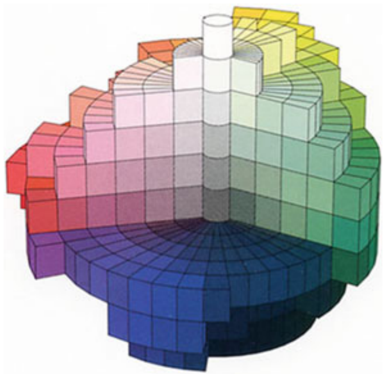


Fig. 7 Munsell's color tree, representing the three attributes of color

The neutral colors are all aligned on the central axis, and they do not have neither a hue nor a saturation degree. For this reason, neutral color is also called *achromatic*.

3.2 Color Temperature

Consider a continuous spectrum source (e.g., an halogen bulb) that behaves as a blackbody. A blackbody is an ideal body which absorbs all radiation impinging on it ("black" for the absence of reflection) and, because of its inner thermal equilibrium, emits all the absorbed radiation. The emitted radiation depends only on the blackbody temperature, so that if temperature increases, the chromaticity of radiation passes from a red color ($\sim$1,000 K) to white ($\sim$5,000 K) to blue ($\sim$6,000 K). In reality, a perfect blackbody does not exist, but some sources, as filament bulbs, can be approximated to it at a certain temperature. It is so possible to define the color temperature as the temperature that a blackbody should have in order to emit a radiation of a certain chromaticity. The variation of the chromaticity of the radiation emitted by a blackbody at different color temperatures is reported on the CIE xyY diagram of Fig. 8.

As we can see, the so-called *warm lights* (longest wavelengths) have *low* color temperature, while *cold lights* have *high* color temperature.

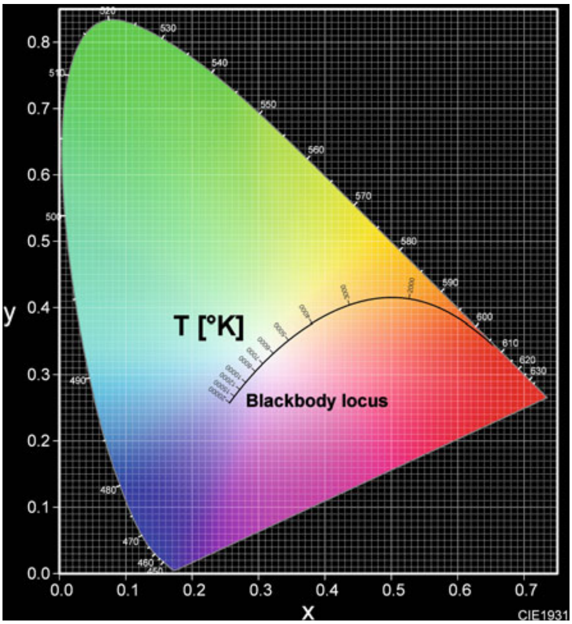


Fig. 8 Path of chromaticity of a blackbody at different temperatures

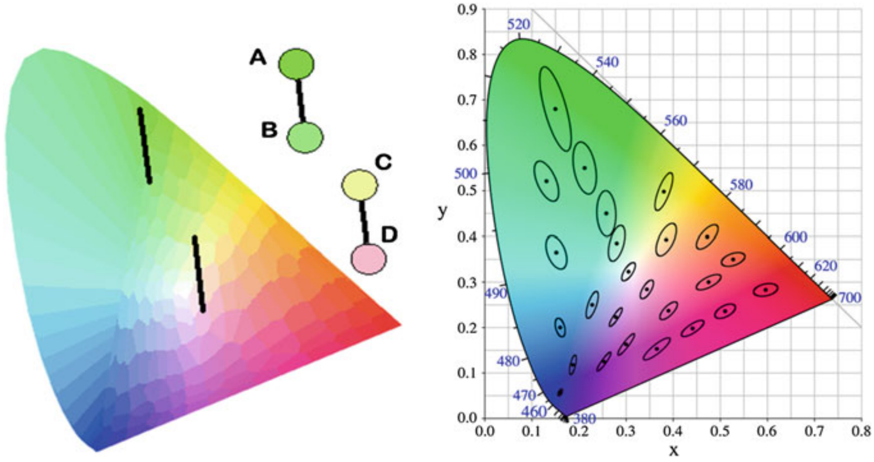


Fig. 9 Limits of CIE xyY diagram

4 Limits of CIE 1931 Color Diagram

Consider the two points A and B and other two points C and D at the same distance of Fig. 9 (left).

It appears clear that a limit of the CIE xyY diagram is its nonlinearity. In fact, even if the distances between points are the same, the degree of difference between colors changes. In particular, A and B are similar, but C and D are two colors completely different.

Another consequence of the non-uniformity of the diagram is that regions exist in which the colors are indistinguishable to the average humane eye. These regions are called Macadam's ellipses and are shown in Fig. 9 (right). Inside an ellipse, colors are indistinguishable and its contour represents the just noticeable difference of chromaticity.

4.1 CIE 1976 Color Spaces

In order to correct these limits, in 1976, CIE proposed two alternatives to the xyY diagram: CIELUV and CIELAB spaces.

It is common experience that colors change according to the illuminant, so that CIE introduced standard illuminants, whose chromatic coordinates are placed on the blackbody locus on the CIE xyY diagram.

In these systems, the illuminant becomes the reference because all colors are related to the white point, defined so that its percentage luminance factor is 100 ($Y_n = L_n = 100$) and its chromaticity (x_n, y_n) is equal to the illuminant's one.

4.1.1 CIELAB

The first quantity of the new tridimensional system is lightness L*, function of the test color luminance Y in relation to the white reference Y_n. L* has got values from 0 to 100 which yield, respectively, black and diffuse white.

The other two coordinates a^* and b^* are determined by a nonlinear transformation in the tristimulus space (X, Y, Z) so that

$$L^* = f\left(\frac{Y}{Y_n}\right); \quad a^* = f\left(\frac{X}{X_n}, \frac{Y}{Y_n}\right); \quad b^* = f\left(\frac{Y}{Y_n}, \frac{Z}{Z_n}\right); \tag{9}$$

and indicate the position of a color stimulus between the complementary couples red/green and yellow/blue.

The CIELAB color space can be so represented as in Fig. 10 by a diagram where the positive axes a^* and b^* represent, respectively, red and yellow stimuli, while in the negative direction, they represent green and blue stimuli.

At the cross point of the two axes, the vertical axis L^* represents the lightness of a stimulus. If we consider a plane at constant lightness, it is possible to define two polar coordinates, the *hue angle* and the *chroma.*

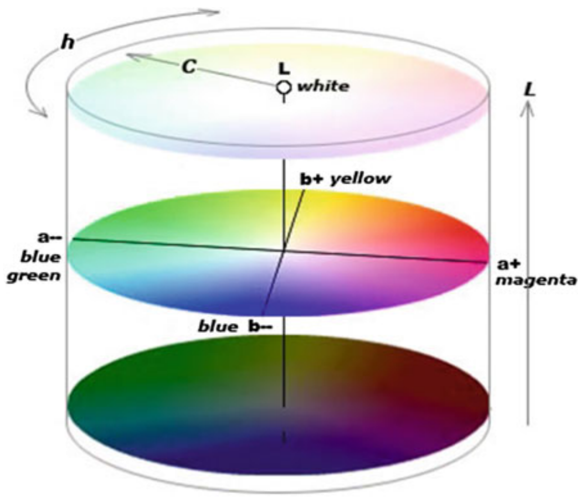
Hue angle h_{ab}, in degree, replaces the concept of dominant wavelength

$$h_{ab} = \tan^{-1}(b^*/a^*) \tag{10}$$

Chroma is the Euclidean distance from the test chromaticity and the illuminant chromaticity. It replaces the concept of saturation

$$C^*_{ab} = \sqrt{a^{*2} + b^{*2}} \tag{11}$$

Fig. 10 CIELAB color space



Given two stimuli $\left(L_1^*, a_1^*, b_1^*\right)$ and $\left(L_2^*, a_2^*, b_2^*\right)$, it is possible to define their color difference as

$$\Delta E_{\mathrm{ab}}^* = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2}. \tag{12}$$

4.1.2 CIELUV 1976

The new coordinates are L^*, u^*, and v^*.

L^*, lightness is the same as in CIELAB space, u^*, and v^* are defined as

$$u* = 13L * \left(u' - u_n'\right); \quad v* = 13L * \left(v' - v_n'\right) \tag{13}$$

where (u', v') are the coordinates of the test stimulus and $\left(u_n', v_n'\right)$ are the reference coordinates.

In CIELUV space, not only hue angle and chroma but also saturation can be defined, so that a correlation between chroma and saturation can be found:

$$h_{\mathrm{uv}} = \tan^{-1}(v^*/u^*) \tag{14}$$

$$C_{\mathrm{uv}}^* = \sqrt{u^{*2} + v^{*2}} = L^* \times s_{\mathrm{uv}} \tag{15}$$

$$s_{\mathrm{uv}} = 13\sqrt{u^{*2} + v^{*2}} \tag{16}$$

From these equations is obvious that the difference from chroma and saturation is that chroma depends on luminance, differently from saturation.

The color difference between two stimuli can be defined as in CIELAB space as

$$E_{\mathrm{ab}}^* = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2} \tag{17}$$

Further Reading

1. Oleari C (1998) Misurare il colore. Spettrofotometria, fotometria e colorimetria. Fisiologia e percezione. SIOF-Hoepli (in Italian)
2. Schanda J (2007) Colorimetry. Understanding the CIE System. Wiley, New York
3. DeCusatis C (1997) Handbook of applied photometry. American Institute of Physics Press, New York
4. Wolfe WL (1998) Introduction to radiometry: tutorial text in optical engineering, vol TT29. SPIE Press, Bellingham
5. Parr AC, Datla RU, Gardner JL (2005) Introduction to optical radiometry. Elsevier Academic Press, Amsterdam, 1 Jan 2005
6. Palladino P (2005) Manuale di Illuminazione. Tecniche nuove (in Italian)

7. Ryer A (1998) Light measurements handbook. International light Inc, Newburyport
8. Ashdown I, Eng P (1996) Photometry and radiometry: a tour guide for computer graphics enthusiasts. LC, FIS

Websites

1. http://www.avmagazine.it
2. http://www.light-measurement.com/basic-radiometric-quantities/
3. http://www.cie.co.at
4. http://en.wikipedia.org
5. http://www.philips.it/

http://www.springer.com/978-1-4471-6632-0

Sustainable Indoor Lighting
Sansoni, P.; Mercatelli, L.; Farini, A. (Eds.)
2015, X, 355 p. 212 illus., 161 illus. in color. With online files/update., Hardcover
ISBN: 978-1-4471-6632-0