

Preface

Many problems of growing interest in science, engineering, biology, and medicine are modeled with systems of differential equations involving delay terms. In general, the presence of the delay in a model increases its reliability in describing the relevant real phenomena and predicting its behavior. Besides, the introduction of history in the evolution law of a system also augments its complexity since, opposite to Ordinary Differential Equations (ODEs), Delay Differential Equations (DDEs) represent infinite dimensional dynamical systems. Thus their time integration and the study of their stability properties require much more effort, together with efficient numerical methods.

Since the introduction of the delay terms in the differential equations may drastically change the system dynamics, inducing dangerous instability and loss of performance as well as improving stability, analyzing the asymptotic stability of either an equilibrium or a periodic solution of nonlinear DDEs is a crucial requirement. Several monographs have been written on this subject and the theory is well developed. By the Principle of Linearized Stability, the stability questions can be reduced to the analysis of linear(ized) DDEs. In the literature, a great number of analytical, geometrical, and numerical techniques have been proposed to answer such questions. Part of these techniques aim at analyzing the distribution in the complex plane of the eigenvalues of certain infinite dimensional linear operators, in particular the solution operators associated to the linear(ized) problem and their infinitesimal generator.

This monograph does not aim to be a survey, but presents the authors' recent work on the numerical methods for the stability analysis of the zero solution of linear DDEs, which consist in applying pseudospectral techniques to discretize either the solution operator or the infinitesimal generator. The eigenvalues of the resulting matrices are then used to approximate the exact spectra. The purpose of the book is to provide a complete and self-contained treatment, which includes the basic underlying mathematics and numerics, examples from applications and, above all, MATLAB programs implementing the proposed algorithms. MATLAB is a high-level language and interactive environment, which is nowadays well developed and widely used for a variety of mathematical problems arising from

both theory and applications. Advanced students and researchers in applied mathematics, in dynamical systems, and in various fields of science and engineering concerned with delay systems are encouraged to experience the practical aspects. Having at disposal MATLAB codes to test the theory and to analyze the performances of the methods on given examples, they can tackle the numerical stability analysis of their own delay models by easily modifying these codes. Readers can also appreciate the possible application of the latter to the stability analysis of equilibria and periodic solutions of nonlinear DDEs as well as to trace bifurcation diagrams and stability charts for DDEs with varying parameters.

To furnish a solid foundation and a complete understanding of the performances of the algorithms, neither the theoretical nor the numerical analysis can be left aside. A motivated introduction to the theory of semigroups with a number of proofs is given, but the emphasis is on the (unifying) idea of using pseudospectral techniques for the numerical stability analysis of linear or linearized DDEs. Therefore, a detailed presentation of the discretization schemes is given. The monograph is completed with a fully developed error analysis, complemented with numerical results on test problems, and models from applications.

After reading the book, one should have reviewed (or acquired) the essential background on the theory of semigroups to understand the main features of the dynamical systems described by DDEs. This is the starting point for the construction of the numerical methods. Readers interested in the numerical analysis can find a complete and detailed error analysis, while readers interested more in models or applications can appreciate the role of the numerical analysis in the derivation of accurate and efficient approximations techniques. Finally, all of them should have learned how to use and modify the MATLAB codes to try new investigations (possibly reading only the first part of Chaps. 7 and 8). Eventually, such codes are made freely available, [48].

Udine, Italy, September 2014
Trieste, Italy
Udine, Italy

Dimitri Breda
Stefano Maset
Rossana Vermiglio

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A Numerical Approach with MATLAB

Breda, D.; Maset, S.; Vermiglio, R.

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