

Preface

Instabilities and Boundary Value Problems

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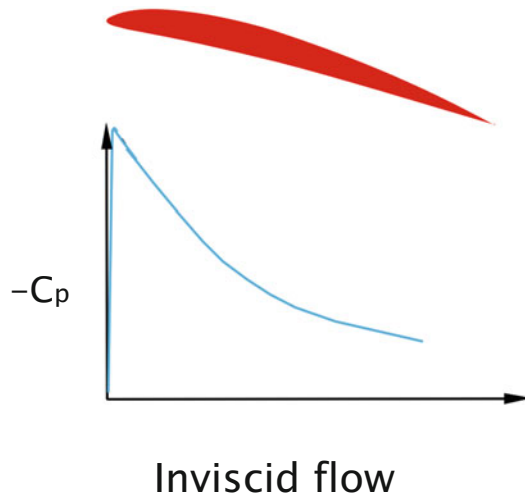
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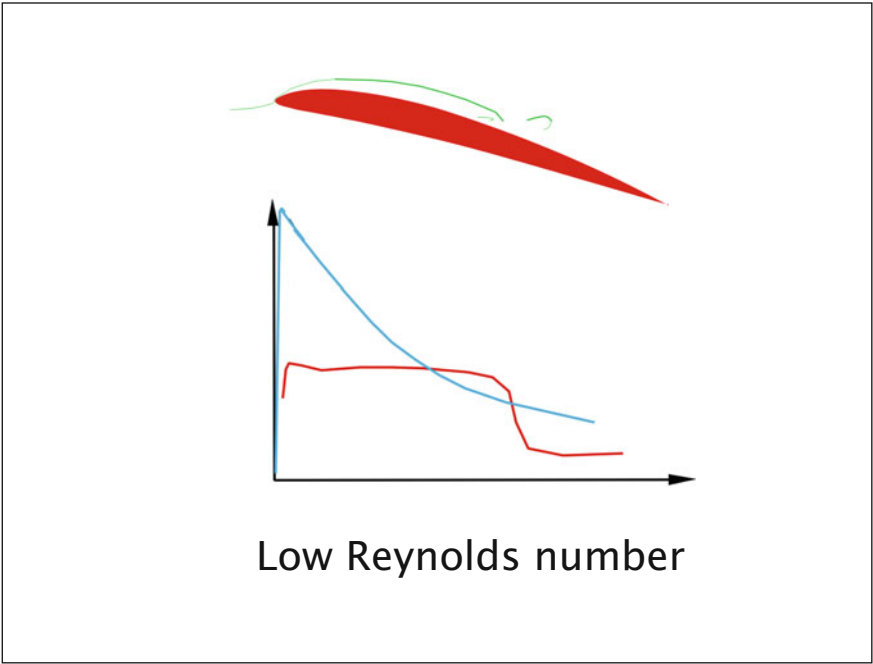
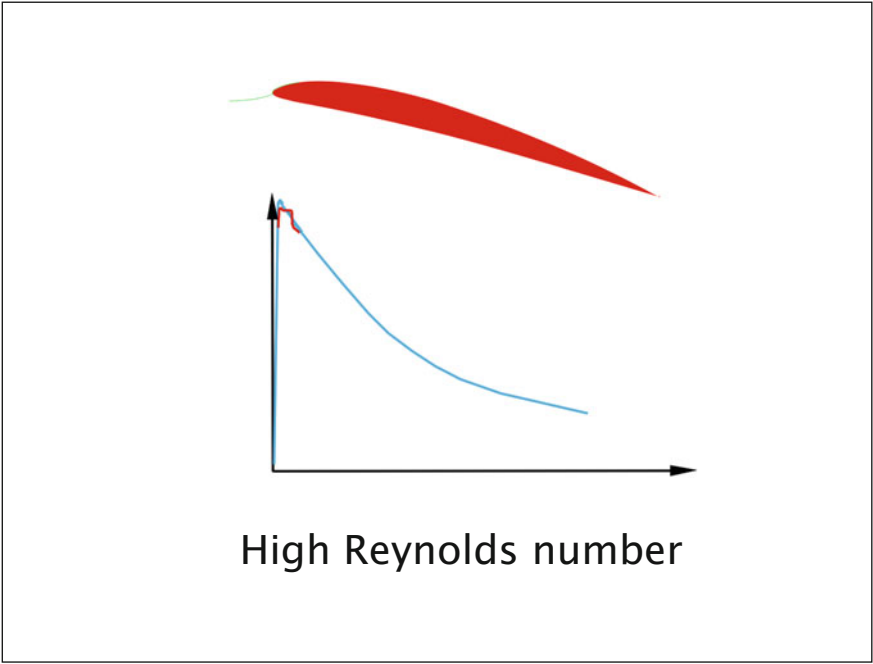
In 1954 I started my research on separation bubbles.

A sharp nosed aerofoil has a steep adverse pressure gradient aft of the nose that causes the boundary layer to separate. At high Reynolds numbers the separated shear layer will become turbulent and reattach forming a 'short bubble'.

At lower Reynolds numbers the flow may fail to reattach, or it may form a 'long bubble'.

This so-called bursting of the bubble causes a drastic loss of lift and is called a leading edge stall.



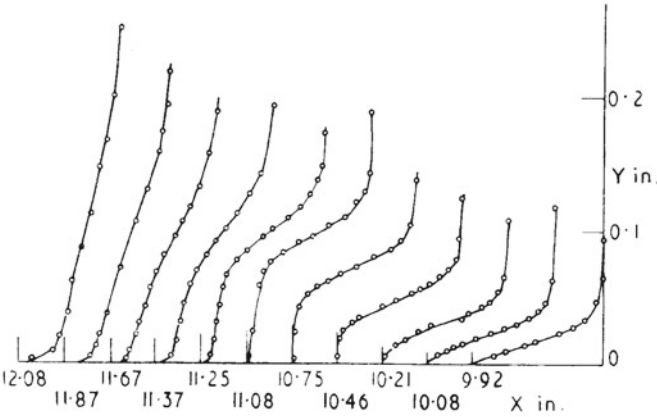
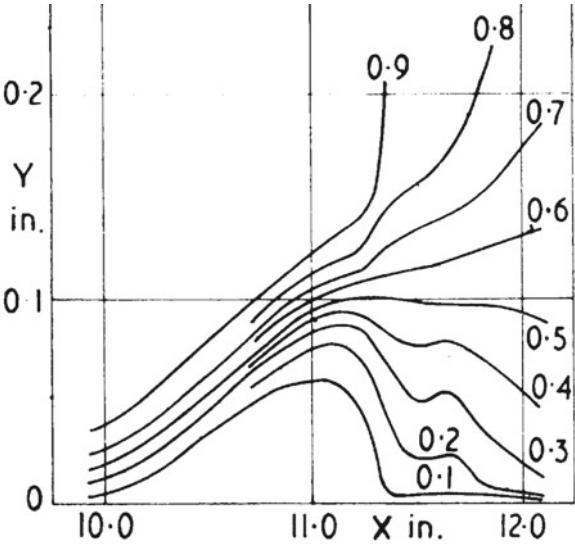


I was asked to find the cause of this devastating form of stall on swept wings. I considered the problem of zero sweep.

The next slide shows contours of the velocities measured with a hot-wire anemometer through a short bubble. The hot-wire signals clearly indicated roughly periodic oscillations in the shear layer. These oscillations increased in amplitude as the wire was moved downstream and eventually the signals became chaotic in the turbulent zone near reattachment.

Was the growth of perturbations the key to the problem?

Velocity contours through the bubble



(111)

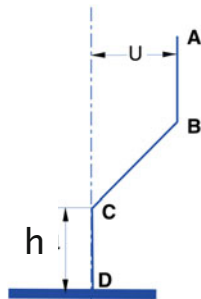
Velocity profiles

The simplest theoretical approach assumed a base flow that was parallel. This flow seems far from parallel until one looks at the contour plot scaled correctly.



The local profile shapes were approximated by three straight lines. The inviscid dispersion could then be obtained.

$$(\phi'' - \alpha^2 \phi)(U - c) - U''\phi = 0$$



$$\phi \sim e^{-\alpha y}$$

$$\phi \sim e^{-\alpha y}, e^{\alpha y}$$

$$\phi \sim e^{-\alpha y}, e^{\alpha y}$$

At B ϕ is continuous, but ϕ' has a jump.

$$\phi'(B^-) = \phi'(B^+) + \phi(B)/(U - c)$$

Straight line velocity profile

If we neglect any viscous effects and critical layer curvature we reduce the characteristic function to :-

$$(\omega - \frac{1}{2})(\alpha - \omega - \frac{1}{2}) = \frac{1}{4} e^{-2\alpha h}$$

The solution is dominated by the first term

$$\omega = \frac{1}{2}$$

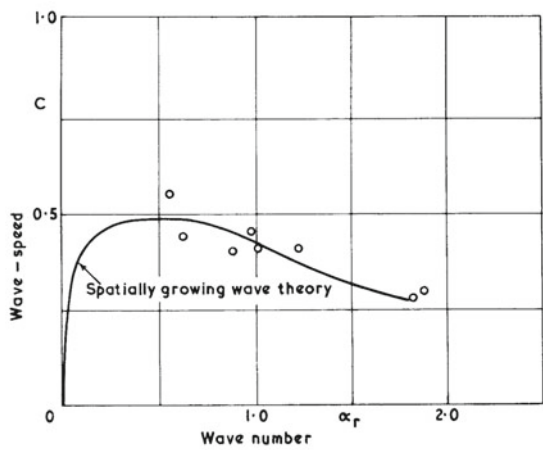
Travelling waves could be excited by sound and a controlled sound was introduced to create regular waves suitable for measurement.

The wavenumbers and frequencies were roughly given by the simple model treated as a 'locally parallel' flow.

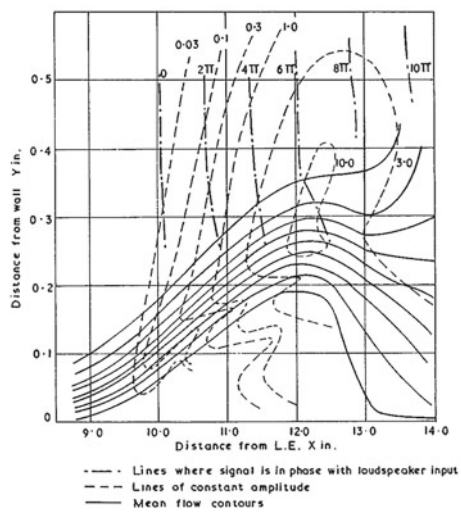
Growth rates and wavenumbers were deduced from the perturbations outside the boundary layer where the solution was simply -

$$e^{-\alpha y}$$

where the wavenumber is complex.



Measured and theoretical dispersion



Phase and amplitude of excited waves

I discussed this work with Dr Stuart and his colleagues at the NPL in (1957).

We didn't speak the same language.

They spent quite a long time with me and tried to be helpful, but could not accept the idea of looking at slices of a developing flow and they certainly didn't like a complex wavenumber.

They even questioned using such an approach on Blasius flow as had been done by Schlichting etc. Their focus was on 'Closed' systems in classic geometries.

I argued that the Schubauer's experiment could be modelled by boundary conditions chosen to represent a ribbon.

So I tackled that problem, but could not evaluate the integrals involved.

The solution arose in two parts: -

The near-field portion, that decayed and the Eigen-solution - the downstream T-S wave - *when the group velocity was positive*.

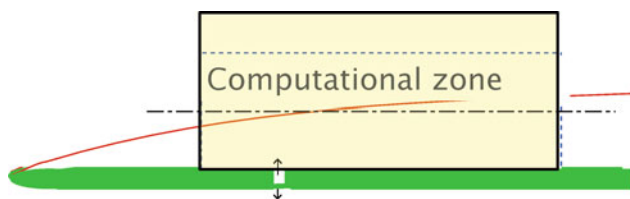
Even then I recall that Dr Stuart said that this solution had nothing to do with the flow **Stability!**

Of course he was correct, but I believe that the sort of situation that arises in ‘open systems’, like boundary layers, involve forced excitations.

At Cambridge around 1990 I realised that one could readily evaluate the integrals numerically in a boundary value problem.

I tackled a number of problems with some very able MPhil students.

The simplest case was that posed by a two-dimensional suction slit in a Blasius boundary layer.



A suction slit may be represented by

$$u(0, x) = 0$$

$$v(0, x) = Q\delta(0),$$

Providing the transform -

$$\hat{v}(0, \alpha) = Q$$

Model of a suction slit

- 1) Linearize the perturbation.
- 2) Treat the base flow as parallel, $U(y)$ only.

The governing equation for the transform of the stream function reduces to:-

$$(\phi'' - \alpha^2 \phi)U - \phi U'' = \frac{i}{\alpha R} (\phi'''' - 2\alpha^2 \phi'' + \alpha^4 \phi)$$

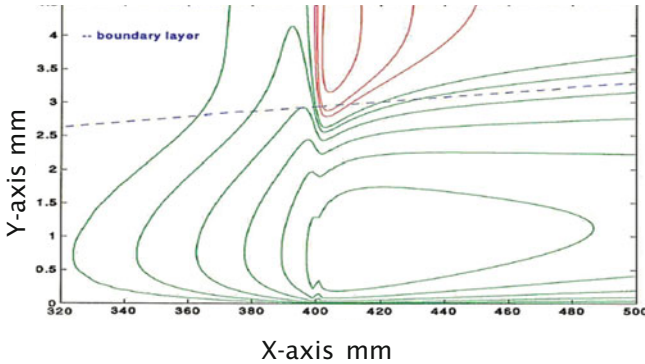
a reduced form of the *Orr-Sommerfeld* equation.

There are no eigen solutions so the perturbation consists solely of the near field.

$$u(y, x) = \frac{1}{2\pi} \int \frac{\hat{u}(y, \alpha)}{\hat{v}(0, \alpha)} e^{i\alpha x} d\alpha$$

Integrated over alpha along the real axis from minus alphamax to plus alphamax, where alphamax is chosen to provide solutions at the required physical spacing.

Contours of velocity perturbation over on an X~Y plane



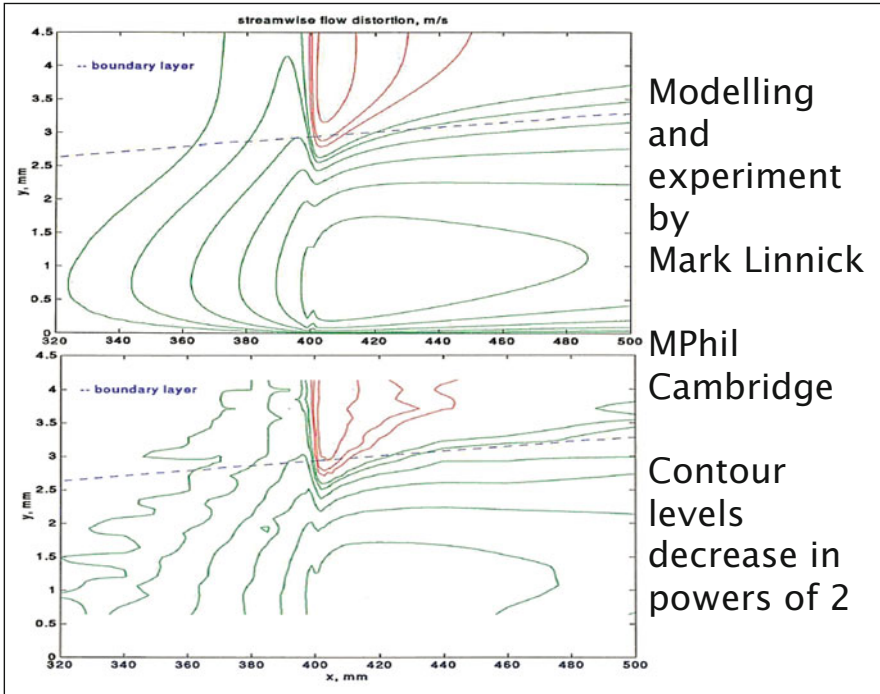
Velocity perturbations induced by suction

When I discussed this work in Cambridge I was told that these approximations were invalid.

Firstly, it was inconsistent to treat the local Reynolds numbers as finite, while at the same time as infinite so as to form a parallel flow.

Secondly, I could not use a spatial Fourier representation involving small wavenumbers because the boundary layer scale would change significantly over a wavelength. The Fourier inversion was therefore invalid.

There was a Third one as well??

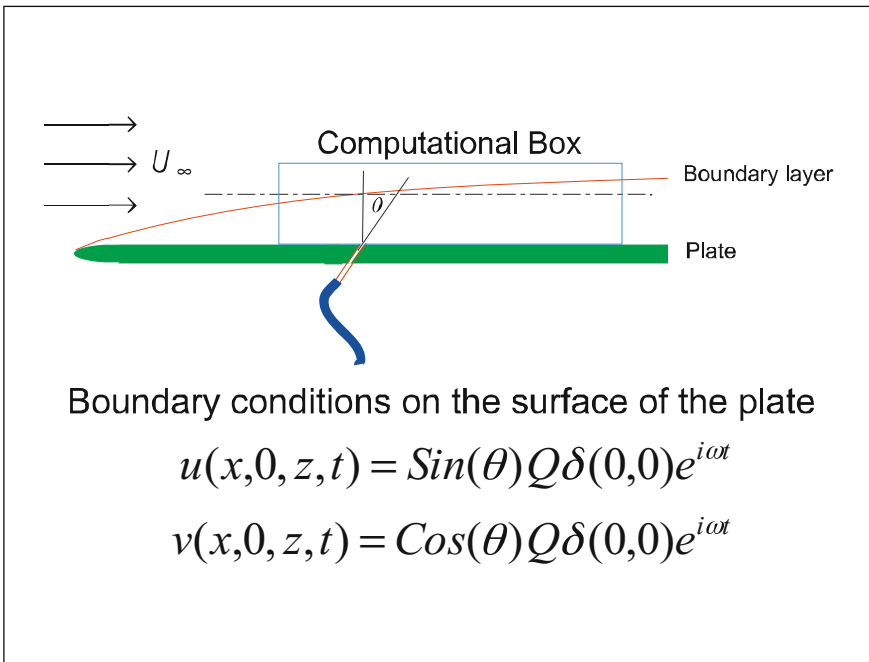
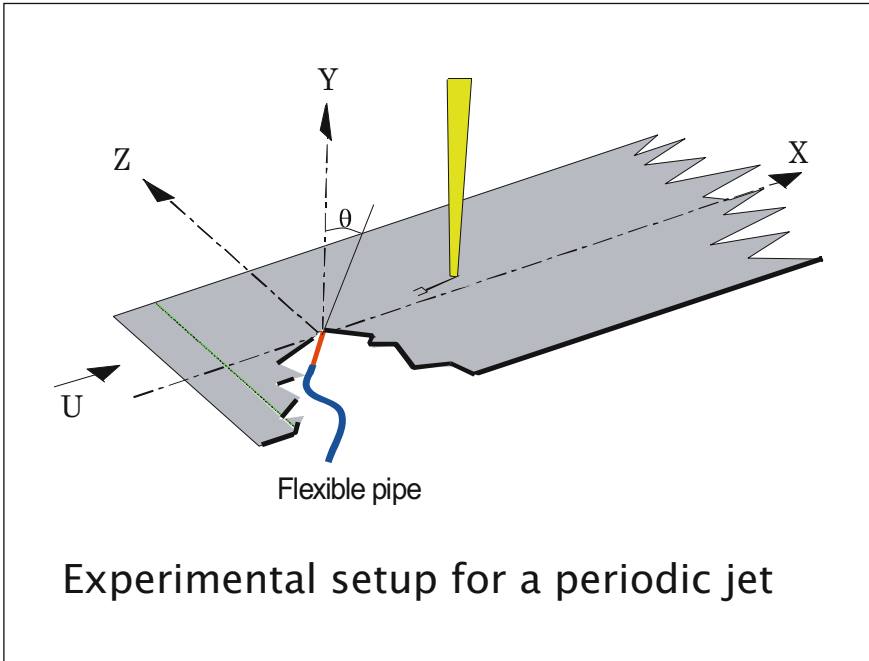


It turned out that the experiment validated the model very well indeed. Why were the doubters so wrong?

We can tackle the slow mean flow variation by either an iterative scheme using the P.D.E. or by modelling the base flow as a series of parallel segments coupled together.

The major element in T-S waves arises through stream wise coupling at different stations or by a transmission factor across discontinuities. The errors are small.

At Queen Mary we went on look at periodically excited jets.

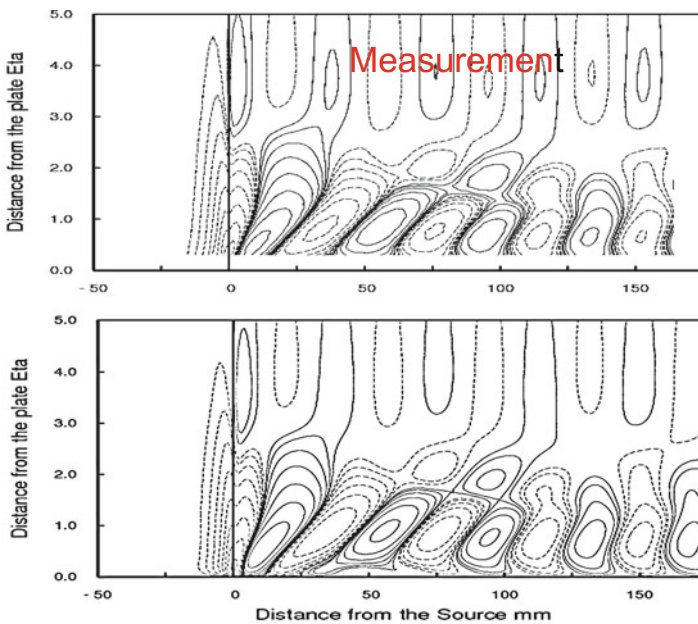


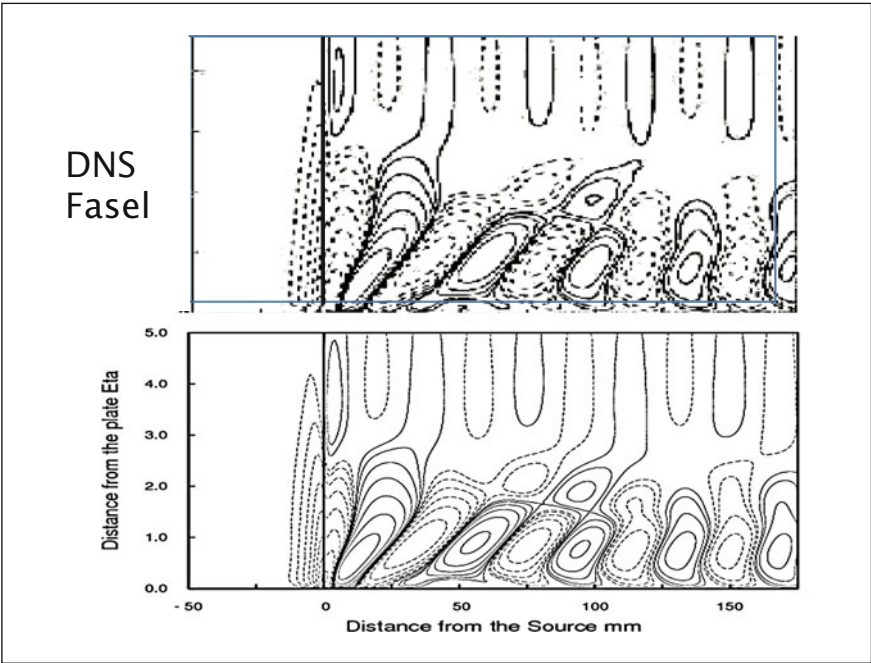
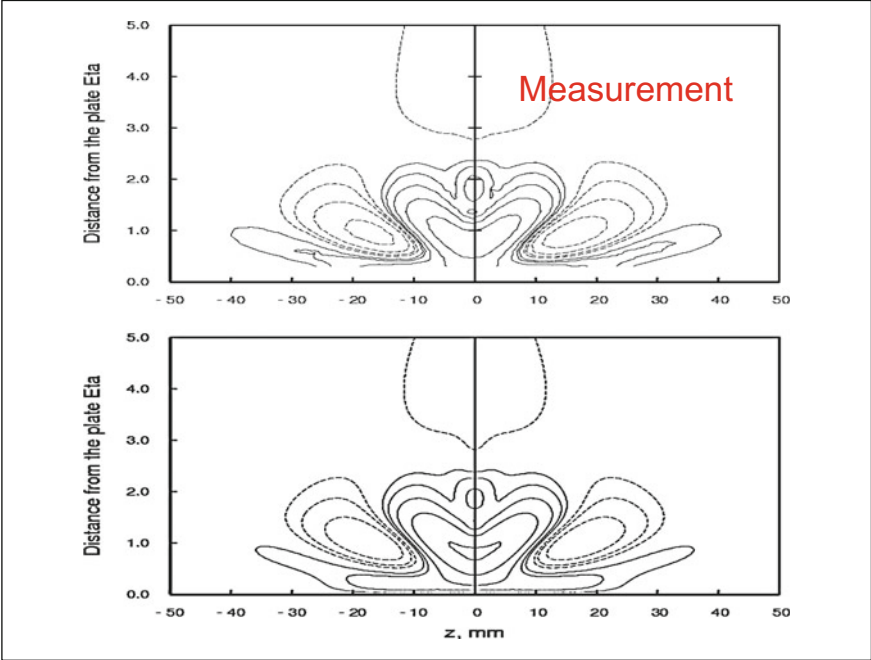
The experiments on periodic jets were carried out by an MSc student, Emmanuel Caloupis, with the support of EOARD in co-operation with Hermann Fasel at the University of Arizona.

Mike G did the calculations.

Again a hot-wire was traversed across the boundary layer and ensemble averaged plots of the wavy motion obtained. The next slide shows an $x\sim y$ slice of the flow field taken through the source.

Then $z\sim y$ cuts through the wedge shape zone are plotted.

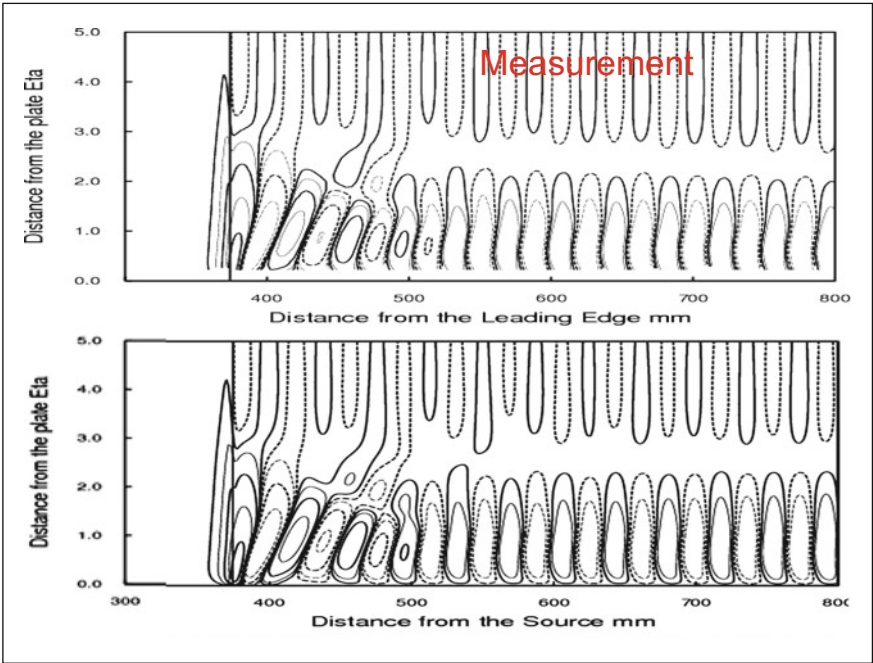


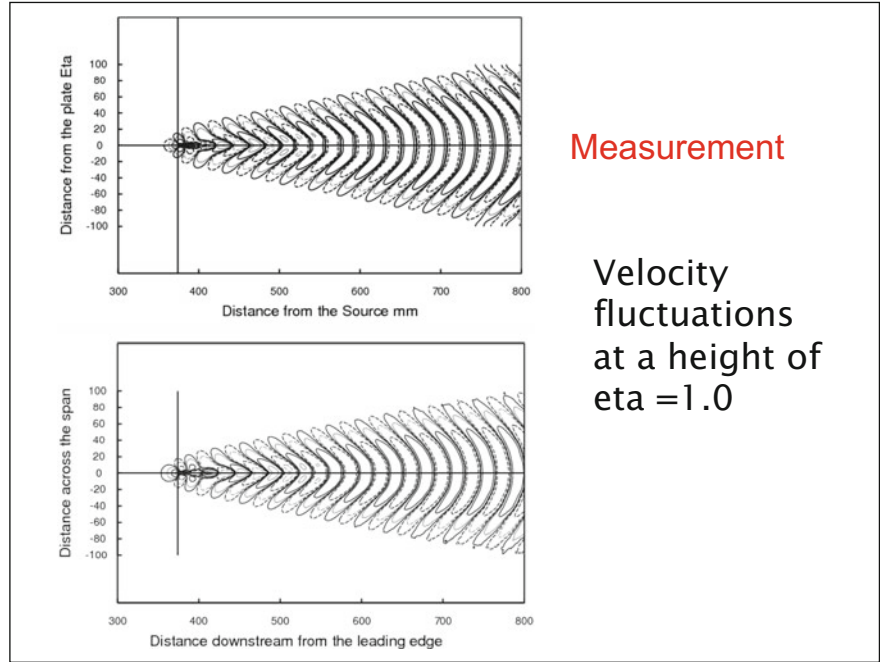


The flow was also evaluated using a DNS code by A PhD student working with Hermann Fassel at the University of Arizona in Tucson.

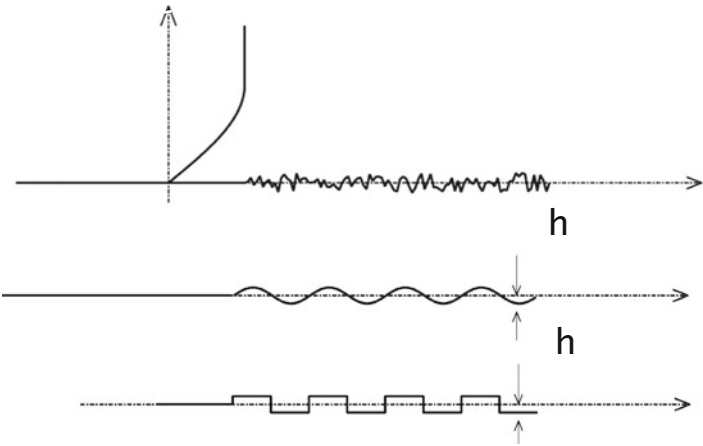
The agreement with our calculations and experiment must surely give support to the validity of the simple model.

Further work was done by myself over much longer fetches. When plotted using non-dimensional measures of the distance from the wall some compensation for flow divergence is approximately catered for and we get good correlation over long distances.





Similar modelling methods have now been applied to wall ripples to represent roughness.



Similar treatment has now been applied to wall ripples to model surface roughness.

The overall mean flow is modelled by:-

Base boundary layer plus a summation of solutions of the governing equations for u & v .

The weights are chosen so that u & v are zero on the boundary.

A *best fit* solution at a large number of collocations points is found by SVD.

Two fundamental solutions, A & B, for the transform of velocities components are:-

$$\begin{aligned} & {}_A\hat{u}_k(y), {}_A\hat{v}_k(y) \\ \text{and} \quad & {}_B\hat{u}_k(y), {}_B\hat{v}_k(y) \end{aligned}$$

With boundary conditions:-

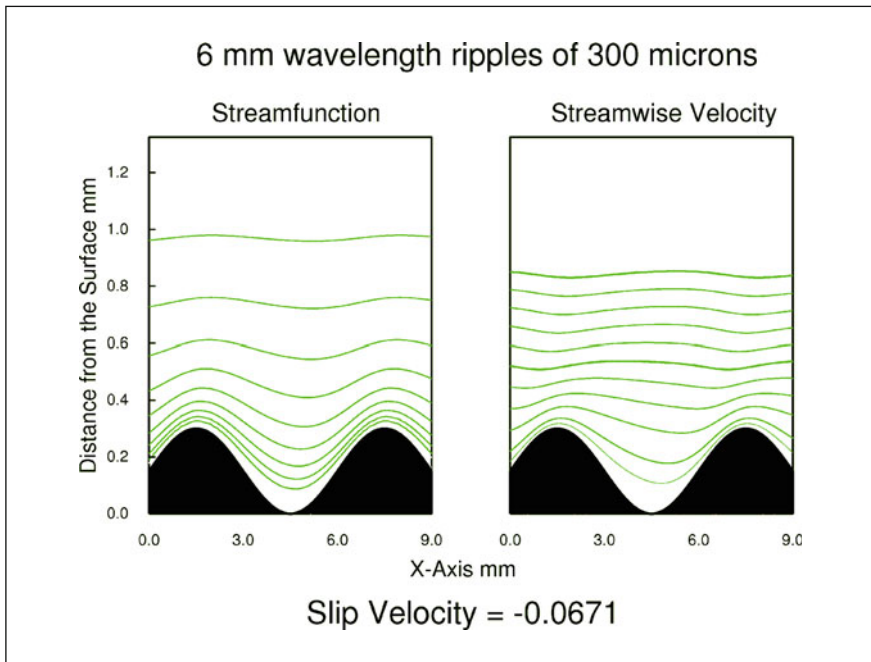
$$\begin{aligned} & {}_A\hat{u}_k(0) = 1.0, \quad {}_A\hat{v}_k(0) = 0 \\ \text{and} \quad & {}_B\hat{u}_k(0) = 0, \quad {}_B\hat{v}_k(0) = 1.0 \end{aligned}$$

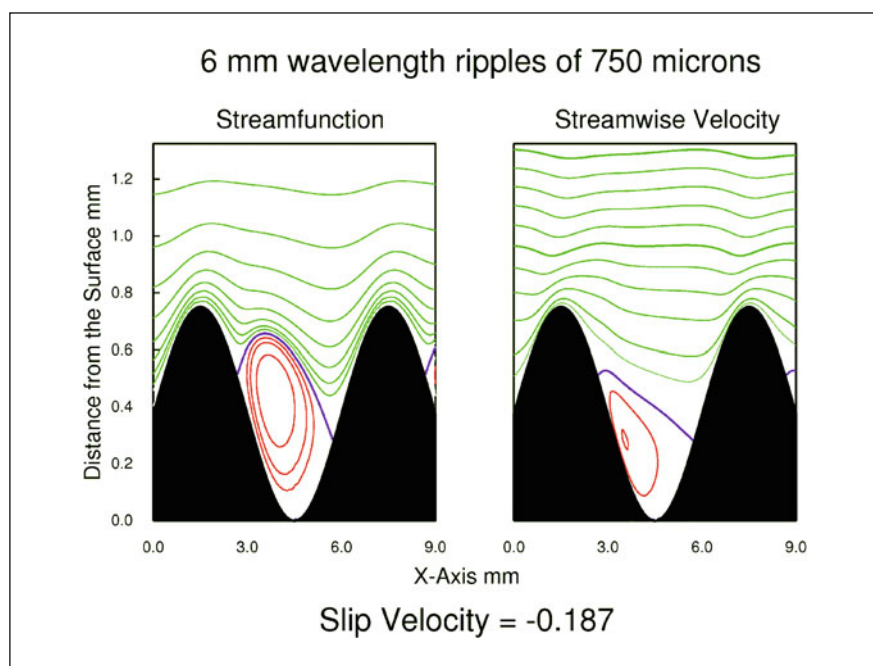
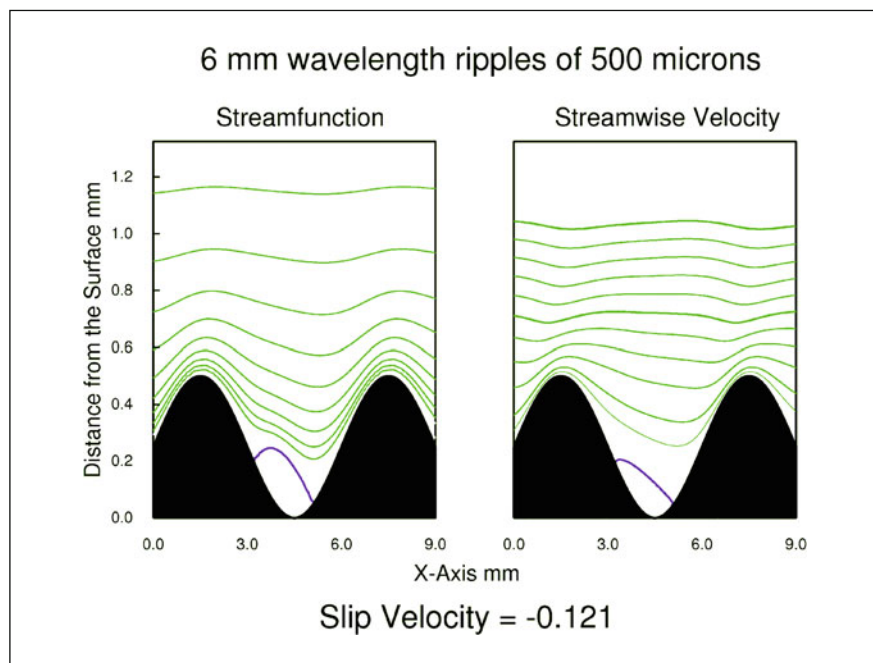
The form of solution is then -

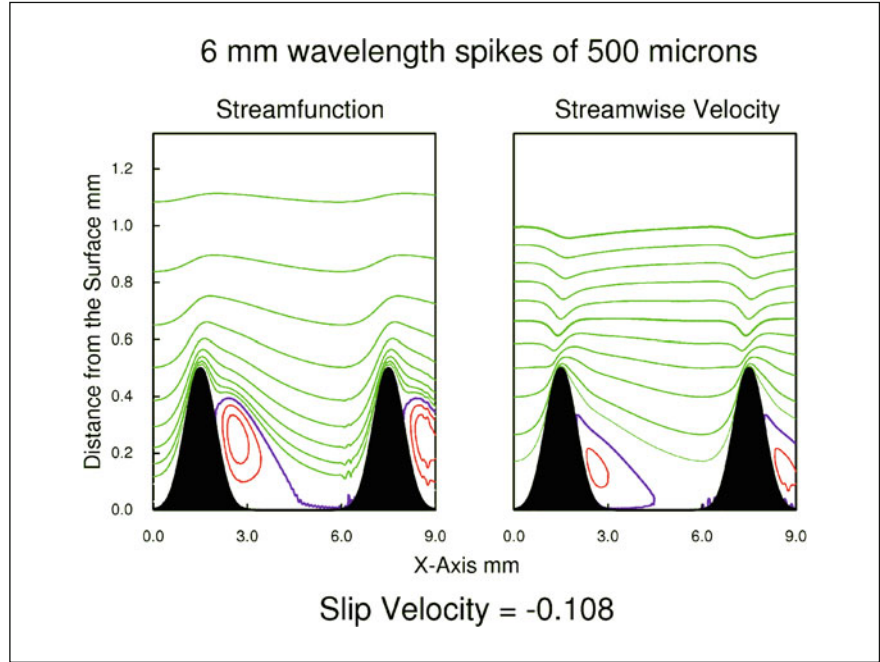
$$u(x, y) = U(y) + \sum A_k e^{ikx} \hat{u}_k(y) + \sum B_k e^{ikx} \hat{u}_k(y)$$

Similarly for the normal velocity component

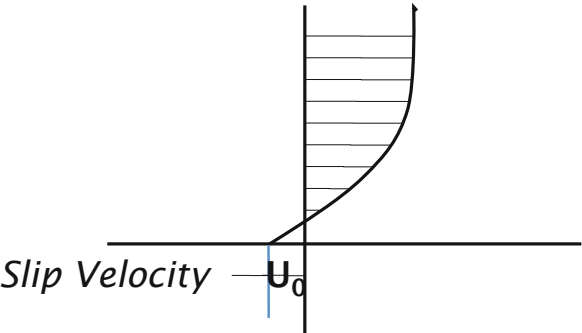
The coefficients are found by SVD so that the proper hydrodynamics boundary conditions apply.







The best fit is obtained with a slip flow boundary condition for the boundary layer.



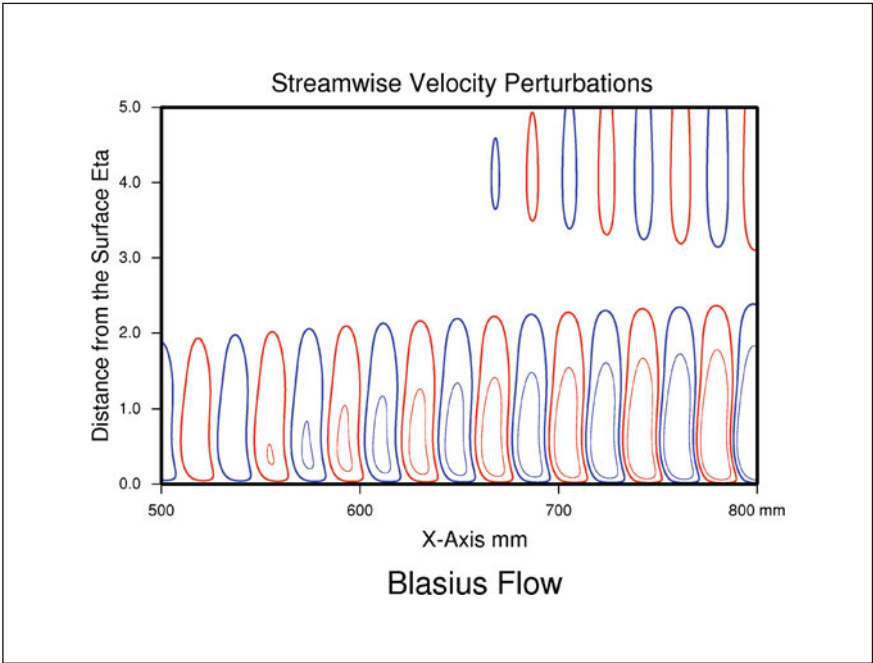
The behaviour is similar to that seen by a moving observer.

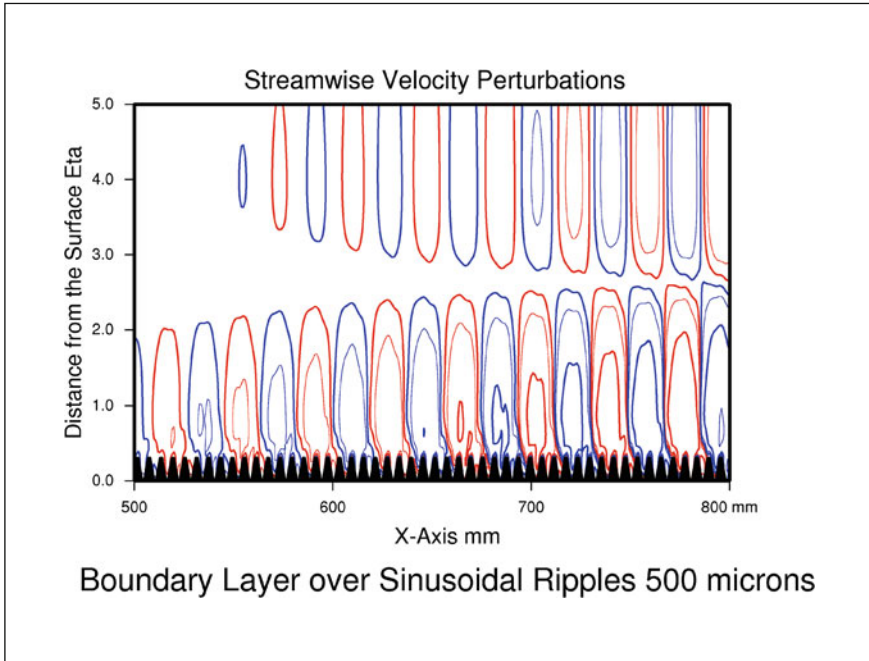
The behaviour of excited two-dimensional periodic travelling waves can be tackled in a similar way using the new boundary layer with the negative slip velocity. Again the perturbations velocities are made zero on the boundary.

The final two frames show the solutions obtained.

The model is being extended to cater for oblique waves so that the experiments from a point source can be properly compared with the theory.

The method will then be capable of treating fully three-dimensional roughness.





Conclusions

The linearized locally parallel flow model provides an excellent representation of the perturbations when the flow is slowly developing..

For even quite large flow non-uniformity we get the correct physical picture and often adequate quantitative values. We are of course looking at ‘Open systems’ that require some form of excitation, like turbulence or noise.

But now people are going full circle and again considering stability using global models that seem appropriate for ‘closed systems’.

Instability and Control of Massively Separated Flows
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