Preface

About the Book

This textbook is a treatment of the structure of abstract spaces, in particular linear, topological, metric, and normed spaces, as well as topological groups, in a rigorous and reader-friendly fashion. The assumed background knowledge on the part of the reader is modest, limited to basic concepts of finite dimensional linear spaces and elementary analysis. The book's aim is to serve as an introduction toward the theory of Hilbert spaces and the theory of operators.

The formalism of Hilbert spaces is fundamental to Physics, and in particular to Quantum Mechanics, requiring a certain amount of fluency with the techniques of linear algebra, metric space theory, and topology. Typical introductory level books devoted to Hilbert spaces assume a significant level of familiarity with the necessary background material and consequently present only a brief review of it. A reader who finds the overview insufficient is forced to consult other sources to fill the gap. Assuming only a rudimentary understanding of real analysis and linear algebra, this book offers an introduction to the mathematical prerequisites of Hilbert space theory in a single self-contained source. The final chapter is devoted to topological groups, offering a glimpse toward more advanced theory. The text is suitable for advanced undergraduate or introductory graduate courses for both Physics and Mathematics students.

The Structure of the Book

The book consists of six chapters, with an additional chapter of solved problems arranged by topic. Each chapter is composed of five sections, with each section accompanied by a set of exercises (with the exception of Chap. 6, which is shorter, and thus contains a single batch of exercises located at the end of the chapter). The total of 210 exercises and 50 solved problems comprise an integral part of the book designed to assist the reader, challenge her, and hone her intuition.

Chapter 1 contains a general introduction to real analysis and, in particular, to each of the subjects presented in the chapters that follow. Chapter 1 also contains a Preliminaries section, intended to quickly orient the reader as to the notation and concepts used throughout the book, starting with sets and ending with an axiomatic presentation of the real numbers.

Chapter 2 is devoted to linear spaces. At the advanced undergraduate level the reader is already familiar with at least some aspects of linear spaces, primarily finite dimensional ones. The chapter does not rely on any previous knowledge though, and is in that sense self-contained. However, the material is somewhat advanced since the focus is infinite dimensional linear spaces, which are technically relatively demanding.

Chapter 3 is an introduction to topology, a subject considered to be at a rather high level of abstraction. The main aim of the chapter is to familiarize the reader with the fundamentals of the theory, and in particular those that are most directly relevant for real analysis and Hilbert spaces. Care is taken to finding a reasonable balance between the study of extreme topology, i.e., spaces or phenomena that one may consider pathological but that hone the topological intuition, and mundane topology, i.e., those spaces or phenomena one is most likely to find in nature, but which may obscure the true nature of topology.

Chapter 4 is a study of metric spaces. Once the necessary fundamentals are covered, the main focus is complete metric spaces. In particular, the Banach Fixed-Point Theorem and Baire's Theorem are proved and completions are discussed, topics which are indispensable for Hilbert space theory.

Chapter 5 introduces and studies normed spaces and Banach spaces. Starting with semi-normed spaces the chapter establishes the fundamentals, and goes on to introduce Banach spaces, treating the Open Mapping Theorem, the Hahn-Banach Theorem, the Closed Graph Theorem, and, alluding to Hilbert space theory proper, the Riesz Representation Theorem.

Chapter 6 is a short introduction to topological groups, emphasizing their relation to Banach spaces. The chapter does not assume any knowledge of group theory, and thus, to remain self-contained, it presents all relevant group-theoretic notions. The chapter ends with a treatment of uniform spaces and a hint of their usefulness in the general theory.

A Word About the Intended Audience

The book is aimed at the advanced undergraduate or beginning postgraduate level, with the general prerequisite of sufficient mathematical maturity as expected at that level of studies. The book should be of interest to the student knowing nothing of

Hilbert space theory who wishes to master its prerequisites. The book should also be useful to the reader who is already familiar with some aspects of Hilbert space theory, linear spaces, topology, or metric spaces, as the book contains all the relevant definitions and pivotal theorems in each of the subjects it covers. Moreover, the book contains a chapter on topological groups and a treatment of uniform spaces, a topic usually considered at a more advanced level.

A Word About the Authors

The authors of the book, a physicist and a mathematician, by writing the entire book together, and through many arguments about notation and style, hope that this clash between the desires of a physicist to quickly yet intelligibly get to the point and the insistence of a mathematician on rigor and conciseness did not leave the pages of this book tainted with blood, but rather that it resulted in a welcoming introduction for both physicists and mathematicians interested in Hilbert space theory.

Prof. Carlo Alabiso obtained his Degree in Physics at Milan University, Italy, and then taught for more than 40 years at Parma University, Parma, Italy (with a period spent as a research fellow at the Stanford Linear Accelerator Center and at Cern, Geneva). His teaching encompassed topics in Quantum Mechanics, special relativity, field theory, elementary particle physics, mathematical physics, and functional analysis. His research fields include mathematical physics (Padé approximants), elementary particle physics (symmetries and quark models), and statistical physics (ergodic problems), and he has published articles in a wide range of national and international journals as well as the previous Springer book (with Alessandro Chiesa), Problemi di Meccanica Quantistica non Relativistica.

Dr. Ittay Weiss completed his B.Sc. and M.Sc. studies in Mathematics at the Hebrew University of Jerusalem and he obtained his Ph.D. in mathematics from Universiteit Utrecht in The Netherlands. He spent an additional 3 years in Utrecht as an assistant professor of mathematics where he taught mathematics courses across the entire undergraduate spectrum both at Utrecht University and at the affiliated University College Utrecht. He is currently a mathematics lecturer at the University of the South Pacific. His research interests lie in the fields of algebraic topology and operad theory, as well as the mathematical foundations of analysis and generalizations of metric spaces.

A Word of Gratitude

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