Preface

Fourier analysis is one method of investigating the origin of functions and their properties by using Fourier series and Fourier transform(s). The Fourier series and Fourier transforms were introduced by the mathematician Jean Baptiste Joseph Fourier at the beginning of the nineteenth century; they are widely applied in the engineering and science fields. One-dimensional waveforms (as functions of time or position) and two-dimensional images (as functions of two positional axes) are the main subjects studied nowadays using Fourier analysis. Fourier analysis is an important method used to analyze complex sound waveforms in the field of acoustical engineering. This is the reason why this book was originally listed in the series of books published by the Acoustical Society of Japan. However, since Fourier analysis is also applicable in many other engineering sciences, these two books, *Digital Fourier Analysis: Fundamentals, and Digital Fourier Analysis: Advanced Techniques*, are useful to readers in broader fields.

The Fourier transform itself does not fit well with analog processing because it requires too much numerical processing, such as multiplications and summations. That is the reason waveform analysis during the analog age could not make full use of the Fourier method of analysis. Fourier analysis was more valuable as the basis of theoretical analysis than for its practical applications during the analog era. However, the digitally processed Fourier transform became a reality with the emergence of the digital computer in the middle of the twentieth century. The later development of the Fast Fourier transform (FFT) algorithm in 1965, and the subsequent inventions of microchips for signal processing accelerated the application of Fourier analysis-based signal processing.

In the twenty-first century, Fourier analysis technology is widely used in our daily activities, and as a natural consequence, the technology is "hidden in a black box" in most of its applications. Even experts in the field use this technology without knowing details of how Fourier techniques are implemented. However, engineers, who wish to play important roles in developing future technologies, must do more than just deal with black boxes. Engineers must understand the basis of the present Fourier technology in order to create and build up new technologies based on it.

This book is intended so that high-school graduates or first or second grade college students with basic knowledge of mathematics can learn the Fourier analysis without too much difficulty. In order to do that, explanations of equations start from the very beginning and details of derivation steps are also described. This is very rare for this kind of specialized book.

This book also deals with advanced topics so that engineers presently involved in signal processing work can get hints to solve their own specific problems. Ways of thinking that lie behind or lead to theories are also described that are a must to apply theories to practical problems.

This work comprises two volumes. Seven chapters are included in Volume I, titled "Digital Fourier Analysis: Fundamentals." Volume II, titled "Digital Fourier Analysis: Advanced Techniques" contains six chapters. As the titles indicate, more advanced topics are included in Volume II. In this sense, the former may be classified as a text for undergraduate courses and the latter for graduate courses. Notice, however, that Volume I includes some advanced topics, whereas Volume II contains necessary items needed for a better understanding of Volume I.

Following is a brief explanation of each chapter. First, the contents of Volume I are briefly described.

Chapter 1 commences with an explanation of the impulse as being the limit of the summation of cosine waves with ascending frequencies. This chapter shows that all waveforms can be synthesized by use of Fourier series, i.e., that sine and cosine waves are the basis of waveform analysis. It then gives a geometric image to Euler's formula by explaining that the projections of a constantly rotating vector around the origin of the rectangular coordinate system onto their real and imaginary axes are the cosine and sine functions, i.e., the real and imaginary parts of a complex exponential function, respectively. The reader will be naturally guided to the concepts of instantaneous phases and instantaneous frequencies through this learning.

Chapter 2 starts with the theory on how to determine coefficients of the Fourier series of a periodic function. It investigates the properties of the Fourier series, showing why high order coefficients are needed for waveform synthesis and what kind of properties the Fourier series of even and odd function waveforms have. Then it shows that the Fourier series expansion becomes the Fourier transform pair when the period is made infinitely large.

Chapter 3 deals with problems encountered when one tries to express a continuous waveform by a sequence of numbers in order to numerically compute the Fourier transform. For that purpose, it investigates the most important issue in digitization: how to handle the sampling time based on the knowledge of the Fourier series; and guides the reader automatically to the sampling theorem. Then the relation between a continuous waveform and the discrete numerical sequence, which is the sampled version of the waveform, is discussed.

Chapter 4 guides you to the definition of discrete Fourier transform (DFT) and inverse Discrete Fourier transform (IDFT). The DFT transforms a numerical sequence with a finite length to another numerical sequence with the same length, and the IDFT transforms the latter sequence back to the former sequence. Then

this chapter clarifies that these sequences are periodic and they have the length (data number) of the sequence as their periodicity. For later applications, the Discrete Cosine transform (DCT) is derived from basic concepts. The DCT describes the relationship between time domain and frequency domain functions using only cosine functions.

Chapter 5 explains a principle of the Fast Fourier transform (FFT) which drastically decreases the number of multiplications and summations required in the computation of the DFT. The FFT is an innovative numerical calculation method which has greatly expanded the range of application of Fourier analysis.

Chapter 6 discusses several items such as: (1) properties of the spectrum given by the DFT of a *N*-sample sequence (waveform) taken from a long chain of data; (2) its relation with the spectrum of the original data; (3) the relationship between sampling time and frequency resolution, and (4) the reason why new frequency components that do not exist in the original waveform are produced by the DFT, and so on.

Chapter 7 studies details of various weighting functions (time windows) applied to waveforms in order to obtain stable and accurate spectra (frequencies and amplitudes) by the DFT approach.

The explanation of Volume I ends here. But, as recognized by readers, the description is insufficient for use of Fourier Analysis in practical applications. More knowledge described in Volume II will be required for in-depth understanding of the descriptions in Volume I and for the application of Fourier analysis to a wide area.

The first chapter of Volume II guides the reader through the use of a convolution of two sequences to be calculated from an input and the system's impulse response. It becomes clear that the Fourier transform of a convolution can be expressed as a multiplication of the two respective Fourier transforms, and this leads to the exploration of a new way that a convolution in the time domain can be calculated in the frequency domain. Since an issue based on DFT periodicity is raised at this time, this chapter discusses the issue and explains in detail how to obtain the correct result.

In Chap. 2, the correlation function, that quantitatively expresses the degree of similarity between two time sequences, is derived. The difference between the correlation and convolution functions is that the directions of the time axes of one of two time sequences in the process of multiplication and summation calculation are opposite to each other. The Fourier transform of the correlation function of two time sequences is given as a cross-spectrum of the two functions in the frequency domain, which will be discussed in the next chapter.

Chapter 3 introduces a Cross-spectrum method that uses multiplication of spectra. The Cross-spectrum technique is a powerful method for uncovering an original function as an inverse process based on the convolution or the correlation function. This is a good example of the DFT's usefulness. DFT periodicity is the most important factor. This chapter illustrates the kind of problems related to Cross-spectrum analysis, and discusses how to avoid errors, by taking advantage of periodicity.

Chapter 4 introduces the concept of a Cepstrum which is defined as a Fourier transform of the logarithm of a spectrum. This useful method of analysis is based on a little quirky idea. Cepstrum analysis is a powerful method of signal analysis for detecting hidden information that is not visible from the Fourier transform of a time domain signal.

Chapter 5, at first, analyzes the problem that occurs when a waveform is depicted as a rotating vector in order to obtain its envelope. Since an orthogonal waveform of the original waveform is needed to get a rotating vector, the question of how to derive the orthogonal function in the frequency region is discussed. The calculation of the orthogonal function in the time region by applying the inverse Fourier transform to the orthogonal function in the frequency region results in the Hilbert transform. While the Hilbert transform is a demodulation of the amplitude-modulated wave to get an envelope as a length of rotating vector, it is also, a demodulation of the frequency-modulated wave, producing an instantaneous frequency which is the rotating speed of the vector.

Chapter 6 touches upon two-dimensional DFT and DCT methods. At first, a definition of the two-dimensional DFT is given. When the reader tries to obtain two-dimensional spectra of images with basic patterns, one can easily guess its output from the relation between the one-dimensional waveform and its spectrum. The reader will understand that one-dimensional Fourier transform procedure is an important base. By showing definitions of DCT and samples of two-dimensional DCT spectra of simple images, the concept of how information compression by DCT takes place will be explained with concrete examples.

One of the features of this book is that it contains a number of figures that have an interactive supplement. (The supplementary files can be downloaded from http://extras.springer.com). Figures with this feature are indicated by their caption, which includes the file name of the corresponding animation file. To view the animation, click the corresponding exe file to start the program, and then click the green "start" button after data input and/or selection of conditions. Then the program starts the calculation based on the theory, input data, and conditions. The reader may see unexpected results occasionally. As they have their own causes or reasons, it will be worthwhile for the reader to think of them for a deeper understanding. Note that the programs are written in Visual Basic and may not work on all computers.

I would like to emphasize the following through my long experience as a writer of this book and also as a user of this book in my classes and other lectures. The reader will lose more than he/she earns if he/she prematurely thinks that he/she has understood one topic after running a related program and briefly looking at the result. The reader must run programs with various data and conditions and look at the corresponding results and then he/she must think how they are related with each other. With the attached programs, the reader can do these easily while having some fun.

Very few references are listed at the end of this book compared to the contents of this book. This is because most of the theories are described from the beginning, and as a result this book became self-contained. Theory-oriented readers should

refer to books such as [4–9] in Reference. Since the Fourier analysis techniques are developing day by day, the readers should refer to current journals in the related area.

Finally, although I would like to express my sincere appreciation to all of those who gave me tremendous encouragement and cooperation to write this book, I must apologize that I cannot list up all of their names. My excuse is that so many people assisted me in writing this book.

This book, originally published in Japanese, was translated first by Dr. Hideo Suzuki, a former professor of Chiba Institute of Technology, Mr. Jin Yamagishi, a technology management consultant of JINY Consultant Inc., and myself, and then, very carefully checked and corrected by Dr. Harold A. Evensen of Michigan Technological University in USA and Dr. Leonard L. Koss of Monash University in Australia.

I would like to express my sincere appreciation to all of those who contributed to publishing this book.

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