

Preface

The subject of this monograph is the homotopy theory of diagrams of spaces, chain complexes, spectra, and generalized spectra, where the homotopy types are determined locally by a Grothendieck topology.

The main components of the theory are the local homotopy theories of simplicial presheaves and simplicial sheaves, local stable homotopy theories, derived categories, and non-abelian cohomology theory. This book presents formal descriptions of the structures comprising these theories, and the links between them. Examples and sample calculations are provided, along with some commentary.

The subject has broad applicability. It can be used to study presheaf and sheaf objects which are defined on the open subsets of a topological space, or on the open subschemes of a scheme, or on more exotic covers. Local homotopy theory is a foundational tool for motivic homotopy theory, and for the theory of topological modular forms in classical stable homotopy theory. As such, there are continuing applications of the theory in topology, geometry, and number theory. The applications and extensions of the subject comprise a large and expanding literature, in multiple subject areas.

Some of the ideas of local homotopy theory go back to the work of the Grothendieck school in the 1960s. The present form of the theory started to emerge in the late 1980s, as part of a study of cohomological problems in algebraic K -theory. Within the framework of this theory, these K -theory questions have now been almost completely resolved, with a fusion of ideas from homotopy theory and algebraic geometry that represents the modern face of both subjects. The theory has broadened the scope of the applications of homotopy theory, while those applications have led to a rethinking of what homotopy theory and particularly stable homotopy theory should be. We also now have a good homotopy theoretic understanding of non-abelian cohomology theory and its applications, and this theory has evolved into the modern theories of higher stacks and higher categories.

The foundational ideas and results of local homotopy theory have been established in a series of well-known papers which have appeared over the last 30 years, but have not before now been given a coherent description in a single source.

This book is designed to rectify this difficulty, at least for basic theory. It is intended for the members of the Mathematics research community, at the senior graduate

student level and beyond, with interests in areas related to homotopy theory and algebraic geometry. The assumption is that the reader either has a basic knowledge of these areas, or has a willingness to acquire it.

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