

Preface

This volume contains a selection of invited contributions from international experts on topics which were discussed at the INdAM Conference Analysis and Geometry in Control Theory and its Applications, held in Rome on 9–13 June 2014. All the contributors attended that event and were among more than 70 participants. The success of the conference was made possible by the invaluable support – both financial and logistic – provided by the Istituto Nazionale di Alta Matematica F. Severi. We would like to express our warmest thanks to both the Director and the staff of the Institute for the excellent scientific environment that they provided. In the spirit of INdAM workshops, the aim of the conference was to create an atmosphere favorable to interactions between young researchers and experienced scholars, as well as among members of communities working in different strands of Control Theory.

Optimal Control Theory developed rapidly during the second half of the twentieth century, as it represented a subject of great interest for application to engineering and economics, as well as a domain which needed solid mathematical theorization. Much progress has been made, and the effects of the many achievements are seen in everyday life, for instance when we take a flight, drive our car, or decide where to direct our investments. Nevertheless, the request to handle new and more complex problems, nonlinear models, and composite structures is providing the motivation for intensive ongoing research and, in our opinion, this will continue to be the case for years to come.

Reporting on the progress of research in all the various domains of interest to Control Theory would be very hard and we shall not attempt to do so. Rather, this volume focuses on the remarkable amount of work that has recently been undertaken to address two essential difficulties present in optimal control problems, namely the lack of smoothness and the lack of controllability. Crucial mathematical notions for the analysis of optimality conditions – from the celebrated Pontryagin maximum principle to the essential tools of Dynamic Programming – are given in a nonsmooth context in order to cover a sufficiently large range of applications. Similarly, as is well known, in order to be controlled even the simplest systems may require the use of composite strategies (modelled by iterated Lie brackets of the associated vector

fields). In response to such needs, two lines of research have been developed over the years – one related to set-valued analysis and the other to geometric techniques – independently at first and then with closer and closer connections to one another. This has led to a much better understanding of the behavior of control systems. The reader might wish to refer to the survey by H. Sussmann on “Geometry and Optimal Control” (in *Mathematical Control Theory*, pp 140–198, Springer, New York, 1999), in which nonsmoothness and chart invariance were already presented as basic ingredients of the *same*, geometric, approach, rather than being considered as alternative viewpoints.

The Editors feel that meetings designed to gather a wide range of experts in mathematical Control Theory should be organized in the future, especially with the aim of further encouraging the interplay of geometric and variational/PDE approaches. Several mathematicians have contributed to such progress and it is not our purpose here to give what would very likely be an incomplete list. But we are pleased to take the opportunity provided by their anniversaries to dedicate this volume to two of them, who have most deeply influenced the subject and boosted research in the two directions we mentioned above: Hélène Frankowska and Héctor Sussmann. The papers published in the present volume cohere with this aim. The contributions range from theoretic aspects of optimal control to a rigorous mathematical treatment of some applications.

Topics covered in this volume include analysis of fine regularity properties of the minimum time function (P. Cannarsa and T. Scarinci) and second-order optimality conditions under general control constraints (H. Frankowska and N.-P. Osmolovskii). The results shown in these papers are in the framework of the approach based on Variational Analysis. Further papers are concerned with a study of ω -limits of discontinuous differential inclusions, which is motivated by stabilizing feedbacks (A.-L. Donchev, M.-I. Krastanov, and V.-M. Veliov), and with the existence and uniqueness of a weak solution for first-order mean field game systems with local coupling (P. Cardaliaguet). Here the emphasis is on local vs. nonlocal coupling. Generalized Hopf-Lax formulas (motivated by some economic models) are given for a class of PDEs involving time averaging (J.-P. Aubin and L. Chen). Sophisticated aspects of the geometric approach are clearly presented in three papers dealing with perturbations of the metric associated with energy minimizing problems in orbital transfers with low propulsion (B. Bonnard, H. Henninger, and J. Rouot), optimal control of nonholonomic mechanical systems (A. Bloch, L. Colombo, R. Gupta, and D.-M. De Diego), and optimal cancer treatment protocols that administer agents at less than maximum tolerated dose rates (U. Ledzewicz and H. Schättler).

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