Chapter 2
Background and Preliminaries

Abstract In this chapter, fundamental definitions and terminology are given to the reader regarding the closed-loop control system. The analysis of the control loop takes place in the frequency domain and, therefore all necessary transfer functions of the control loop are presented in Sect. 2.2. The important aspect of internal stability of a control loop is presented in Sect. 2.3, whereas in Sect. 2.4 the property of robustness in a control loop is analyzed. In Sect. 2.5, a clear definition of the type of the control loop is given, since in Part II, the proposed theory is dedicated to the design of type-I, type-II, and type-III, … type-p control loops. Last but not least, in Sect. 2.6, the definitions of sensitivity and complementary sensitivity functions are presented so that the tradeoff feature in terms of controller performance that these two functions introduce is made clear to the reader. Finally, in Sect. 2.7, the principle of the Magnitude Optimum criterion is presented and certain optimization conditions are proved that comprise the basic tool for all control laws' proof throughout this book. These optimization conditions serve to maintain the magnitude of the closed-loop frequency response equal to the unity in the widest possible frequency range as the Magnitude Optimum criterion implies. In the same section, the Magnitude Optimum criterion is proved to be considered as a practical aspect of the H_∞ design control principle.

2.1 Definitions and Preliminaries

The core of a closed-loop control system is namely the plant or the process, see Fig. 2.1. The plant receives signals from the outer world, commonly known as inputs, depicted by $u(t)$ in Fig. 2.1, and acts at the same time to the outer world with its response, known as output, $y(t)$. Moreover, the whole process can also be described by its states $x(t)$, which along with the inputs $u(t)$, determine the response $y(t)$ of the plant itself.

Ideally, there are two fundamental requirements of a process in any real-time application:

1. From a plant, it is required that its output $y(t)$ must track perfectly its input $u(t)$.
2. The aforementioned output tracking of the input $u(t)$ must also be repetitive and for several different input signals $u(t)$.

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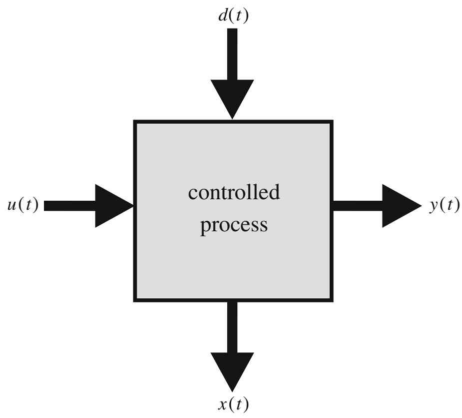

Fig. 2.1 The plant or process

Of course, these two aforementioned requirements are practically impossible to be satisfied at the same time in real-world plants and applications, since the existence of disturbances $d(t)$ alters the behavior of the process during its operation.

In real-world problems, disturbances $d(t)$ are classified into two categories. The first category involves disturbances coming from the process itself, known as internal disturbances.[1]

The second category includes any external or exogenous disturbance that can be relevant basically to the environmental conditions the process is located at, i.e., varying loads acting as input signals to the output of the process, noise coming from the measuring equipment, etc.

With respect to the above, it is without any doubt apparent that during the plant's operation, perfect tracking of the output $y(t)$ for repetitive and different input signals $u(t)$ can only be satisfied if fast suppression of internal and external disturbances is achieved.

For achieving fast suppression of disturbance in a closed-loop control system, the solution of the well-known principle of negative feedback,[2] widely used in mechanical, chemical, and electronic engineering is adopted. The introduction of negative feedback in a control loop leads to a control system presented in Fig. 2.2 the basic elements of which are (1) the process (plant), (2) the measuring equipment, (3) the reference signal, (4) the comparator, and (5) the controller along with the actuator unit.

In such a control loop, the path that connects the reference input $r(t)$ with the output of the control loop $y(t)$ is called *forward path*. This path includes the (1) process, (2) the actuator or power part unit, (3) the controller, and the (4) comparator.

[1] I.e., rise of temperature during a motor's operation, aging of materials after a certain time (for example, copper conductors in a squirrel-cage induction motor).

[2] Negative feedback is present to the water clock invented by Ktesibios (Greek inventor and mathematician in Alexandreia, 285–222 BC) and in the steam engine governor patented by James Watt in 1788.

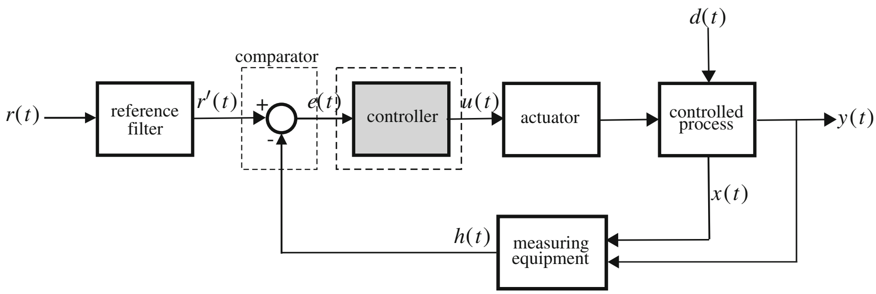

Fig. 2.2 General form of a closed-loop control system with negative feedback. The path that connects the reference input $r(t)$ with the output control loop $y(t)$ is called forward path. The path that connects the measuring equipment with the states and the output of the process is the feedback path

The path connecting the measuring equipment with the states and the output of the process is called *feedback path*. The logic for the existence of these two paths is as follows:

All information $h(t)$ that can be potentially accessed in the process either from the output $y(t)$ or the states $x(t)$, is collected by the measuring equipment and is transferred to the input of the comparator. This information $h(t)$, is then compared with a reference signal $r(t)$ that describes the desired behavior of the process. This comparison takes place within the comparator unit, the output of which is the error $e(t) = r(t) - h(t)$. This error enters the controller unit, passes through the actuator, and finally enters the input $u(t)$ of the process. The goal of $u(t)$ is to make the output of the process $y(t)$ track perfectly the reference signal $r(t)$.

With respect to the above, it becomes apparent that the aforementioned goal has to be achieved by the controller unit, which basically, given the error $e(t)$ and the presence of any disturbance $d(t)$ entering the plant, tries to calculate the proper $u(t)$ command signal such that the output $y(t)$ tracks perfectly the reference signal $r(t)$. In principle and as previously mentioned, both *perfect* disturbance rejection and *perfect* tracking of the reference at the same time cannot take place. Therefore, the design of a control action has to take always into account this compromise and deliver this command signal to the plant, which satisfies certain constraints according to the application.

In many industry applications, a control engineer sets as a first priority to design such a control unit able for fast suppression of disturbances. The reason for this is due to the nature of these signals which often enter the control loop suddenly and without any prediction. Thus, tracking of the reference signal is set as a second priority for the control action's design, since $r(t)$ does not change frequently while its value is known a priori before setting the control loop into operation.

Finally, once disturbance rejection is achieved for improving reference tracking, many are the times when the reference signal is filtered to avoid high overshoot at step changes of $r(t)$, see Fig. 2.2. This control scheme is also known in the literature as a two degree of freedom controller (2DoF).

2.2 Frequency Domain Modeling

In this section, we refer to the closed-loop control system presented in Fig. 2.3 where $G(s)$, $C(s)$ stand for the process and the controller transfer functions, respectively. Output of the control loop is defined as $y(s)$ and k_h stands for the feedback path for the output $y(s)$.

Signal $r(s)$ is the reference input to the control loop, $d_\mathrm{o}(s)$ and $d_\mathrm{i}(s)$ are the output and input disturbance signals, respectively, and $n_\mathrm{r}(s)$, $n_\mathrm{o}(s)$ are the noise signals at the reference input and the process output, respectively. Finally, k_p stands for the plant's dc gain at steady state.[3]

- **Closed-loop transfer function**

$$T(s) = \frac{y(s)}{r(s)} = \frac{F_\mathrm{fp}(s)}{1 + F_\mathrm{ol}(s)}, \tag{2.1}$$

where

$$F_\mathrm{fp}(s) = \frac{y(s)}{e(s)} = k_\mathrm{p}C(s)G(s), \tag{2.2}$$

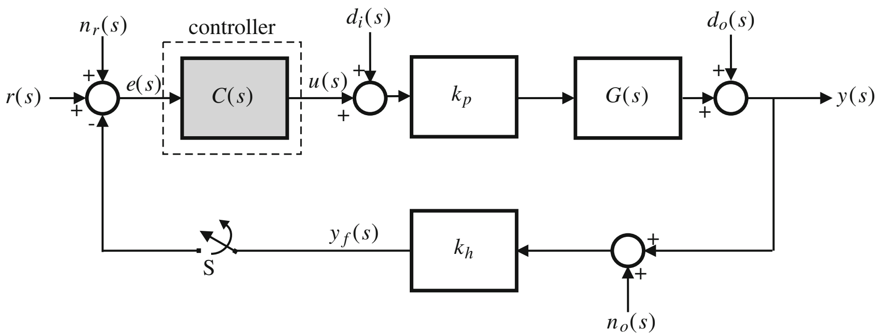

Fig. 2.3 Block diagram of the closed-loop control system. $G(s)$ is the plant transfer function, $C(s)$ is the controller transfer function, $r(s)$ is the reference signal, $y(s)$ is the output of the control loop, $y_\mathrm{f}(s)$ is the output signal after k_h, $d_\mathrm{o}(s)$ and $d_\mathrm{i}(s)$ are the output and input disturbance signals, respectively, and $n_\mathrm{r}(s)$ and $n_\mathrm{o}(s)$ are the noise signals at the reference input and process output, respectively. k_p stands for the plant's dc gain and k_h is the feedback path

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[3] In the case of electric motor drives, for example, k_p stands for the proportional gain introduced by the power electronics circuit, which finally applies the command signal $u(t)$ to the plant which in this case is the electric motor. For voltage source inverters, the command signal $u(t)$ is voltage. In the sequel it is explained that the gain introduced by the actuator has to be linear and proportional so that the command signal of the controller remains unaltered. In the specific case of electric motor drives, the power part introduces also a time delay with time constant T_d which corresponds to the time the controller decides the command $u(t)$, until the time it is finally applied by the power electronic circuit. Therefore in this case, the model of the actuator is given as $k_\mathrm{p}\mathrm{e}^{-sT_\mathrm{d}}$

is the *forward path transfer function* and

$$F_{\mathrm{ol}}(s) = \frac{y_{\mathrm{f}}(s)}{e(s)} = k_{\mathrm{h}} k_{\mathrm{p}} C(s) G(s), \tag{2.3}$$

is the *open-loop transfer function*.

- **Output sensitivity or sensitivity function**

$$S(s) = \frac{y_{d_{\mathrm{o}}}(s)}{d_{\mathrm{o}}(s)} = \frac{1}{1 + F_{\mathrm{ol}}(s)}, \tag{2.4}$$

which expresses the variation of the output $y_{d_{\mathrm{o}}}(s)$, in the presence of output disturbance $d_{\mathrm{o}}(s)$.

- **Input sensitivity function**

$$S_{\mathrm{i}}(s) = \frac{y_{d_{\mathrm{i}}}(s)}{d_{\mathrm{i}}(s)} = \frac{k_{\mathrm{p}} G(s)}{1 + F_{\mathrm{ol}}(s)} = k_{\mathrm{p}} G(s) S(s), \tag{2.5}$$

which expresses the variation of the output $y_{d_{\mathrm{i}}}(s)$, in the presence of input disturbance $d_{\mathrm{i}}(s)$.

- **Control (command) signal sensitivity function**

$$S_u(s) = \frac{u(s)}{d_{\mathrm{o}}(s)} = -\frac{k_{\mathrm{h}} C(s)}{1 + F_{\mathrm{ol}}(s)} = -k_{\mathrm{h}} C(s) S(s), \tag{2.6}$$

which expresses the variation of the command signal $u(s)$ of the controller in the presence of output disturbance $d_{\mathrm{o}}(s)$.

In general, if we consider that all inputs of the control loop are acting at the same time, then after applying the theorem of superposition among (2.1), (2.4), and (2.5), it becomes apparent that the output of the control loop is determined as

$$y(s) = T(s)[r(s) + n_{\mathrm{r}}(s) - k_{\mathrm{h}} n_{\mathrm{o}}(s)] + S(s)[d_{\mathrm{o}}(s) + k_{\mathrm{p}} G(s) d_{\mathrm{i}}(s)]. \tag{2.7}$$

2.3 Internal Stability

The problem of stability in a control loop is considered of highest priority in many real-world applications. Loss of stability in an industrial plant may lead often to damage of expensive components or even to loss of human life. Therefore, control engineers are often willing and determined to spend much effort on designing stable control loops so that the aforementioned cases are avoided.

A classic reference that remains modern till date, is the paper by Gunter Stein, "Respect the Unstable", which describes accurately the importance of stability in modern control systems, see [2].

Definition 1 Any closed-loop control system is said to be internally stable if for any bounded signal entering the control loop, all other generated responses (states, output) remain bounded.

Definition 2 A linear time-invariant system (LTI) is said to be internally stable, if and only if, every transfer function from whichever input to whichever output within the control loop is stable. In other words, every transfer function from whichever input to whichever output within the control loop must introduce poles only in the left-half plane (LHP).

From the control loop structure presented in Fig. 2.3, it is seen that the difference between the reference signal $r(s)$ and the output of the control loop $y(s)$ is expressed by the error signal $e(s)$, because $e(s) = r(s) - y(s)$. Since $r(s)$ is bounded and $r(s) = e(s) + y(s)$, for checking the internal stability of the control loop, it is sufficient to track either the response of the output signal $y(s)$ or the error signal $e(s)$. Assuming a stable controller design of $C(s)$ it is apparent that $u(s)$ is also stable, since $u(s) = C(s)e(s)$. As a result, for checking the internal stability of the control loop, it is again sufficient to track either the response of the output signal $y(s)$ or the controller's command signal $u(s)$ in the presence of the bounded signal $r(s)$.

The same investigation has to take place also for the affect of the disturbance signals $d(s)$ which enter the control loop either on the input $d_{\mathrm{i}}(s)$ or the output $d_{\mathrm{o}}(s)$ of the process. Therefore, it is necessary to investigate the effect of the signals $d_{\mathrm{i}}(s)$ or $d_{\mathrm{o}}(s)$ on the response of $u(s)$, since both $d_{\mathrm{i}}(s)$ and $d_{\mathrm{o}}(s)$ are bounded.

For investigating the way how signals $y(s)$, $u(s)$ are affected in the presence of the reference signal $r(s)$ and disturbance $d(s)$ ($d_{\mathrm{i}}(s)$ or $d_{\mathrm{i}}(s)$), the internal stability matrix of (2.8) is introduced

$$\begin{bmatrix} y(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} T(s) & k_{\mathrm{p}}G(s)S(s) \\ C(s)S(s) & -k_{\mathrm{h}}T(s) \end{bmatrix} \begin{bmatrix} r(s) \\ d_{\mathrm{i}}(s) \end{bmatrix}. \tag{2.8}$$

From (2.8), it is concluded that internal stability for the control loop of Fig. 2.3 is guaranteed only if each one of the transfer functions $T(s)$, $S_{\mathrm{i}}(s)$, $S_u(s)$ is stable. For the definition of $T(s)$, $S_{\mathrm{i}}(s)$, $S_u(s)$ see accordingly (2.1), (2.5), and (2.6). After algebraic manipulation of (2.8), it is seen that

$$y(s) = r(s)T(s) + d_{\mathrm{i}}(s)k_{\mathrm{p}}G(s)S(s) \tag{2.9}$$

which is valid if we set in the general expression of $y(s)$ (2.7), $n_{\mathrm{r}}(s) = 0$, $n_{\mathrm{o}}(s) = 0$, and $d_{\mathrm{o}}(s) = 0$. Moreover, from (2.8) it is seen that

$$u(s) = r(s)C(s)S(s) - k_{\mathrm{h}}d_{\mathrm{i}}(s)T(s) \tag{2.10}$$

which is also valid. If $d_\mathrm{i}(s) = 0$ and assuming then that $r(s)$ is the only active input in the control loop, it is necessary to prove that $u(s) = r(s)C(s)S(s)$. This is proved from Fig. 2.3, since if $d_\mathrm{i} = d_\mathrm{o} = n_\mathrm{r}(s) = n_\mathrm{o}(s) = 0$ then $e(s) = r(s) - k_\mathrm{h}k_\mathrm{p}G(s)u(s)$

$$\frac{u(s)}{C(s)} = r(s) - k_\mathrm{h}k_\mathrm{p}G(s)u(s), \tag{2.11}$$

or

$$u(s) = r(s)C(s) - k_\mathrm{h}k_\mathrm{p}G(s)u(s)C(s), \tag{2.12}$$

or

$$u(s) = r(s)C(s) - u(s)k_\mathrm{h}k_\mathrm{p}G(s)C(s), \tag{2.13}$$

or finally

$$u(s)[1 + k_\mathrm{h}k_\mathrm{p}C(s)G(s)] = r(s)C(s). \tag{2.14}$$

From (2.14) and along with (2.4) it is apparent that $u(s)\frac{1}{S(s)} = r(s)C(s)$ or finally

$$u(s) = r(s)C(s)S(s). \tag{2.15}$$

In a similar fashion, it can be proved that $u(s) = -d_\mathrm{i}(s)k_\mathrm{h}T(s)$ assuming all other inputs within the control loop are set to zero.

From Fig. 2.3 it is obvious that

$$u(s) + d_\mathrm{i}(s) = -\frac{u(s)}{k_\mathrm{p}k_\mathrm{h}C(s)G(s)} \tag{2.16}$$

or

$$u(s) + \frac{u(s)}{k_\mathrm{p}k_\mathrm{h}C(s)G(s)} + d_\mathrm{i}(s) = 0. \tag{2.17}$$

From (2.17) it is seen that

$$u(s)\left[\frac{1}{k_\mathrm{h}}\left(\frac{1 + k_\mathrm{p}k_\mathrm{h}C(s)G(s)}{k_\mathrm{p}C(s)G(s)}\right)\right] + d_\mathrm{i}(s) = 0 \tag{2.18}$$

or finally along with the use of (2.1)

$$u(s)\frac{1}{k_\mathrm{h}T(s)} + d_\mathrm{i}(s) = 0, \tag{2.19}$$

which is equal to

$$u(s) = -k_\mathrm{h}d_\mathrm{i}(s)T(s). \tag{2.20}$$

2.4 Robustness

Robust performance is of primary importance when designing a control law. In other words, it is related to the ability of the controller to deliver the necessary command signal to the plant, which both makes the plant achieve perfect tracking of the reference along with satisfactory disturbance rejection and regardless of the changes that might take place within the process during its operation.

For measuring robustness, the functions of sensitivity and complementary sensitivity are introduced. The sensitivity function for two functions F, S is given as

$$S_G^F(s) = \frac{\mathrm{d}F/F}{\mathrm{d}G/G} = \frac{G}{F}\frac{\mathrm{d}F}{\mathrm{d}G} \tag{2.21}$$

see [3]. By applying the aforementioned definition to the sensitivity of the closed-loop transfer function T with respect to changes in the transfer function of the process G see Fig. 2.3, results in

$$S_G^T(s) = \frac{G}{T}\frac{\mathrm{d}T}{\mathrm{d}G} = \frac{1}{1 + k_\mathrm{p}k_\mathrm{h}C(s)G(s)} = \frac{1}{1 + F_\mathrm{ol}(s)} = S(s). \tag{2.22}$$

Further to (2.22), by applying (2.21) to the sensitivity of the closed-loop transfer function T with respect to changes in the feedback path k_h, results in

$$S_{k_\mathrm{h}}^T(s) = \frac{k_\mathrm{h}}{T}\frac{\mathrm{d}T}{\mathrm{d}k_\mathrm{h}} = -\frac{k_\mathrm{p}k_\mathrm{h}C(s)G(s)}{1 + k_\mathrm{p}k_\mathrm{h}C(s)G(s)} = \frac{F_\mathrm{ol}(s)}{1 + F_\mathrm{ol}(s)}. \tag{2.23}$$

If the magnitude of the open-loop transfer function $|F_\mathrm{ol}(s)|$ is fairly high compared to unity ($|F_\mathrm{ol}(s)| \gg 1$) then (2.22) and (2.23) are transformed into

$$S_G^T(s) = \frac{G}{T}\frac{\mathrm{d}T}{\mathrm{d}G} \ll 1, \tag{2.24}$$

and

$$S_{k_\mathrm{h}}^T(s) = \frac{k_\mathrm{h}}{T}\frac{\mathrm{d}T}{\mathrm{d}k_\mathrm{h}} \approx 1. \tag{2.25}$$

Equation (2.24) reveals that possible changes on the model G of the process do not affect seriously the behavior of the closed-loop transfer function T and therefore of the closed-loop control system. Moreover, from (2.25), it is concluded that any variation that takes place in the feedback path k_h, is transferred directly and without any change to the output of the closed-loop control system T.

With respect to the above, it is apparent that the sensitivity of the units located in the forward path of the closed-loop control system is directly transmitted to the feedback path. As a result, when designing a closed-loop control system, extra care must be taken by the control engineer for the sensitivity of the feedback path. After summing up together (2.1) and (2.22), it is seen that

$$k_h T(s) + S(s) = 1. \tag{2.26}$$

Note at this point that (2.26) is the fundamental equation that connects the sensitivity S with the transfer function of the closed-loop control system T, via the feedback path k_h. In case of unity feedback systems $k_h = 1$, (2.26) is rewritten as follows:

$$T(s) + S(s) = 1, \tag{2.27}$$

which is considered as one more fundamental relation in a closed-loop control system, see [4–7, 9].

2.5 Type of Control Loop

Preliminary definitions regarding the type of control loop are given in this section. According to Fig. 2.3, the error $e(s)$ is defined by $e(s) = r(s) - y(s) = (1 - T(s)) r(s) = S(s)r(s)$. If the closed-loop transfer function $T(s) = \frac{y(s)}{r(s)}$ from reference $r(s)$ to output $y(s)$ while all other inputs in the control loop are assumed zero is defined as

$$T(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \tag{2.28}$$

then the resulting error $e(s)$ is given as

$$e(s) = \left(\frac{a_n s^n + \cdots + c_m s^m + \cdots + c_1 s + c_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \right) r(s) \tag{2.29}$$

where $c_j = (a_j - b_j)$ $(j = 0 \ldots m)$. According to the final value theorem and if $e(s)$ is stable, $e(\infty)$ is equal to

$$e(\infty) = \lim_{s \to 0} s \left(\frac{a_n s^n + \cdots + c_2 s^2 + c_1 s + c_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \right) r(s). \tag{2.30}$$

If $r(s) = \frac{1}{s}$ then

$$e(\infty) = \lim_{s \to 0} \left(\frac{c_0}{a_0} \right) \tag{2.31}$$

which becomes zero when $c_0 = 0$ or when $a_0 = b_0$. Hence, sensitivity $S(s) = \frac{y(s)}{d_o(s)}$ [4] and closed-loop transfer function $T(s)$ are defined as

[4] $S(s)$ stands for the sensitivity of the closed-loop control system and is defined as $S(s) = \frac{y(s)}{d_o(s)}$ when $r(s) = n_r(s) = d_i(s) = n_r(s) = 0$.

$$T(s) = \frac{s^m b_m + \cdots + s^2 b_2 + s b_1 + a_0}{s^n a_n + \cdots + s^2 a_2 + s a_1 + a_0}, \tag{2.32}$$

$$S(s) = s \frac{\begin{pmatrix} a_n s^{n-1} + a_{n-1} s^{n-2} + \cdots + (a_m - b_m) s^{m-1} \\ + (a_{m-1} - b_{m-1}) s^{m-2} + s(a_2 - b_2) + a_1 - b_1 \end{pmatrix}}{s^n a_n + s^{n-1} a_{n-1} + \cdots + s^2 a_2 + s a_1 + a_0}, \tag{2.33}$$

respectively. If (2.32) and (2.33) hold by the closed-loop control system is said to be of type-I. In a similar fashion, if $r(s) = \frac{1}{s^2}$ then the velocity error is equal to

$$e(\infty) = \lim_{s \to 0} \left(\frac{a_n s^n + \cdots + c_m s^m + \cdots + c_1 s + c_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \right) \frac{1}{s} \tag{2.34}$$

which becomes finite if $c_0 = 0$ or $a_0 = b_0$. As a result, the final value of the error is given as

$$\lim_{t \to \infty} e_{\text{vss}}(t) = \lim_{s \to 0} \left(\frac{c_1}{a_0} \right) = \lim_{s \to 0} \left(\frac{a_1 - b_1}{a_0} \right) \tag{2.35}$$

and becomes zero when $c_1 = 0$ or when $a_1 = b_1$. In this case, the closed-loop control system is said to be of type-II.[5] Sensitivity $S(s)$ and closed-loop transfer function $T(s)$ take the following form, respectively:

$$T(s) = \frac{s^m b_m + s^{m-1} b_{m-1} + \cdots + s a_1 + a_0}{s^n a_n + s^{n-1} a_{n-1} + \cdots + s a_1 + a_0}, \tag{2.36}$$

$$S(s) = s^2 \frac{a_n s^{n-2} - \cdots - b_m s^{m-2} - b_{m-1} s^{m-3} + \cdots + a_2 - b_2}{s^n a_n + s^{n-1} a_{n-1} + \cdots + s^2 a_2 + s a_1 + a_0}. \tag{2.37}$$

According to the above analysis, a closed-loop control system is said to be of type-p when sensitivity $S(s)$ and complementary sensitivity $T(s)$ have the following form:

$$S(s) = s^p \frac{\begin{pmatrix} a_n s^{n-p} + a_{n-1} s^{n-1-p} - \cdots - b_m s^{m-p} \\ - b_{m-1} s^{m-1-p} + a_p - b_p \end{pmatrix}}{s^n a_n + s^{n-1} a_{n-1} + \cdots + s^2 a_2 + s a_1 + a_0} \tag{2.38}$$

[5] In grid-connected power converters and when vector control is followed for regulating the DC link voltage to be utilized by the motor connected converter, there is one inner loop for regulating the current of the power converter and one outer loop for regulating its DC link voltage. In this case, the inner current control loop is of type-I, since in its open-loop transfer function there exists only one integrator coming from the current PI control action, whereas the outer control loop is of type-II, since the open-loop transfer function introduces two integrators, one coming from the DC link voltage PI control action and another coming from the capacitor bank path ($\frac{1}{sC}$). A case of type-II control loop in the field of electric motor drives is the speed control loop in vector-controlled or direct torque-controlled drives. In this case, one integrator comes from the speed PI control action and another integrator comes from the inertia ($\frac{1}{sJ}$) of the shaft of the motor the speed of which is controlled.

and

$$T(s) = \frac{b_m s^m + \cdots + a_p s^p + a_{p-1} s^{p-1} + \cdots + a_1 s + a_0}{a_n s^n + \cdots + a_p s^p + a_{p-1} s^{p-1} + \cdots + a_1 s + a_0}, \tag{2.39}$$

respectively. Also, one could argue according, to (2.38), that type-p control loops are characterized by the order of zeros at $s = 0$ in the sensitivity function $S(s)$, see (2.33), (2.37) and (2.38). In a similar fashion, the type of the control loop is automatically defined by the closed-loop transfer function $T(s)$ when observing the terms of s^j $(j = 0 \ldots p-1)$ both in the numerator and the denominator's polynomial.

2.6 Sensitivity and Complementary Sensitivity Function

The calculation of the magnitude of (2.27) results in

$$|T(s) + S(s)| = 1. \tag{2.40}$$

Ideally, in a closed-loop control system it is necessary to have the magnitude of S sufficient small, or in other words

$$|S(s)| \ll 1, \tag{2.41}$$

so that optimal disturbance rejection is achieved. However, perfect tracking of the reference signal $r(s)$ by the output $y(s)$ of the control loop requires also that

$$|T(s)| \approx 1. \tag{2.42}$$

At this point, it would be necessary to recall that relation

$$y(s) = T(s)[r(s) + n_{\mathrm{r}}(s) - k_{\mathrm{h}} n_{\mathrm{o}}(s)] + S(s)[d_{\mathrm{o}}(s) + k_{\mathrm{p}} G(s) d_{\mathrm{i}}(s)] \tag{2.43}$$

holds by within the closed-loop control system of Fig. 2.3. From (2.43), it is apparent that if sensitivity S is large enough, any disturbance signal ($d_{\mathrm{o}}(s)$ or $d_{\mathrm{i}}(s)$) entering the control loop is amplified, and as a result the output of the control loop $y(s)$ can hardly track the reference signal $r(s)$. To this end, the main problem which a control engineer faces when designing an output feedback control loop, is that in such a system, it is impossible to have perfect tracking of the reference signal $r(s)$ along with optimal disturbance and noise rejection at the same time.

Looking further on this statement, one can claim that the aforementioned conclusion is not 100 % correct, if we consider the frequency spectrum of both the noise and disturbance signals that enter the control loop. Often in many real-time applications, the reference signal $r(s)$ along with disturbances $d_{\mathrm{o}}(s)$ (i.e., load disturbance in electric motor drives operation) that appear at the output of the process, are signals of low frequency. By contrast, noise signals come basically by measuring equipment and most of the time contain high-frequency components.

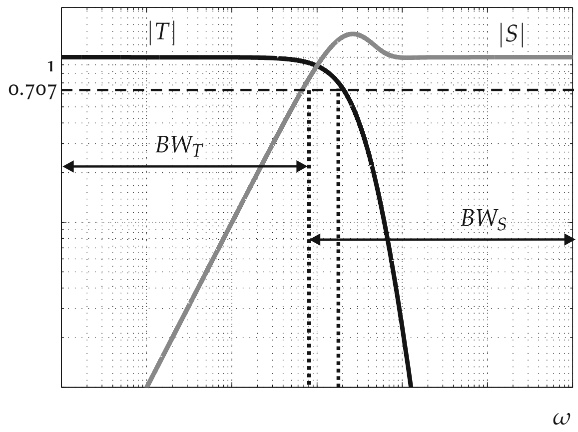

Fig. 2.4 Typical frequency response of sensitivity S and complementary T

Taking into account these facts, it becomes apparent that if the magnitude of T remains equal to unity in the widest possible frequency range, then complementary sensitivity is low enough, see Fig. 2.4 and therefore low-frequency disturbances are not amplified by low sensitivity S in the low-frequency region. As a result, satisfactory tracking of the reference can be achieved while disturbances are suppressed.

On the other hand, since noise signals appear in the higher frequency region, they cannot be amplified by the low complementary sensitivity T since it is close to zero in the high-frequency region; see Fig. 2.4. Finally, no disturbances can be amplified by the high magnitude of the complementary sensitivity S, since they do not exist in this high-frequency region.

However, it has to be pointed out that a high magnitude of T does not necessarily mean that the magnitude of S is low, since the relation (2.40) between T, S is a relation between vectors, see also Fig. 2.5. The aforementioned statement is true only in the

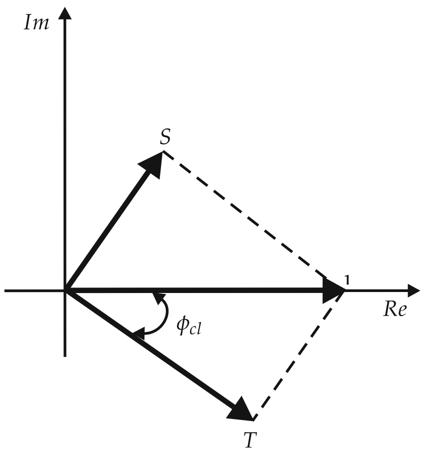

Fig. 2.5 Geometric interpretation of $|T(s) + S(s)| = 1$ in the complex plane

case where the angle ϕ_{cl} of T is very low. As a result, it becomes apparent that optimal disturbance rejection along with perfect reference tracking can be achieved only when

$$T(j\omega) \approx 1\angle 0^{\circ}. \tag{2.44}$$

Since practically this kind of design cannot be achieved, control engineers have to design control loops such that the frequency response of the closed-loop control system does not exhibit any resonance all over the low- and high-frequency regions.

2.7 The Magnitude Optimum Design Criterion

Further to the requirements defined by (2.42) and (2.44) in a closed-loop control system, in this section the principle of the Magnitude Optimum criterion is introduced. The target of the Magnitude Optimum (Betragsoptimum) criterion is to maintain the amplitude $|T(j\omega)|$ of the closed-loop frequency response equal to unity in the widest possible frequency range. This target can be mathematically expressed by

$$|T(j\omega)| \simeq 1. \tag{2.45}$$

The aforementioned equation can be considered as a practical implementation of the H_{∞} controller design principle, see [8], since as mentioned in Chap. 1, the H_{∞} design principle tries to optimize the amplitude of the closed-loop transfer function regardless of the resulting order of the controller. For this reason, most often times, the order of the controller of such a solution is so high that it makes its practical implementation unattractive or even sometimes unfeasible.

Back to Fig. 2.2 again, it is assumed that the transfer function of the closed-loop control system is given as

$$T(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_2 s^2 + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0} \quad (n \geq m). \tag{2.46}$$

The H_{∞} controller design principle can be mathematically described as

$$H_{\infty} = \min \left[\lim_{n \to \infty} \sqrt[n]{\int_0^{\infty} |T(j\omega)|^n \, \mathrm{d}\omega} \right]. \tag{2.47}$$

A typical frequency response of $|T(j\omega)|$ involving its maximum $T_{\max}$ at a certain resonance frequency is presented in Fig. 2.6a. In this case, a good approximation of the area of the frequency response $|T(j\omega)|$ is given as

$$E = T_{\max} \Delta\omega. \tag{2.48}$$

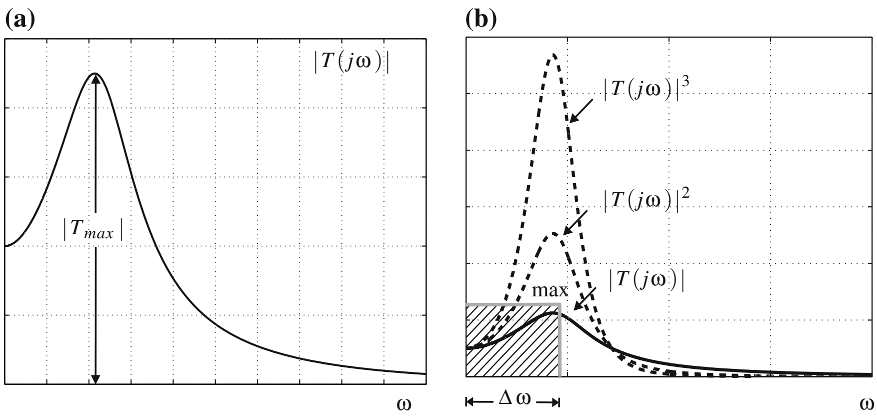

Fig. 2.6 **a** Frequency response of $|T(j\omega)|$ with resonant peak $T_{\max}$. **b** Frequency response of $|T(j\omega)|^n$ for various values of parameter n, $n = 1, 2, \ldots$

In a similar way, for calculating the surface of $|T\ (j\omega)|^n$, we can rewrite according to

$$E = T^n_{\max}\Delta\omega. \tag{2.49}$$

For calculating the surface of $|T(j\omega)|^n$, we can also write

$$E = \int_0^\infty |T(j\omega)|^n \,\mathrm{d}\omega. \tag{2.50}$$

Note that (2.49) is equal to (2.50) in case where n becomes sufficiently high and the term $\Delta\omega$ becomes sufficiently small. Strictly speaking, (2.49) is equal to (2.50) when $n \to \infty$ and $\Delta\omega \to 0$. For this reason, after taking the lim of both (2.49) and (2.50) when $n \to \infty$, we can rewrite

$$\lim_{n\to\infty} \int_0^\infty |T(j\omega)|^n \,\mathrm{d}\omega = \lim_{n\to\infty} (\Delta\omega T^n_{\max}). \tag{2.51}$$

The algebraic manipulation of (2.51) results in

$$\lim_{n\to\infty} \sqrt[n]{\int_0^\infty |T(j\omega)|^n \mathrm{d}\omega} = \lim_{n\to\infty} \left(T_{\max}\sqrt[n]{\Delta\omega}\right). \tag{2.52}$$

From (2.52) it is obvious that, if $n \to \infty$ then

$$\lim_{n\to\infty} \sqrt[n]{\Delta\omega} = 1 \quad \forall\Delta\omega. \tag{2.53}$$

Substituting (2.53) into (2.52) results in

$$\lim_{n\to\infty} \sqrt[n]{\int_0^\infty |T(j\omega)|^n \, \mathrm{d}\omega} = \lim_{n\to\infty} (T_{\max}) . \tag{2.54}$$

Therefore, for minimizing (2.54), we can rewrite

$$H_\infty = \min\left(\lim_{n\to\infty} \sqrt[n]{\int_0^\infty |T(j\omega)|^n \, \mathrm{d}\omega} \right) = \min\left(\lim_{n\to\infty} (T_{\max}) \right) . \tag{2.55}$$

For this, we have to invent a systematic approach that satisfies the condition

$$H_\infty = \min(T_{\max}). \tag{2.56}$$

Such a systematic strict mathematical approach of optimizing (2.56) can be found in [10] the final result of which is graphically depicted in Fig. 2.7.

Since in this book, the goal is to present tuning rules for the PID controller which is often met and applicable in the majority of industrial applications see [1], a less strict mathematical optimization is presented for forcing the magnitude of the closed-loop frequency response $|T(j\omega)|$ equal to unity in the widest possible frequency range.

From (2.46) it becomes apparent that if we substitute $s = j\omega$ results in

$$|T(j\omega)| = \left| \frac{N(j\omega)}{D(j\omega)} \right| . \tag{2.57}$$

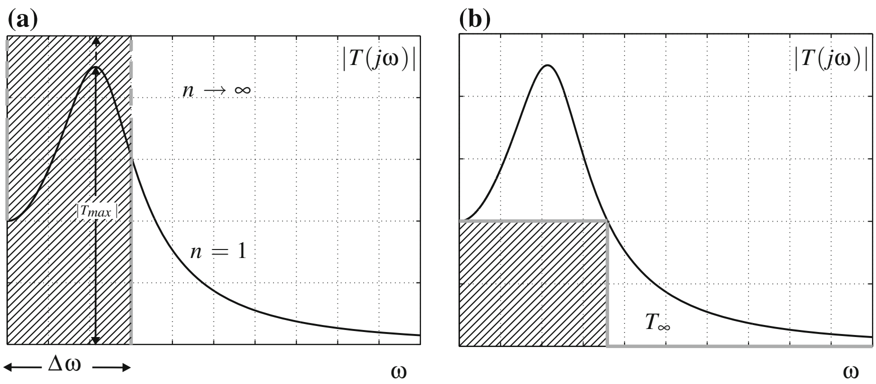

Fig. 2.7 **a** Frequency response of $|T(j\omega)|^n$ for various values of parameter n and when $n \to \infty$. **b** Desired frequency response of $T(j\omega)$ after minimization of any resonant peak at any resonance frequency

Calculating $|N(j\omega)|^2$, $|D(j\omega)|^2$, results in

$$|N(j\omega)|^2 \simeq \cdots + B_8\omega^{16} + B_7\omega^{14} + B_6\omega^{12} + B_5\omega^{10} \tag{2.58}$$
$$+ B_4\omega^8 + B_3\omega^6 + B_2\omega^4 + B_1\omega^2 + B_0 \tag{2.59}$$

$$|D(j\omega)|^2 \simeq \cdots + A_8\omega^{16} + A_7\omega^{14} + A_6\omega^{12} + A_5\omega^{10} \tag{2.60}$$
$$+ A_4\omega^8 + A_3\omega^6 + A_2\omega^4 + A_1\omega^2 + A_0\,, \tag{2.61}$$

respectively. For forcing $|T(j\omega)| \approx 1$ in the widest possible frequency range

$$A_i = B_j \quad \forall i, j \quad (i = 0, n)\ (j = 0, m) \tag{2.62}$$

must hold by. In Appendix A it is proved that for setting $A_i = B_j$, $\forall i, j$ $(i = 0, n)$ $(j = 0, m)$ results finally in

$$a_0 = b_0 \tag{2.63}$$
$$a_1^2 - 2a_2a_0 = b_1^2 - 2b_2b_0 \tag{2.64}$$
$$a_2^2 - 2a_3a_1 + 2a_4a_0 = b_2^2 - 2b_3b_1 + 2b_4b_0 \tag{2.65}$$
$$a_3^2 + 2a_1a_5 - 2a_6a_0 - 2a_4a_2 = b_3^2 + 2b_1b_5 - 2b_6b_0 - 2b_4b_2 \tag{2.66}$$
$$\begin{pmatrix} a_4^2 + 2a_0a_8 + 2a_6a_2 - 2a_1a_7 \\ -2a_3a_5 \end{pmatrix} = \begin{pmatrix} b_4^2 + 2b_0b_8 + 2b_6b_2 - 2b_1b_7 \\ -2b_3b_5 \end{pmatrix} \tag{2.67}$$

Equations (2.63)–(2.67) comprise the basis for the proof of every optimal control law for every type of control loop presented in the sequel within this book.

2.8 Summary

In this chapter, preliminary definitions of the operation of closed-loop control systems were presented in Sect. 2.1. It was shown how the problem of perfect reference tracking is in conflict with any kind of disturbance entering the control loop from the outer world. To justify this statement, in Sect. 2.2, the closed-loop control system was presented in a more concrete mathematical modeling by the frequency domain approach. With respect to this approach, basic transfer functions of the control loop were presented, which serve as proof of the proposed PID control law, which follows in the next chapters for any type-I, type-II, and type-III control loops.

Given the aforementioned necessary definitions regarding the transfer functions involved within a closed-loop control system, in Sects. 2.3 and 2.4 the important aspect of internal stability and robustness in a control loop were covered. In Sect. 2.5, a mathematical approach was presented relevant to the type of feedback control loop. This section aims at giving the reader quick hints on how to easily identify, given the closed-loop transfer function of a control system, its exact type (type-I, type-II, and type-III).

In Sect. 2.6 it was shown why it is important to keep the magnitude of the closed-loop control system equal to unity in the widest possible frequency range ($|T(j\omega)|$), since under certain circumstances this principle leads to satisfactory disturbance rejection both at the input and output of the process. This section is also the connecting ring to the principle of the Magnitude Optimum criterion which is finally presented in Sect. 2.7. The principle of the Magnitude Optimum criterion is considered as a practical aspect of the H_∞ and is used to deploy the proposed PID control laws presented in the following chapters. Finally, certain optimization conditions are derived in Sect. 2.7 which serve as the basis for the explicit definition of the PID control action irrespective of the process complexity.

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