Preface

This second edition, entitled *Logic, Mathematics, and Computer Science: Modern Foundations with Practical Applications*, has been adapted from *Foundations of Logic and Mathematics: Applications to Computer Science and Cryptography*, © 2002 by Birkhäuser, from which Chapters 1–5 have been retained but extensively revised. Chapters 6 and 7 have been added.

This text discusses the foundations where logic, mathematics, and computer science begin. The intended readership consists of undergraduate students majoring in mathematics or computer science who must learn such foundations either for their own interest or for further studies. For a motivated reader, there are no technical prerequisites: you need not know any technical subject to start reading this text.

Although the text does not focus on the history and philosophy of the foundations, the material cites copious references to the literature, where the reader may find additional historical context. Consulting such references is neither suggested nor necessary to study the theory or to work on the exercises, but individual citations document the material by original sources, and all the citations together provide a guide to the variations and chronological developments of logic, mathematics, and computer science. For example, Chapter 1 traces the origin of Truth tables to Charles Sanders Peirce's unpublished 1909 *Logic Notebook* on philosophy and points out their applications over one half of a century later to the design of computers for use on Earth and on board the Apollo lunar spacecraft.

Along informal arguments, this text also shows the corresponding purely symbolic manipulations of formulae, because they clarify the reasoning [11] and can reveal hitherto hidden logical properties, such as the mutual independence of different patterns of reasoning, or the impossibility of some proofs within some logics:

> As for algebra [of logic], the very idea of the art is that it presents formulae which can be manipulated, and that by observing the effects of such manipulation we find properties not to be otherwise discerned (Charles Sanders Peirce [104, p. 182]).

If professionals are unable to learn some topics by any means other than the pure manipulation of symbols, then it would seem unfair to claim that all learning must be intuitive and hide from students such purely manipulative but successful methods.

The selection of topics also reflects major accomplishments from the twentieth century: the foundation of all of mathematics, and later computer science, as well as computer-assisted proofs of mathematical theorems, on a formal logic based on only a few axioms, transformation rules, and postulates for set theory [47, 50, 54, 105, 139]. Also, while not written in formal logic, Nobel-Prize winning applications to the social sciences rely on the same foundations, as shown in Chapter 7.

To introduce the foundations of logic, the provability theorem in Chapter 1 provides an algorithm to design proofs in propositional logic. Chapter 1 also explains the concept of undecidability with multi-valued ("fuzzy") logic and presents a proof of unprovability. Chapter 2 introduces logical quantifiers. A working knowledge of logical quantifiers is crucial for the study of basic concepts in modern mathematical analysis and topology, such as the uniform convergence of a sequence of equicontinuous functions. Continuing with the foundations of mathematics, Chapter 3 presents a version of the Zermelo–Fraenkel set theory. At the juncture of mathematics and computer science, Chapter 4 develops the concepts of definition and proof by induction. Chapter 4 then uses induction with set theory to define the integers and rational numbers and derive the associative, commutative, and distributive laws, as well as algorithms, for their arithmetics. To give readers some idea of topics at an intermediate level, Chapter 5 shows that in a well-formed theory some paradoxes do not occur, while Chapter 6 completes the foundations of set theory with the axiom of choice.

No extragalactic asteroid has yet been found with the universal laws of logic engraved in it. Consequently, not just one logic but many different logics have been invented. Different logics lead to different mathematics and different computer sciences. However, the acid test for adopting a particular logic is its ability to make predictions that are born by subsequent experiments. Formal logic is thus a mathematical model of rational thought processes. In this aspect, logic, mathematics, and computer science are experimental sciences. Only one logic has passed all such tests, which is the one used throughout this text. Other logics are outlined in Chapter 1 as a pedagogical device and to show some of their shortcomings.

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