

Preface

This volume celebrates the seventieth birthday of Professor Ermanno Lanconelli, whose scientific activity has strongly influenced the recent research on partial differential equations (PDEs) with non-negative characteristic form, and on the related potential theory. Beyond his distinguished scientific contributions, Ermanno Lanconelli has also been responsible for forming a prestigious school of mathematicians who share with him his infectious love for Mathematics.

The first notable contribution in Ermanno's scientific activity was a representation formula on the level sets of the fundamental solution of the heat equation, inspired by the work of Bruno Pini. This technique, until then used only for harmonic functions, has since been extended to increasingly large classes of equations, including the totally degenerate ones.

The scientific thought of Ermanno Lanconelli reached its full maturity with his works on totally degenerate equations. In the early 1980s, he introduced an original geometric approach for study of solutions to Grushin-type equations. Thereafter, he investigated the link between the geometric properties of the vector fields and the fundamental solutions of the associated second order operators in a long series of papers, culminating in a monograph, which is now considered one of the foundational references for potential theory in this setting.

He also addressed problems related to non-linear PDEs and proved a fundamental result for curvature-type equations, which opened up a new direction in the study of differential equations with non-linearity in the vector fields.

As a mathematician, Lanconelli has been constantly motivated by a strong desire to develop unifying techniques in the analysis of problems related to differential equations that classically were approached with separate and independent methods.

This volume contains 18 contributions that cover a wide range of topics that characterize Ermanno's scientific production. It brings together a selection of invited contributions from the main speakers at the conference "Geometric methods in PDEs: Indam Meeting on the occasion of the 70th birthday of Ermanno Lanconelli" and presents a wide cross-section of the most recent contributions on linear and non-linear differential equations and also on geometric problems that give rise to differential equations.

The first group of contributions in the volume deals with various kinds of functional inequalities: Friedrichs-type commutator lemmas, sharp inequalities of Hardy and Moser–Trudinger types, and Lusin theorems for BV functions.

Several contributions focus on the regularity theory of linear PDEs. They touch on Harnack-type estimates for equations associated with harmonic maps, subelliptic Fefferman–Phong type inequalities, estimates for parabolic equations involving Ornstein–Uhlenbeck terms, and the problem of existence and regularity of a fundamental solution for sum of squares of vector fields.

A third group of contributions deals with non-linear PDEs. Existence and multiplicity results for non-local eigenvalue problems are established; uniqueness problems for subelliptic semilinear and quasilinear equations are studied; existence and non-existence results for differential inequalities in Carnot groups are given; and gradient estimates with rigidity results for parabolic Modica-type PDEs are proven.

Some other contributions in the volume are concerned with fully non-linear PDEs of elliptic type, focusing on local and global gradient estimates for non-negative solutions and $C^{1,\gamma}$ regularity estimates for equations with sublinear first-order terms.

Also included are contributions concerning, first, the existence of solutions for a model to design reflectors and, secondly, some div-curl inequalities in Carnot groups.

Finally, the volume includes two surveys, the first of which is on free boundary problems. The second has been written by Ermanno Lanconelli's former students as a tribute to his career and to thank him for the guidance that he has provided throughout their research activities.

Beyond the scientific contents mentioned above, Andrea Bonfiglioli, Giovanna Citti, Giovanni Cupini, Maria Manfredini, Annamaria Montanari, Daniele Morbidelli, Andrea Pascucci, Sergio Polidoro, and Francesco Uguzzoni conceived this volume in order to offer researchers who have enjoyed collaborating with Ermanno the opportunity to share their scientific experiences.

The Editors

Geometric Methods in PDE's

Citti, G.; Manfredini, M.; Morbidelli, D.; Polidoro, S.;

Uguzzoni, F. (Eds.)

2015, XIII, 373 p. 8 illus., 7 illus. in color., Hardcover

ISBN: 978-3-319-02665-7