Chapter 2
Can There Be Any Relationships Between Mathematics and Architecture?

Mario Salvadori

My dear friend Kim,
Ladies and Gentlemen of the Congress,
Signore e Signori,
Señores y Señoras
Mes Dames et Messieurs,
Meine Damen und Herren,

I am greatly honoured to be asked to present a lecture at this congress on the theme of Mathematics and Architecture, because I happen to be a mathematician and because, although academically untrained in the difficult discipline of architecture, it has been my good luck to collaborate as a structural engineer in the creation of architectural buildings of all types and all over the world with architects like Gropius, Breuer, Saarinen and many others, during the 30 years I spent in a well known architectural engineering office in the United States.

Let me say that, as I prepared myself for today's demanding assignment, I ran immediately into a basic question. Since there is not just *one* mathematics, but many, and there is not *one* architecture but many, *which* mathematics should I discuss in relationship to *which* architecture?

I am afraid that, to clear these doubts, I will have to give the architects in this hall some idea of how many mathematics there have been, there are and there will be in

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Mario Salvadori (1907–1997).

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the future, and to the mathematicians sitting next to them how many architectures there have been, there are and there will be in the future. But, in order not to bore both groups of specialists, I will try to be brief and simple.

To start with, we all know that there is the mathematics we use to go to market, but that kind of mathematics is an obvious practical derivation from a more complex field of higher mathematics called *number theory*. Then there is the kind of mathematics used by all technologists, which is extremely useful to all of them but has little to do with the *real* mathematics. Finally there is the mathematics used by the great scientists, that has allowed them to better understand the universe, of which we are such a minimal part, and amazes all of us poor mortals with the astonishing results it allows us to obtain. Yet, even *this* is not the highest level of mathematics, which is usually called *pure mathematics*.

The essential character of *pure* mathematics is its purity and derives from the fact that it is the fruit of our minds and has no relationship *whatsoever* to what some people call "reality". Just as we may *think* of a green cow, although there are no green cows in our world, mathematics, *pure* mathematics, is the purest product of our mind and has nothing to do with nature or the constructs of man.

Let me illustrate what I mean by a simple example. You are all familiar with the geometry of Euclid that asserts how from a point *outside* a given line one can draw a *single* line *parallel* to the given line. When discussing Euclidian geometry I like to ask one of my young students: "And how long are the lines Euclid is talking about?" He or she usually answers: "They last forever", or: "They are infinite." Upon hearing this statement I like to remind the student that we both live on earth and that, if we keep going on for ever, we will describe a *circle* and *not* a straight line. The student becomes confused and I then explain that the concepts of point and of line used by Euclid are purely *abstract* concepts and have nothing to do with the earth on which we live, because straight lines cannot exist on a round earth. And to convince my students of the total abstraction of mathematics I mention that towards the end of the last century one Russian and one Hungarian mathematician invented a new geometry in which not one but *two* lines could be drawn parallel to a given line and, as if this were not enough, in 1907 the German mathematician Riemann invented a geometry in which an *infinite* number of lines can be drawn parallel to a given line.

I seem to hear some of our architects say: "Well, if mathematicians like to play with abstractions and have a good time with them, let them. But *we* are interested in 'reality'" (whatever this word means to them). To which I answer that if Riemann had not invented the Riemannian geometry Einstein could not have invented his general theory of relativity, which has solved a number of mysteries unsolvable by the apparently "more real" Euclidian geometry.

In one word, mathematics is a fruit of the human spirit, like poetry, and, like poetry, it is one of the infinitely beautiful fruits of the human spirit because it is totally free, it is abstract.

Let me now touch upon some characteristics of architecture. I know that there are in this room many, highly knowledgeable, ladies and gentlemen who have investigated the architectures of the past, those of the present and even tried to guess those of the future. They know the *theory* of architecture better than many famous architects and I have the greatest respect for their knowledge. But I must *unequivocally* state that they are *not* real architects because the basic characteristic of architecture is that it is among the most *concrete* of all human endeavours and, may I add, that (in my humble opinion) it is also one of the most demanding human endeavours, if not *the* most demanding. I know by experience that architecture is much more difficult than mathematics, whose freedom of invention is only bound by the needs of logic, while architecture is bound by innumerable laws, opinions, traditions and, above all, by the whim of man.

What I am trying to say is that no architecture exists unless it is *architected,* that is, *concretely* built without any reference to *pure ideas,* but constrained by the laws of nature and, above all, I repeat, by the whim of man, the most unsatisfied animal of the animal kingdom.

I am therefore asking myself, and all of you: "How can there be a relationship between the totally abstract real mathematics and the totally concrete real architecture?"

This is where I seem to hear a murmur in this hall that says: "But, come on, you have forgotten geometry! If you don't wish to consider other possible relationships, how can you ignore the importance of geometry in architecture?"

Ladies and gentlemen, here again I must state that there are at least two aspects of geometry: one that is obviously of interest to the architects and of no interest to the mathematicians, and one that does just the opposite. The architect may be interested in a geometrical shape called a triangle, but the mathematician doesn't care about *the shape* of the triangle. What excites him is that *whatever the shape of the triangle* the sum of its three angles adds always to 180°. And he is not even interested in the shape of a so called *right triangle,* but only in the fact that the sum of the square of its two sides, *whatever the shape and dimension of the right triangle,* always adds up to the square of its longest side, its hypotenuse! And (may I add in parentheses) that it took over 300 years of very hard work by the greatest mathematicians in the world to prove that, calling a and b the sides of a right triangle and c its hypotenuse, by Pythagoras theorem:

$$a^2 + b^2 = c^2,$$

but that there will never, *never* be a right triangle *with integer sides* for which:

$$a^3 + b^3 = c^3,$$

or any other exponent *larger than two.* And let me finally add that all the mathematicians in the world rejoice that one of them, just about a year ago, succeeded, after 7 years of concentration on this unique problem, to solve the so-called *Fermat's last theorem,* where everybody else had failed in the preceding 300 years.

Ladies and Gentlemen, having "proved" that to look for relationships between as abstract a science as mathematics and as concrete an art as architecture is theoretically inconceivable, allow me now to take off my mathematical hat and put on my engineering hat. As soon as I change hats, I realize that all my disquisitions on the impossibility of relating mathematics and architecture vanish and, as a technologist, I must agree with you that the relationships between mathematics and architecture are so many and so important that, if mathematics had not been invented, architects would have had to invent it themselves.

But there are so many architects and investigators of architecture in this hall that I would not dare to address a topic they are more than capable and eager to present themselves.

I therefore apologize for my long initial statement, but with an explanation. I love mathematics and architecture equally, but find architecture so difficult that, as the lazy man I am. I prefer to limit my activity in this field to helping the architects. And I confess that, were it not for my technological knowledge, I would not dare touch the sublime beauty and the scary difficulties of architecture, pacifying my big ego with the thought that the architects of today perhaps could not architect without the contributions of great engineers like Pier Luigi Nervi and so many more of us, even if we are not of Nervi's calibre.

Thank you very much for your courteous patience.

Biography Mario Salvadori (1907–1997) earned doctoral degrees in both civil engineering and mathematics from the University of Rome in 1930 and 1933 respectively. An outspoken critic of Fascist regime, he left Italy in 1938 for New York at the recommendation of his teacher and friend, Enrico Fermi. After the war, he took up teaching at Columbia University, where he would become a professor in 1959 in the School of Architecture, Planning and Preservation; he taught at Columbia for 50 years. From 1954 to 1960, Salvadori worked as a consultant and then principal at the engineering firm Weidlinger Associates. He was a partner until 1991, when he became honorary chairman. As a structural engineer, Salvadori became known for the design of thin concrete shells. As he reached retirement age, he began volunteering to work with under-privileged minority students from inner-city New York public schools. He is the author of numerous books, including *Mathematics in Architecture* (1968), *Why Buildings Stand Up* (1980) and *Why Buildings Fall Down* (1992).

Further Reading

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