Chapter 2
Statistical Characterization of Single PPTA Fiber Tensile Properties from High Strain Rate Tests

J.H. Kim, N.A. Heckert, W.G. McDonough, K.D. Rice, and G.A. Holmes

Abstract Single [poly (*p*-phenylene terephalamide)] PPTA fiber tensile strengths were measured under quasi-static and high strain rate loading conditions, and poly (methyl methacrylate) (PMMA) and rubber as gripping materials were used to investigate gripping effects for the tests. To incorporate the strength distributions of single PPTA fibers into a rate dependent stochastic strength model, it is important to estimate uncertainties of the model parameters as well as the best-fitting-distribution for the parameter estimation. We demonstrated the appropriateness of a Weibull model for the tensile strengths obtained by the quasi-static test and preliminary results for the corresponding Weibull shape parameters with approximately ± 20 % parameter confidence intervals. These results will be used to characterize of the strengths obtained by the high strain rate test using the Weibull model.

Keywords Single fiber tensile test • PPTA fiber • Statistical analysis • High strain rate • Direct fiber grip

2.1 Introduction

Soft body armors have been used to protect the human body from the ballistic impact. The impact and perforation of fabrics in the body armors depend on several parameters including the material properties of the yarns, fabric structure, the projectile velocity etc. When a projectile strikes a fabric of body armor, longitudinal and transverse waves propagate from the impact zone, and these create fiber deformations in several different directions indicating tension along the fiber's axis, transverse compression, and fiber deflection. Numerous studies have been carried out on the impact behaviors of soft body armors during ballistic events, however, most of the studies on the influence of materials tensile properties on ballistic performance are conducted using the quasi-static properties [1].

Until recently, most fiber strengths obtained by single fiber tensile tests have been performed under many orders of magnitude slower loading conditions compared to ballistic impact. In order to measure fiber strengths under loading rates comparable to those of ballistic impact, a miniaturized Kolsky bar has been developed [2] and a direct fiber gripping method to increase test throughput has been adopted after a comparison study for gripping methods [3, 4].

Fiber strengths obtained by the single fiber tensile test typically exhibit large variation, so statistical analyses are often carried out to model dispersions of strength data. Many Weibull analyses for single fiber strengths obtained under the quasi-static loading conditions have been carried out; however fiber strengths for high strain rates tests are rarely reported.

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Main objectives of this study are investigating stochastic the behavior of single PPTA fiber strengths obtained under the quasi-static and high rate loading tests. We focus on the Weibull distribution to model the strength dispersions, after examining distributions graphically.

2.2 Stochastic Fiber Fracture Model

A stochastic fiber fracture model using the two-parameter Weibull distribution has been proposed to predict ultimate strengths of various types of fibers [5]. The average tensile strength (σ_f) of the individual fibers with a length (L) can be given by:

$$\sigma_f = \gamma \left(\frac{L}{L_0} \right)^{-1/\beta} \Gamma \left(1 + \frac{1}{\beta} \right), \tag{2.1}$$

where γ and β are the Weibull scale and shape parameters respectively, and Γ is the gamma function. $\Gamma(1 + 1/\beta) \approx 0.95 \pm 0.03$ in the case of the Weibull shape parameter β values varying from 5 to 30 [5]. L_0 is a reference length (1 mm in this study). Equation (2.1) is typically used for estimating fiber strengths obtained by quasi-static tests. Assuming the same linear elastic behaviors of the fibers until rupture for both quasi-static and high strain rate loadings, the relation of the Weibull parameters between quasi-static and high strain rate tests as a function of strain rate can be given by [6]:

$$\begin{cases} \beta_s = \beta_h \\ \gamma_s = \left[1 + E \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ref}} \right) \right] \left[1 + \varepsilon \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ref}} \right) \right] \gamma_h \end{cases} \tag{2.2}$$

where the subscripts s and h represent quasi-static and high strain rate loading conditions, and $\dot{\varepsilon}$ and $\dot{\varepsilon}_{ref}$ represent the input strain rate and the reference strain rate (i.e. the strain rate for quasi-static loading) respectively. The ratio of the mean strengths for the two cases is $\gamma_s \left(\frac{L}{L_0} \right)^{-1/\beta_s} \Gamma \left(1 + \frac{1}{\beta_s} \right) / \gamma_h \left(\frac{L}{L_0} \right)^{-1/\beta_h} \Gamma \left(1 + \frac{1}{\beta_h} \right)$. So if two cases (i.e., quasi-static and high strain rate) are adequately modelled by the two-parameter Weibull and the shape parameters are essentially equivalent, then the ratio simplifies to the ratio of the scale parameters.

2.3 Experimental Procedure

2.3.1 Single Fiber Tensile Tests

PMMA [poly (methyl methacrylate)] and rubber were used as clamp materials for the single fiber tensile tests to investigate gripping effects. The authors will utilize the term "PMMA and rubber grips" to refer a fiber grip made by two different materials set. For the quasi-static loading, a single fiber was clamped in the grips of a screw-driven machine with approximately 1 mN of pretension using a weight. Open/close motions of the grips were controlled by a pneumatic controller. Strain-to-failure was obtained by the displacement of the actuator, and the tensile stress was obtained by the force history and the cross sectional area of the fiber. Fiber lengths of 2, 5, and 10 mm were chosen to be the gauge lengths respectively. For the high rate loading, the miniaturized Kolsky bar was used in conjunction with a quartz-piezoelectric load cell due to very small transmitted force signal through a single PPTA fiber. A laser optical system [7] was used to measure the displacement of the Kolsky bar. A thin laser line generated by 100 mW laser illuminates a target that is attached to the gripping area of the Kolsky bar. The intensity of the refocused beam from the laser line is increased as the end of the bar moves in uniaxial tension and the relation between the bar location and the laser intensity is used to calibrate the laser intensity. Fiber lengths with 2, 5, and 8 mm were used as the gauge lengths of the high rate tests. Both tensile test results as a function of strain rate will be demonstrated in the presentation.

2.4 Results and Discussion

In this section, the procedures of the statistical analyses for the tensile strength data are briefly described and the statistical analysis results are summarized for each step.

2.4.1 Non-parametric Analysis of the Tensile Strengths

The fiber tensile strengths obtained by PMMA and rubber grips were compared graphically using kernel density plots. The kernel density estimate is defined as

$$\mathrm{f(y)} = \frac{\sum_{i=1}^{n} K\left\{\frac{(y-Y_i)}{h}\right\}}{nh}, \tag{2.3}$$

with K, h, Y_i, and n denoting the kernel function, the window width, the ith data point and the number of data points, respectively. The histogram is a simple kernel density estimator where h corresponds to the bin width, but typically the kernel density plot can show the underlying structure in the data more clearly than a histogram, particularly for modest sample sizes. Kernel density plots provide indications of such features as (1) the center of the data, (2) the spread of the data, and (3) the skewness of the data. Because of these advantages, we used it for estimating the strength distributions graphically. Figure 2.1 shows the kernel density plots of the tensile strengths for the PMMA and rubber grip tests using 2 mm fibers under the quasi-static loading condition. Similar widths of the kernel density plots for both grip tests indicate comparable strength distributions for the tests, but with possibly distinct modes of peak locations.

2.4.2 Distributional Fits: Parameter Estimates and Confidence Limits

Since the parameters of the two-parameter Weibull distribution are used in estimating average fiber strengths, one should estimate the Weibull parameters and uncertainties (confidence intervals) for the parameter estimates. The cumulative distribution function of the two-parameter Weibull distribution is given by:

$$F = 1 - exp\left(-\frac{L}{L_0}\left(\frac{x}{\gamma}\right)^{\beta}\right), \tag{2.4}$$

where x is fiber strength and other parameters are the same with those in Eq. (2.1). Although the Weibull plot is frequently used to estimate the parameters, we used maximum likelihood (ML) method. Since the Weibull shape parameter is

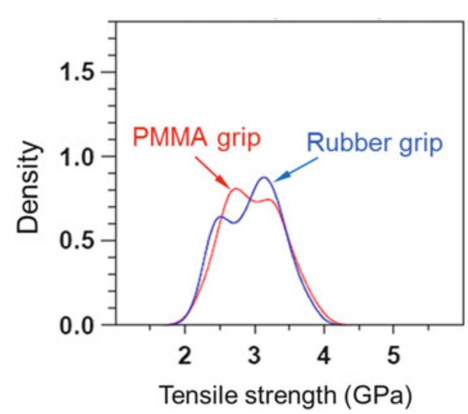

Fig. 2.1 Kernel density plot for the fiber strengths obtained by PMMA and rubber grips

correlated to the dispersion of the data, we focus on the shape parameter and its confidence interval. The shape parameters for the strength data (Fig. 2.1) obtained by the ML method varied from six to eight with confidence intervals approximately ±20 %.

2.4.3 Assessing Goodness of Fit

The two-parameter Weibull distribution is typically used to analyze dispersions of fiber strength data without investigating goodness of fit. A primary analytical method to assess goodness-of-fit is the Anderson–Darling (AD) test. Using PPTA fibers similar to the fibers used in this study, fiber strengths with 2 mm were previously measured for the PMMA grip under the quasi-static loading condition. A–D tests were carried out, which rejected the best-fitting assumption for the two-parameter Weibull distribution (1.2 A–D and 0.76 critical values). More goodness-of-fit analyses are being carried out with the PPTA fibers for the PMMA and rubber grips and will be presented in the future.

2.5 Concluding Remarks

Single PPTA fiber strengths were measured using the PMMA and rubber grip methods under quasi-static and high strain rate loading conditions. To validate a model, an important procedure is to confirm the best-fitting distribution as well as the parameter estimates. Since we are investigating dispersions of fiber strengths obtained by the high rate tests which are rarely reported in literatures, a procedure for assessing the strength distributions with the two-parameter Weibull is demonstrated for each step. Detailed statistical investigations for the strengths will be used to characterize PPTA fiber tensile properties as a function of loading rate and gripping method.

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