

Investigations of Stability and Transition of a Jet in Crossflow Using DNS

A. Peplinski, P. Schlatter and D. S. Henningson

Abstract We study the stability of a jet in crossflow at low values of the jet-to-crossflow velocity ratio R focusing on direct numerical simulations (DNS) and the global linear stability analysis adopting a time-stepper method. For the simplified setup neglecting a meshed pipe in the simulations, we compare results of the fully-spectral code *SIMSON* with the spectral-element code *Nek5000*. We find the calculated critical value R for the first bifurcation to be dependent on the numerical method used. This result is related to a large sensitivity of the eigenvalues and to the large spatial growth of the corresponding eigenmodes, making the use of periodic domains, even with the fringe method, difficult. However, we observe a similar sensitivity to reflection from the outflow boundary in the inflow/outflow configuration as well. We apply in our studies both modal and non-modal analyses investigating transient effects and their asymptotic fate, and we find transient wavepacket that develop almost identically in the stable and unstable cases. Finally, we compare these results with the simulation including the pipe in the computational domain finding the latter one to be much more unstable.

1 Introduction

The jet in crossflow (JCF) refers to fluid that exits a nozzle and interacts with the boundary layer flowing across the nozzle. This case has been extensively studied both experimentally and theoretically over the past decades due to its high practical relevance. Smoke and pollutant plumes, fuel injection and mixing or film cooling are

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just a few applications. It is considered a canonical flow problem with complex, fully three-dimensional dynamics, making it a perfect tool for testing simulation methods. Recent reviews on this flow configuration are given in Refs. [7, 8].

The JCF can be characterised by three independent non-dimensional parameters: free-stream and jet Reynolds numbers ($Re_{\delta_0^*}$, Re_{jet}) and jet to free-stream velocity ratio R , which is the key parameter in this work. The major flow features are (see e.g. Fig. 1 in Ref. [7]): the counter-rotating vortex pair (CVP) in the far field, the horseshoe vortex developing upstream of the jet orifice, and vortices shed from the shear layers that result from the interaction of the jet with the cross-flow both upstream and downstream of the jet trajectory. Some other features, such as wake vortices or upright vortices, are visible at higher values of the cross velocity ratio R only. As the ratio R increases, the flow evolves from a stable (and thus steady) configuration consisting of (steady) CVPs and horseshoe vortices, through simple periodic vortex shedding (a limit cycle) to more complicated quasi-periodic behaviour, before finally becoming turbulent. In Refs. [2, 6, 13] the stability of the JCF was studied by means of modal analysis. The first linear global stability analysis of this flow at $R = 3$ was presented by Bagheri et al. [2, 13]. For this jet to free-stream velocity ratio, the JCF was found to be dominated by an interplay of three common instability mechanisms: a Kelvin–Helmholtz shear layer instability, a possible elliptic instability of the CVPs, and a near-wall vortex shedding mechanism similar to a von Kármán vortex street. It was also shown that the flow acts as an oscillator, with high-frequency unstable global eigenmodes associated with shear-layer instabilities on the CVP and low-frequency modes resulting in vortex shedding in the jet wake. This work was later extended to the wider range of $R \in (0.55, 2.75)$ [6], focusing on transition from steady to unsteady flow as R is increased. The first bifurcation (appearance of the first unstable eigenmode) was found to occur at $R \approx 0.675$, when shedding of hairpin vortices characteristic of a shear layer instability was observed, and the source of this instability (*wavemaker*) was located in the shear layer just downstream of the orifice. Results of linear stability analysis were consistent with nonlinear direct numerical simulations (DNS) at the critical value of R predicting well the frequency and initial growth rate of the disturbance. It was also concluded that, based on linear analysis, good qualitative predictions about the flow dynamics can be made even for higher R , where multiple unstable eigenmodes are present. The authors pointed out, however, that the critical R cannot be determined exactly due to sensitivity of the results to changes in the domain length as well as to the presence of the fringe region enforcing periodic boundary condition (BC).

In the current study, we follow Ilak et al. [6], focusing on the transition from steady to unsteady flow and, using linear global stability analysis, searching for the value of R at which the first bifurcation occurs. The scope of this work is to test the numerical methods and techniques, and to identify the major difficulties related to linear stability of this type of complex flows. As purely modal analysis is known to fail in predicting the practical critical Reynolds number for transition to turbulence in a number of systems, we apply in our studies both modal and non-modal analyses. A classical example of such a flow is the convectively unstable flat-plate boundary layer [4], which behaves as broadband amplifier of incoming disturbances and is globally

stable according to linear global analysis. However, a global stability analysis based on the asymptotic behaviour of single eigenmodes of the system does not capture relevant dynamics, and transition to turbulence at lower Re occurs due to transient effects. Following Ref. [14] we investigate the linear growth of perturbations in the JCF for a limited time, before the exponential modal behaviour is most dominant, and determine an *optimal initial condition* (initial condition yielding largest possible growth in energy) adopting a time-stepper method.

The previous stability results for this flow [2, 6, 13] were obtained for simplified setups, in which the inflow jet was represented by a Dirichlet BC due to the limitations of the applied pseudo-spectral simulation method. We avoid this limitation using the more flexible spectral-element method (SEM), which provides spectral accuracy while allowing for complex geometries. However, to be able to compare our results with the previous ones we adopt in most of the simulations simplified setup of Ref. [6]. In addition we perform a few simulations with the more realistic one with the pipe included in the computational domain.

This work is an extension to our previous work [11], where preliminary results were presented.

2 Numerical Setup

In this work we perform simulations of JCF for two numerical setups: the simplified (S1, with Dirichlet BC) and the more realistic (S2, with pipe section) one. We adopt the simplified S1 setup similar to Ref. [6], modelling the interaction of a (laminar) boundary layer with a perpendicular (laminar) jet exiting a circular pipe with diameter $D = 3 * \delta_0^*$, where δ_0^* is the displacement thickness at the inflow placed $9.375 * \delta_0^*$ upstream the centre of the pipe orifice; δ_0^* is adopted as length unit. Following Ref. [6] we use both laminar cross-flow and jet inflow profile and, as the jet pipe is absent in the simulations, an inhomogeneous (Dirichlet) BC prescribing the inflow jet profile. This is an important limitation of the problem, modifying the flow pattern at the pipe orifice and affecting its stability properties (see Fig. 4). In addition, the jet profile is modified by a Gaussian function (Eq. 2.1 in [6]) to keep the velocity field smooth in the whole domain. The mass flux of the modified profile is 20.6% lower compared to the parabolic one. The consequence of this setup is that a direct connection to the physical problem is not possible, and our synthetic setup serves as a well-defined test problem for the numerical tools. We will list here only the key parameter of the setup referring the reader to [2, 6] for more details. The flow is fully described by the pipe diameter D , cross-flow Reynolds number $Re_{\delta_0^*} = U_\infty \delta_0^* / \nu$ and jet to cross-flow velocity ratio $R = V / U_\infty$, where U_∞ , V and ν are free-stream velocity, *peak* jet velocity and the kinematic viscosity, respectively. As before [6], we set $Re_{\delta_0^*} = 165$, and choose R as 1.0, 1.4, 1.5 and 1.6.

The more realistic S2 setup is similar to S1, however the pipe of length $20\delta_0^*$ is included in computational domain. In this case there is no need for smoothing of the jet profile by a Gaussian function and we use a parabolic profile as pipe inflow.

Because of that the results of S1 and S2 setups cannot be compared directly. Sample results of simulations with S2 setup are described in Sect. 4.2.

The simulations are performed with two different massively parallel DNS solvers for the incompressible Navier–Stokes equations: SIMSON [3] and Nek5000 [5]. SIMSON was used in Refs. [2, 6, 13] and is a fully-spectral code suitable for stability computations. However, a major constraint is the requirement for periodic BC in streamwise direction necessitating a fringe region (Chap. 4.2.2 in Ref. [3] and references therein) for damping disturbances. We use SIMSON to investigate the influence of the fringe length L_F on the stability results from nonlinear DNS in S1 setup. Fringe parameters are adopted from Ilak et al. [6] with varying L_F set to 15 (as in [6]), 45 and 75 units. Since the fringe region reduces the useful part of the computational domain, we doubled the length of the box as compared to Ref. [6], setting its size to $L_x = 150$, $L_y = 20$, $L_z = 30$, with the resolution of $512 \times 201 \times 144$ modes in the streamwise (x), wall-normal (y), and spanwise (z) directions, respectively.

Nek5000 is a spectral-element code providing spectral accuracy while allowing for complex geometries. In this method the governing equations are cast into weak form and discretised in space by the Galerkin approximation, following the $P_N - P_{N-2}$ approach with the velocity space spanned by N th-order Lagrange polynomial interpolants. In our studies we use Nek5000 to investigate the influence of resolution ($N = 6, 9$ and 12), box length ($L_x = 150$ and 250) and grid structure. Domain decomposition into hexahedral elements is used to reduce resolution where it is not needed. We keep uniform resolution in the pipe vicinity within 5 unit distance from the orifice, and reduce it at larger distance by smooth element stretching (mesh M1). In mesh M2 we double the vertical resolution next to the wall. There are no periodic BC in streamwise direction, however, we found our results to be dependent on the outflow BC unless we set a sponge layer at the outflow as well as making the computational domain longer to reduce reflections from the boundary. The forcing function for the sponge was adopted from the fringe in SIMSON, and the sponge length was set to 25 and 35 units for $L_x = 150$ and 250 , respectively. Nek5000 is also used as time-stepper for solving the linearised Navier–Stokes equations in modal and non-modal linear stability analyses in setups S1 and S2. In the case of modal linear stability analysis and $L_x = 250$ we increase the height of the computational domain $L_y = 30$ to ensure CVPs fit within the box. Detailed descriptions of the implementation of Nek5000 can be found in Ref. [5].

3 Direct Numerical Simulations

We performed a number of nonlinear DNS to investigate the dependence of the critical inflow ratio R value on the various simulation parameters.

The studies results pertaining to the damping sensitivity in the fringe for $R = 1$ were performed with SIMSON and are presented in Fig. 1. The left plot shows the time-dependent amplitude of a single Fourier component in the signal of the streamwise velocity component of a probe located 15 units downstream of the pipe centre for $L_F = 15, 45$ and 75 (curves 1, 2 and 3). The chosen Fourier component

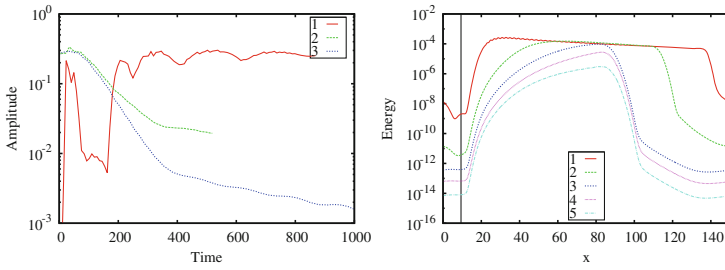


Fig. 1 Temporal evolution of disturbance amplitude (*left*) and energy as function of streamwise position (*right*) of the single most unstable Fourier component for $R = 1$ and different fringe lengths L_F (1, 2, 3) and simulation time (4, 5). Simulation results of SIMSON

is the most dominant one in the flow, and its amplitude at time t is calculated by projection of the signal on sine and cosine functions within the window $(t - T_p/2, t + T_p/2)$ (T_p being the component period) and finally smoothed by Bézier curves. The right plot presents the energy of the same Fourier component of the streamwise velocity component integrated over $y-z$ plane and plotted as a function of streamwise position x . Curves 1 and 2 correspond to $L_F = 15$, $t = 800$ and $L_F = 45$, $t = 500$ respectively, and curves 3, 4 and 5 give the time evolution of energy distribution for $L_F = 75$ and t equal 500, 1000 and 1500 (time is counted from the beginning of simulation). The pipe centre is marked by the vertical line, and the size of the fringe layer is clearly visible. The first simulation was performed with $L_F = 15$ and Blasius boundary layer as initial condition, and run up to $t = 880$, when the probe signal saturated. This state provided the initial conditions for the two other runs. Note that the final velocity field of the first run consists of more than one frequency component, and curve 1 in Fig. 1 gives only an estimate of observed oscillations.

According to Ilak et al. [6] the JCF for $R = 1$ appears globally unstable, however, closer examination of Fig. 1 shows this flow to be convectively unstable, and the misinterpretation of the instability mechanism to stem from insufficient damping in the fringe. The temporal amplitude evolution of the simulation with the shortest fringe features a short phase of approximately exponential decay (after initial transient phase) ending at $t \approx 160$, when the signal from the fringe reaches the pipe after re-entering the domain and triggers instability. It occurs because the damping in the fringe (of the order of 10^5) is too weak compared to the growth rate in the domain leading to nonlinear saturation of the signal. Similar conclusions can be drawn from the second run ($L_F = 45$), where the nonlinear saturation on the energy plot is still visible. To achieve sufficient damping, the fringe with at least $L_F = 75$ (50% of the domain) is required. Then, the signal amplitude decays exponentially and the saturation is no longer visible after 500 time units. However, even in this case, the decay rate after $t = 400$ is relatively low leaving considerable energy in the strongest Fourier mode after $t = 1500$. This proves that methods relying on periodic domains to be unsuited for flow cases with considerable spatial growth rates. To employ those methods one has to ensure the damping within the fringe

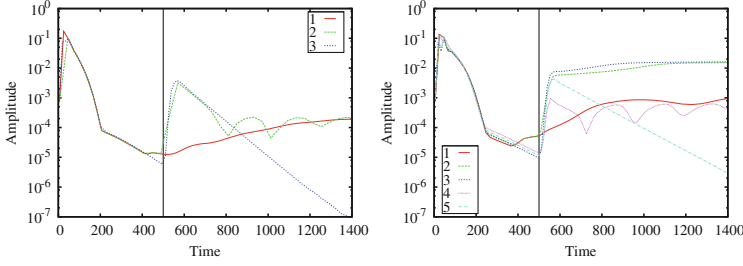


Fig. 2 Time dependent amplitude evolution of the single Fourier component of the velocity probe for $R = 1.4$ (left) and 1.5 (right) for different domain length and grid resolution. See text for description. Runs with Nek5000

region is larger than the expected (physical) spatial growth in the domain to make perturbations re-entering the domain not relevant for the flow dynamics (see as well Ref. [12]).

All subsequent runs are performed with Nek5000, which does not require periodicity in streamwise direction. Nevertheless, even in this case a correct treatment of outflow BC was found to be crucial. The time-dependent amplitudes of a single Fourier component in the signal of the streamwise velocity component of a probe located 15 units downstream of the pipe for $R = 1.4$ and 1.5 are shown in Fig. 2. Curves 1–3 correspond to lower resolution runs with $N = 6$, and curves 4 and 5 show results with higher order $N = 9$ for $R = 1.5$. Two different domain lengths are studied: $L_x = 150$ (curves 1, 2 and 4) and 250 (curves 3 and 5). All DNS start with undisturbed Blasius initial condition, and in runs 2, 3, 4 and 5 the instability is triggered by adding non-symmetric noise of amplitude 10^{-4} to the velocity field at $t = 500$ (marked by vertical line). All presented simulations were performed on the M1 mesh. For all cases, an initial transient phase (ending around $t = 220$) is followed by short phase of nearly exponential decay. Depending on the domain size and whether noise was added or not, this phase ends with a rapid amplitude increase followed by exponential decay/saturation, or with the slow growth of low amplitude oscillations. Strong dependency on grid resolution is clearly visible, as the $R = 1.5$ run is globally unstable for $N = 6$ and stable for $N = 9$. On the other hand, small amplitude ($10^{-4} - 10^{-3}$) oscillations visible at the end of all runs with shorter domain (independently whether noise was added or not) are manifestation of interaction between pipe vicinity and outflow BC. In the lower resolution simulations those oscillations are excited during the phase of exponential decay (at $t \approx 400$ and 370 for $R = 1.4$ and 1.5) and reach a final amplitude of $10^{-4} - 10^{-3}$. In the unstable case $R = 1.5$ (curve 1 in right plot) they are later overtaken by growing unstable modes with similar frequency. Such oscillations are not visible in the simulations with longer domain, where all the disturbances are damped exponentially. Increased resolution allows excitation of those waves (no longer visible in the first decay phase) to be delayed, but it does not reduce significantly the final amplitude. Note that for the low resolution runs at $R = 1.5$ the final saturation is reached faster in the longer domain, and the maximum amplitude reached after the noise is added

for $N = 9$ is higher in the longer domain as well. This illustrates the need for proper non-reflective BC.

4 Modal Analysis

Modal stability analysis is the classical method of hydrodynamics stability, where the critical value of a given parameter, for which a single exponentially growing disturbance exists, is computed. In the linear theory for global analysis, those disturbances, the so-called linear global modes, are associated with eigenmodes of the linearised Navier–Stokes operator. Their identification requires finding the base flow $\mathbf{U}_b$, i.e. stationary solution to the nonlinear Navier–Stokes equations, and next solving an eigenvalue problem of the linearised operator. As the considered flow configuration is usually unstable, one option is to calculate $\mathbf{U}_b$ adopting an additional stabilisation mechanism to eliminate time dependency in a time-stepper. In our calculations selective frequency damping (SFD) [1] was used. To calculate the eigenvalues, the dimension of the state space of the eigenvalue problem $\dim(\mathbf{U}) \sim 10^7$ makes explicit construction of the matrix impossible and requires projection on a low dimension subspace. To achieve this we adopt the Arnoldi algorithm using special matrix-free methods based on time-steppers. Detailed description of the problem with governing equations, implementation in Nek5000, validation and application to JCF can be found in Peplinski et al. [10]. An example of the base flow and spectra (growth rate ω_i vs frequency ω_r) is shown in Figs. 1 and 5 in Ref. [10].

4.1 Setup Without Pipe

Here we present results of the DNS with S1 setup performed on meshes M1 and M2 with domain length $L_x = 150$ for two polynomial orders $N = 6$ and 9, and two velocity ratios $R = 1.5$ and 1.6. To confirm numerical convergence of our results with respect to box length and polynomial order, we performed additional simulations with $L_x = 250$, $L_y = 30$ and resolutions $N = 9$ and 12 for $R = 1.6$. However, since those high resolution runs are computationally very expensive, we calculated only the single strongest mode and we do not present the whole spectra in this case. As the spectra are symmetric with respect to $\omega_r = 0$ we utilise the negative and positive ω_r parts to compare different cases, and we plot only those parts of the spectra matching both direct and adjoint operators (compare positive ω_r part of the left plot in Fig. 5 in Ref. [10]).

Selected results are presented in Fig. 3. In all studied cases increasing resolution decreases the growth rates of all the modes except the ones with zero frequency. The upper left plot presents spectra for $R = 1.5$ and two meshes M1 and M2 (negative versus positive ω_r part of spectra) for two resolutions $N = 6$ (symbols 1 and 3) and 9 (symbols 2 and 4). The importance of high resolution close to the wall is clearly shown, as the low resolution run on mesh M1 is globally unstable, and the run on mesh M2 is stable. On the other hand, most of the converged modes in the spectra of

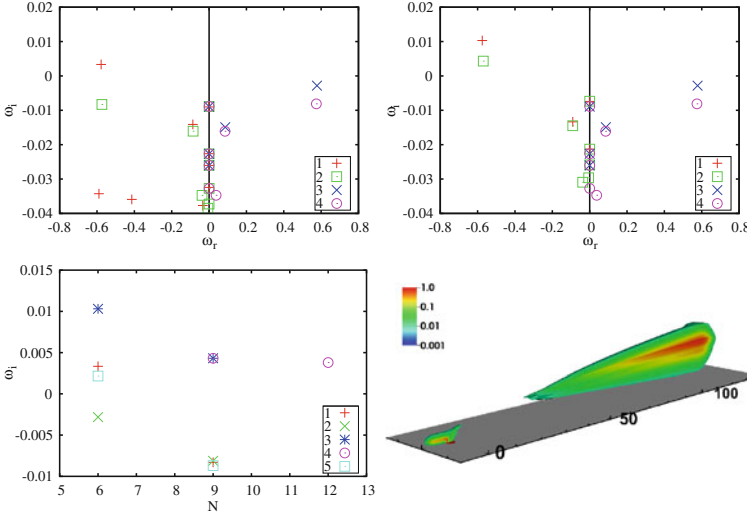


Fig. 3 Upper row sensitivity of the spectra to the mesh structure and resolution for different velocity ratios. *Left* spectra for $R = 1.5$ run on M1 (negative ω_r) and M2 (positive ω_r) meshes. *Right* spectra for $R = 1.6$ (negative ω_r) and $R = 1.5$ (positive ω_r) run on M2 mesh. *Lower left* growth rate of the strongest mode as the function of polynomial order N for the modal analysis and nonlinear DNS. *Lower right* Modulus of the direct (far field) and adjoint (pipe orifice) strongest eigenmode

higher resolution runs match each other for both meshes suggesting the resolution necessary for numerical convergence was reached. Simulations with $R = 1.6$ and 1.5 performed on mesh M2 are presented in the upper right plot (negative versus positive ω_r). As before, symbols 1, 3, and 4, 5 correspond to $N = 6$ and 9 , respectively. The shift of ω_i is larger for higher ω_r , and it seems to be independent of the value of R . The $R = 1.6$ case remains unstable even with the highest resolution and $L_x = 250$. All results are summarised in the lower left plot presenting the growth rate of the strongest mode as a function of N for both linear modal analysis and nonlinear DNS. Symbols 1, 2, 3, and 4 correspond to simulations at $R = 1.5$, mesh M1; $R = 1.5$, mesh M2; $R = 1.6$, mesh M2; $R = 1.6$, mesh M1 with $L_x = 250$, respectively, while symbol 5 gives the growth rate of the strongest Fourier component in DNS for $R = 1.5$ (mesh M1). Higher resolution runs of the linear analysis and nonlinear DNS are consistent with each other in all cases.

The lower right plot in Fig. 3 shows the modulus of the strongest eigenmodes of the direct (far field) and adjoint (pipe vicinity) operators for $R = 1.5$ and $N = 9$. The pipe, marked by a red circle, is located at $x = 0$ and the modes maxima are normalised to unity. The colour scale is logarithmic and the surface of the plotted region corresponds to 1 % of the maximum value illustrating fast decay of the eigenmodes. The total growth of the direct mode (ratio of the maximum to the value at the pipe orifice) is of order 10^9 . The visible strong streamwise separation of the direct and adjoint global modes is induced by the base-flow advection and is a signature of non-normality of the linearised operator. This is important since such a high degree

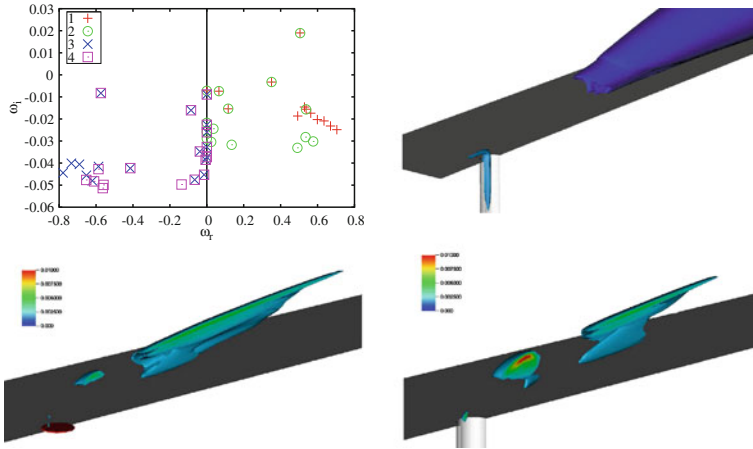


Fig. 4 *Upper left* comparison of the spectra for $R = 1.5$ for setup S1 (negative ω_r) and S2 (positive ω_r). Both direct (symbols 1 and 3) and adjoint (symbols 2 and 4) spectra are plotted. *Upper right* Modulus of the direct (far field) and adjoint (pipe orifice) strongest eigenmode for setup S2. The isosurface corresponds to 20 % of the maximum value. *Lower row* wavemaker for setup S1 (*left plot*) and S2 (*right plot*). The overlap of direct and adjoint modes is re-normalised and the colour scale on both plots is the same. The isosurface corresponds to 20 % of the maximum value

of operator non-normality leads to great sensitivity of the corresponding eigenvalues. It is also well-known that, as a result of non-normality, the perturbation energy may experience strong transient growth even though the flow is globally stable.

4.2 Setup Including Pipe

Simulations with setup S2 were performed for $R = 1.5$ on mesh M1 with $L_x = 150$ and $N = 9$, which are minimal values for box length and polynomial order securing convergence. Results of this run together with comparison to the corresponding S1 simulation are presented in Fig. 4. In the more realistic setup, the $R = 1.5$ case is unstable with relatively large growth rate $\omega_i = 0.019$ suggesting considerable reduction of critical R . It is related to the modified shape of the adjoint eigenmode extending down into the inlet pipe and is clearly visible in the wavemaker plot, as the overlap of direct and adjoint modes next to the pipe orifice and in the shear layer is considerably stronger for setup S2. However, the difference in mass flux can also affect stability, and more studies are necessary to confirm its influence.

5 Non-Modal Analysis

In this section we discuss results of linear optimal disturbances, which is a well established technique to identify the initial condition leading to the largest growth of the disturbance at finite time. We look for the perturbation $\mathbf{u}(t = 0)$ which leads to

maximum energy $\langle \mathbf{u}(T), \mathbf{u}(T) \rangle$ at time T . This problem is equivalent to solving the eigenvalue problem $\lambda \mathbf{u}(0) = \exp(\mathbf{A}^\dagger T) \exp(\mathbf{A} T) \mathbf{u}(0)$, where $\exp(\mathbf{A}^\dagger T) \exp(\mathbf{A} T)$ is the forward and adjoint composite propagator, and $\mathbf{A}$ is a linearised Navier–Stokes operator. The largest eigenvalue can be found iteratively by (matrix-free) power iterations, where the state is first marched forward in time with the standard numerical solver (direct propagator) and then backward with the corresponding adjoint solver (adjoint propagator). The initial condition is white noise and the procedure is repeated until the assumed convergence criterion for $|\mathbf{u}_n(0) - \mathbf{u}_{n-1}(0)|$ is reached, where n is the iteration number. For a more detailed discussion and description of implementation in Nek5000 see Ref. [9].

Our simulations were performed for setup S1 on the fine mesh M2 with domain length $L_x = 150$ and optimisation time $T = 77$ for $R = 1.5$ and 1.6 . The initial state $\mathbf{u}(0)$ for each of step (direct and adjoint) was normalised $\langle \mathbf{u}(0), \mathbf{u}(0) \rangle = 1$, and the assumed convergence criterion was 5×10^{-5} , reached after 36 iterations. Note that the backward time marching in our simulations does not mean negative time steps, so the energy increases in both direct and adjoint stepper phases. To keep direct and adjoint problems consistent we apply sponge layers together with homogeneous Dirichlet BC both at the inflow and outflow in the flow. The results are presented in Figs. 5 and 6.

The left plot in Fig. 5 presents the energy growth with time for direct (curves 1, 3) and adjoint (curves 2, 4) phases for stable ($R = 1.5$; curves 1, 2) and unstable ($R = 1.6$; curves 3, 4) cases. The energy evolution is similar in both cases and its final value differs only by factor of 2 ($E = 8 \times 10^{11}$ and 1.6×10^{12} for $R = 1.5$ and 1.6 respectively) showing the transient growth to be only weakly dependent on R . Similar conclusions can be drawn from the nonlinear DNS, in which the optimal disturbance superposed on a steady base flow is used as initial condition. The evolution of time-dependent amplitude of a single Fourier component in the signal of the velocity probe (located 15 units downstream from the pipe) is presented in the right plot in Fig. 5 (curves 1 and 2 correspond to $R = 1.5$ and 1.6 , respectively). The initial amplitude development is identical both for stable and unstable cases, however, the final (asymptotic) fate is consistent with modal analysis. The amplitude saturation at 10^{-4} for the stable case $R = 1.5$ (curve 1) is caused by interaction with outflow BC discussed in Sect. 3 above.

Figure 6 presents a comparison of the optimal initial conditions for the streamwise velocity component $u_x(0)$ (upper row) and corresponding final wave packet $u_x(T)$ (lower row) for stable $R = 1.5$ and unstable $R = 1.6$ cases. Angled and top view are shown. As the optimal conditions and wave packets are symmetric with respect to the grid symmetry plane ($y = 0$) we plot results of both simulations on a single frame placing the stable case in the front/lower part of the plot. The maximum value of all the functions are normalised to unity, and the plotted isosurface corresponds to 0.2. Both the optimal disturbances and wave packets for stable and unstable cases are almost identical, however they are slightly shifted with respect to each other due to different shape of the base flow. The wave packets differ also by the wavelength, which is minimally shorter for the stable case.

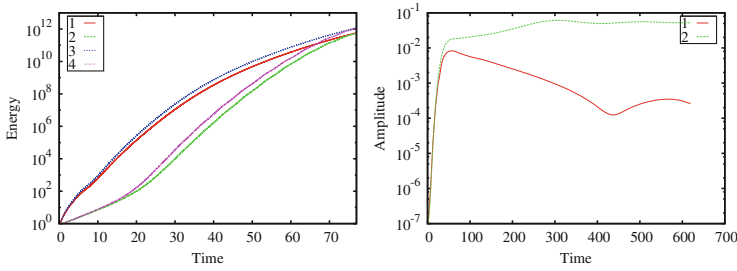


Fig. 5 Energy (*left*) and time-dependent amplitude (*right*) evolution for the transient growth of the optimal disturbance. See text for description

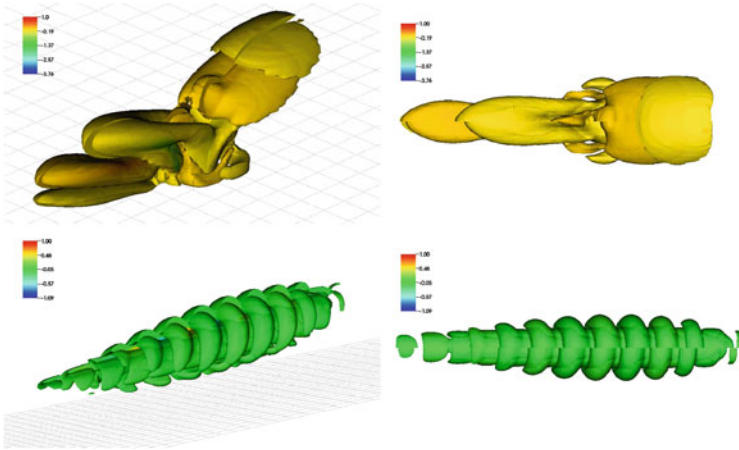


Fig. 6 Comparison of the optimal disturbance (*upper row*) and the resulting wave packet (*lower row*) for the stable (*front*, $R = 1.5$) and unstable (*back*, $R = 1.6$) case

An important advantage of the non-modal analysis is its insensitivity to the out-flow BC, as the travelling wave packet never reaches the outflow. However, in our simulations we could see the influence of the inflow (with respect to the direct operator) BC on the adjoint stepper phase manifesting itself by small oscillations of the energy curve at the end of integration period (barely visible in Fig. 5).

6 Conclusions

In the current work, we performed a stability analysis of the jet in crossflow (JCF) testing the numerical methods and stability techniques. We focus on the calculation of the critical velocity ratio R at which first bifurcation (transition from the steady to unsteady flow) occurs. We performed a number of simulations using different numerical methods and codes, including fully spectral (Fourier-Chebyshev) and spectral element discretisation. We find the JCF to be so sensitive to the simulation

parameters and setup that the critical velocity ratio can be found for a particular numerical setup only. In particular we found that methods based on streamwise Fourier decomposition to be not well suited for the present cases with considerable disturbance growth due to the periodic BC in streamwise direction, as insufficient damping in the fringe region can significantly change the flow dynamics. Even for codes with inflow/outflow BC, we demonstrate the need for proper non-reflective BC, and the sensitivity of the modal stability analysis to the grid resolution. This great sensitivity of the eigenvalue at the bifurcation point indicates that it may not be a particularly interesting quantity to consider, as the flow itself is very sensitive to external disturbances and that transient effects are more relevant than the asymptotic growth rate associated with a particular global mode. We calculated the optimal disturbance finding its growth and shape robust and almost independent on R .

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