

# Determination of $\Delta T$ and Lunar Tidal Acceleration from Ancient Eclipses and Occultations

Mitsuru Sôma and Kiyotaka Tanikawa

**Abstract** In order to investigate the variation of the Earth's rotation speed we have been using ancient solar eclipses and lunar occultations. We are also studying whether or not the Moon's tidal acceleration has been constant from ancient times.

In this paper we show that the records of solar eclipses between 198 and 181 BC in China and in Rome give a value for the lunar tidal acceleration that is consistent with the current one. We also show that the records of lunar occultations of Venus and Saturn in AD 503 and 513 in China are useful for our studies of the Earth's rotation.

## 1 Introduction

Stephenson (1997) published his monumental work on the investigation of changes in the Earth's rotation using pre-telescopic observations of eclipses between 763 BC and AD 1772. He determined the Earth's clock error,  $\Delta T$ , for each of the eclipses.  $\Delta T$  denotes the time difference  $TT - UT$ , where  $TT$  is Terrestrial Time, which is a uniform measure of time, and  $UT$  is Universal Time, which is determined by the rotation of the Earth. We have been trying to determine more precise values of  $\Delta T$  using contemporaneous solar eclipses. In addition we have been trying to investigate whether or not the Moon's tidal acceleration has been constant since ancient times. In this paper we explain how we are conducting our research. Note that the dates of ancient events given in this paper are indicated using the Julian Calendar.

## 2 Tidal Acceleration in the Moon's Motion

For the calculation of the Moon's position the quadratic term in the Moon's longitude plays a very important role. Its gravitational component due to planetary perturbations and the Earth's figure perturbations is well established, and according to Chapront

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M. Sôma (✉) • K. Tanikawa

National Astronomical Observatory of Japan, Mitaka, Tokyo 181-8588, Japan

e-mail: [mitsuru.soma@nao.ac.jp](mailto:mitsuru.soma@nao.ac.jp); [tanikawa.ky@nao.ac.jp](mailto:tanikawa.ky@nao.ac.jp)

et al. (2002) based on the analytical theory ELP2000-96 (Chapront and Chapront-Touzé 1997) it is  $6''.0590 T^2$ , where  $T$  is the time in centuries. It should be noted that the term just mentioned refers to the sidereal motion of the Moon and when we speak of the longitude of the Moon referred to the mean equinox of date, the precessional term should be added. According to Capitaine et al. (2003), the precessional quadratic term is  $1''.1054 T^2$ . Therefore, the quadratic term for the above-mentioned Moon's longitude referred to the mean equinox of date is  $7''.1644 T^2$ . It should be noted that Brown obtained a comparable value,  $7''.14 T^2$ , back in 1919.

There is another component of the quadratic term in the Moon's longitude. This is the tidal component due to the reaction on the lunar motion of the tides raised by the Moon on the Earth, which has to be determined from observations. When we speak of acceleration, the coefficient of the quadratic term in longitude has to be multiplied by 2, and we now denote the lunar tidal acceleration by  $\dot{n}$ . Table 1 lists values of  $\dot{n}$  obtained from past observations.

It should be noted that the large negative values for the lunar tidal acceleration obtained by Van Flandern (1970) and Morrison (1973) were largely due to deficiencies in the planetary terms in Brown's theory of the motion of the Moon (Sôma 1985). The large negative values obtained by Oesterwinter and Cohen (1972) and Van Flandern (1975) may be due to either the short duration of the observations or deficiencies in

**Table 1** Principal values of lunar tidal acceleration obtained so far

| Author(s)                   | Year | Tidal acceleration ( $''\text{cy}^2$ ) <sup>a</sup> |
|-----------------------------|------|-----------------------------------------------------|
| Spencer Jones               | 1939 | -22.44                                              |
| Van Flandern                | 1970 | -56.0                                               |
| Oesterwinter and Cohen      | 1972 | -38                                                 |
| Morrison                    | 1973 | -42.0                                               |
| Van Flandern                | 1975 | -65.0                                               |
| Morrison and Ward           | 1975 | -26.0                                               |
| Muller                      | 1976 | -30                                                 |
| Calame and Mulholland       | 1978 | -24.6                                               |
| Williams et al.             | 1978 | -23.8                                               |
| Ferrari et al.              | 1980 | -23.8                                               |
| Dickey et al.               | 1982 | -23.8                                               |
| Dickey and Williams         | 1982 | -25.12                                              |
| Newhall et al.              | 1988 | -24.90                                              |
| Dickey et al.               | 1994 | -25.88                                              |
| Chapront and Chapront-Touzé | 1997 | -25.64                                              |
| Chapront et al.             | 1999 | -25.78                                              |
| Chapront et al.             | 2000 | -25.836                                             |
| Chapront et al.             | 2002 | -25.858                                             |

<sup>a</sup>Note that these values are twice the coefficient of the  $T^2$  tidal term. The value listed for Spencer Jones (1939) was actually derived by Clemence (1948), and based on Spencer Jones' results

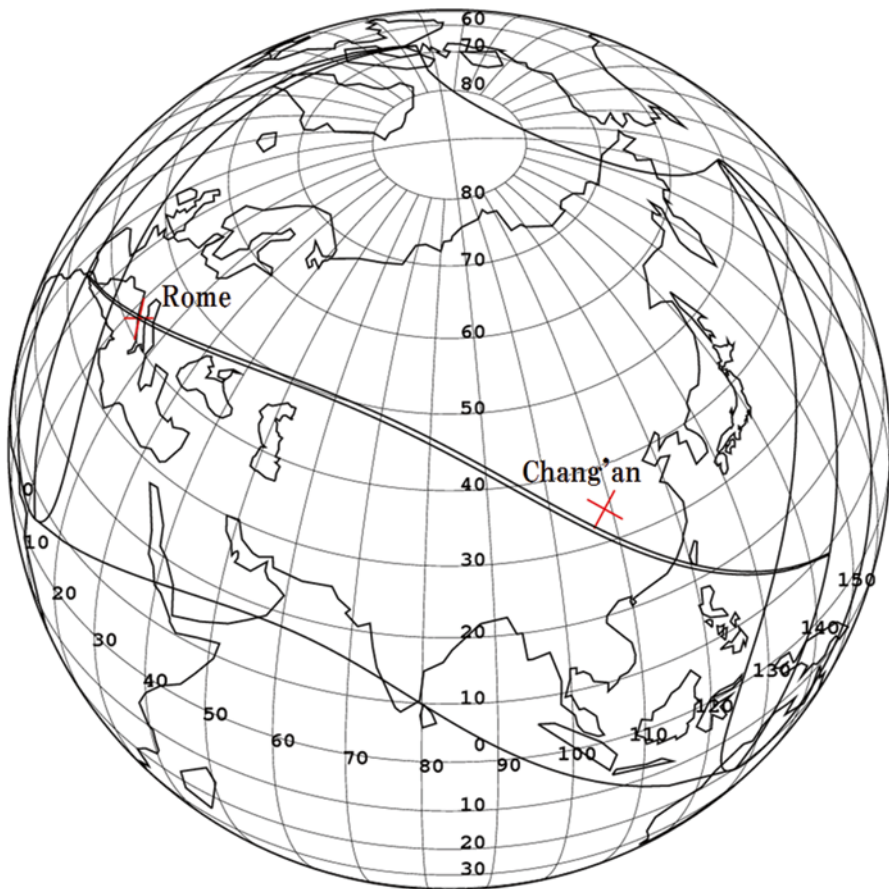
the lunar motion models, or both of these. The values obtained from the lunar laser ranging observations since the 1970s converge well at about  $-25''.9 \text{ cy}^{-2}$ .

The ephemeris of the Sun, Moon and planets that we used for our analyses in this paper was the JPL ephemeris DE406. DE406 covers the interval 3000 BC to AD 3000, and it is consistent with DE405 (Standish 1998), which is the ephemeris used for *The Astronomical Almanac* for 2003 through 2014 and covers the interval AD 1600 to 2200. According to Chapront et al. (2002), the lunar tidal acceleration intrinsic to DE405 and DE406 is  $-25''.826 \text{ cy}^{-2}$ , which agrees well with the recently-derived ones discussed above. The value  $-26'' \text{ cy}^{-2}$  derived by Morrison and Ward (1975) was obtained from comparison of lunar occultations and transits of Mercury since 1677, and the fact that this value is consistent with the recently-determined lunar tidal accelerations derived from the lunar laser ranging observations indicates that the Moon's tidal acceleration has been almost constant since the seventeenth century, but this is no guarantee that the lunar tidal acceleration has been constant since ancient times. Therefore it is important to investigate ancient records of astronomical phenomena and determine whether this value has been constant from ancient times. In the following two sections we will show how we can use data derived from ancient eclipses and occultations to derive values for the lunar tidal acceleration and  $\Delta T$ .

### 3 Solar Eclipses Between 198 BC and 181 BC

We have developed a method whereby the  $\Delta T$  values and the lunar tidal acceleration  $\dot{n}$  are simultaneously determined using records of contemporaneous solar eclipses and lunar occultations. The method is briefly described in Sôma et al. (2004), Tanikawa and Sôma (2004) and Sôma and Tanikawa (2005). Here we give an example of the method using ancient Chinese records of solar eclipses dating between 198 and 181 BC.

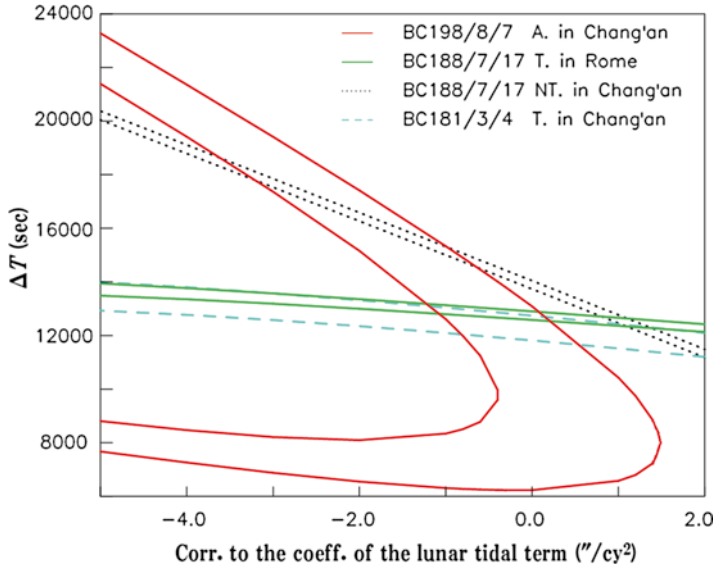
The great Roman historian Titus Livius (59 BC-AD 17), who is known as Livy in English, wrote about the solar eclipse on 17 July 188 BC as follows: "... darkness had fallen between roughly the third and fourth hours of daylight ..." (Yardley 2000: 392). From this we can assume that the eclipse was total in Rome. The same eclipse was also recorded as "... almost total ..." (幾既) in the *Hanshu* (漢書), which is the official history book of the Former Han (前漢, 西漢) Dynasty (202 BC-AD 8) of China. We can assume that the records in the *Hanshu* were based on events observed in Chang'an (長安), the capital of the Han Dynasty. Figure 1 shows the area on the Earth where the eclipse was seen, and the area where the total eclipse was visible is indicated by the narrow band crossing Rome. It was drawn using  $\Delta T = 12,600 \text{ s}$  and adopting the current value for the lunar tidal acceleration. By changing the value of  $\Delta T$  we can move the band either in an eastwards or a westwards direction. From this figure we can readily see that astronomers in Rome and Chang'an could not have witnessed the total eclipse simultaneously if we adopt the current value for the lunar tidal acceleration.



**Fig. 1** Solar eclipse on 17 July 188 BC. This gives the area on the Earth where the eclipse was seen, and the area where the total eclipse was seen is indicated by the *narrow band* crossing Rome. It was drawn using  $\Delta T = 12,600$  s and adopting the current lunar tidal acceleration

Two other solar eclipses within 10 years before and after the 188 BC solar eclipse were recorded in the *Hanshu*. One is the solar eclipse on 7 August 198 BC and the other is the solar eclipse on 4 March 181 BC. They are both recorded as “ji” (既). The ji character usually means a total solar eclipse, but it was also used for an annular eclipse. Modern calculations show that the 198 BC eclipse was annular and the 181 BC eclipse was total, and therefore from these records we can assume that the 198 BC eclipse was annular in Chang’an and the 181 BC eclipse was total, also in Chang’an.

In Fig. 2 we take as the abscissa the correction to the coefficient of the time-square term in the lunar longitude ( $\text{arcsec century}^{-2}$ ) or, in short, the correction to



**Fig. 2** Diagram showing the possible area of the two parameter values from the solar eclipses between 198 and 181 BC

the lunar tidal term, and take as the ordinate the value of  $\Delta T$ . We plot on this plane curves of the boundaries of possible values of the two parameters for events observed at known sites. As noted in Sect. 2, the correction to the lunar tidal acceleration is twice that of the coefficient of the time-square term in lunar longitude. The zero in the abscissa corresponds to the current lunar tidal acceleration  $-25''.826 \text{ cy}^{-2}$  intrinsic to the JPL ephemeris DE406. In this diagram the possible ranges of the correction to the lunar tidal term and the  $\Delta T$  value are obtained as the intersections of the plural bands. This figure indicates that the correction,  $\Delta \dot{n}$ , to the lunar tidal acceleration obtained from all of the eclipse records given above is

$$-2''.42 \text{cy}^{-2} < \Delta \dot{n} < 0''.34 \text{cy}^{-2},$$

which means that all of the eclipse records given above are consistently explained with the current lunar acceleration. If we adopt the current lunar tidal acceleration, we obtain  $\Delta T$  values of  $12,577 \text{ s} < \Delta T < 12,896 \text{ s}$  from the 188 BC eclipse in Rome, and  $11,761 \text{ s} < \Delta T < 12,688 \text{ s}$  from the 181 BC eclipse in Chang'an, and hence as a common range we obtain

$$12,577 \text{ s} < \Delta T < 12,688 \text{ s},$$

for the epoch around 188 BC.

This method of simultaneously determining the  $\Delta T$  and  $\dot{n}$  values is based on the fact that the  $\Delta T$  and  $\dot{n}$  values can be regarded as almost constant during a short

period of time. It should be noted that the  $\dot{n}$  value obtained above from the eclipses of around the year 188 BC is not the lunar tidal acceleration value at the times of the eclipses, but it is the mean lunar acceleration value from that period up to the present. The fact that the lunar tidal acceleration obtained above matches the current one does not necessarily mean that the lunar tidal acceleration has been constant during that period. Therefore, we need to investigate further the lunar tidal acceleration from ancient records of other periods.

## 4 Lunar Occultations and Solar Eclipses Between AD 503 and 522

Lunar occultations of planets and bright stars were often observed and recorded in ancient China, but visible areas of lunar occultations are usually wide, and therefore they rarely can be used to determine  $\Delta T$  values and lunar tidal acceleration. However, there are some occultation records which can play an important role for our studies. The lunar occultations of Venus on 5 August 503 and of Saturn on 22 August 513 recorded in the *Weishu* (魏書), which is the official history book of the Wei (魏) Dynasty (398–534) of China, are such examples because the former was observed just after the Moon rose in the east and the latter just before the Moon set in the west. Therefore, the combination of these can set limits to the Earth's rotation parameter. It should be noted that these two occultations were seen near the horizon and it is sometimes difficult to observe astronomical phenomena near the horizon because the brightness of stars is diminished by the Earth's atmosphere, but under favorable condition Venus and Saturn are bright enough to be observed near the horizon.

Figures 3 and 4 show the areas where the lunar occultation of Venus on 5 August 503 and that of Saturn on 22 August 513 were seen. As shown in Fig. 3, Luoyang (洛陽), the capital of the Wei Dynasty at the time, was near the western edge of the area where the occultation was seen, so the record gives an upper limit to the  $\Delta T$  value. On the other hand, Fig. 4 shows that Luoyang was near the eastern edge of the area where the occultation was seen, so this record gives a lower limit to the  $\Delta T$  value. By combining these two events we can determine limits to the  $\Delta T$  value.

In the same period there were two solar eclipses recorded in China. One was on 18 April 516 and was recorded in a volume titled *Liang Benji* (梁本紀) in the official history book *Nanshi* (南史), and the other was on 10 June 522 and was recorded in a volume called *Wudi Zhong* (武帝中) in an official history book *Liangshu* (梁書). Both were recorded as “ji” (既) in the descriptions of the Liang (梁) Dynasty (502–557). Modern calculations show that the eclipse in 516 was annular while the eclipse in 522 was total. Therefore we can assume that the annular solar eclipse in 516 and the total solar eclipse in 522 were both observed in Jiankang (建康), the capital of the Liang Dynasty at that time. For the areas of visibility for these eclipses see Figs. 5 and 6.



**Fig. 3** Lunar occultation of Venus on 5 August 503. This shows the area where the occultation was seen. The area between the *solid lines* saw the occultation at night while the area between the *broken lines* saw it in the day-time. The area within the *dotted lines* saw either the disappearance or the reappearance at night. The cross indicates the location of the capital, Luoyang, of Wei at the time when the observations were assumed to have been made. The figure was drawn assuming  $\Delta T = 4,500$  sec and the current lunar tidal acceleration

During the 20 years spanning the above-mentioned events the variation in  $\Delta T$  should be small and we can assume almost the same  $\Delta T$  value for all four events. Therefore we also can investigate whether those events can be explained with the current lunar tidal acceleration.

Figure 7 shows the possible areas of the parameter values of  $\Delta T$  and the coefficient of lunar tidal term for the events discussed above. As shown in this figure, the possible common area of the parameter values for the two eclipses in 516 and 522 is included in the possible common area for the occultations in 503 and 513. Therefore we can confirm that the records of these two eclipses are highly reliable.

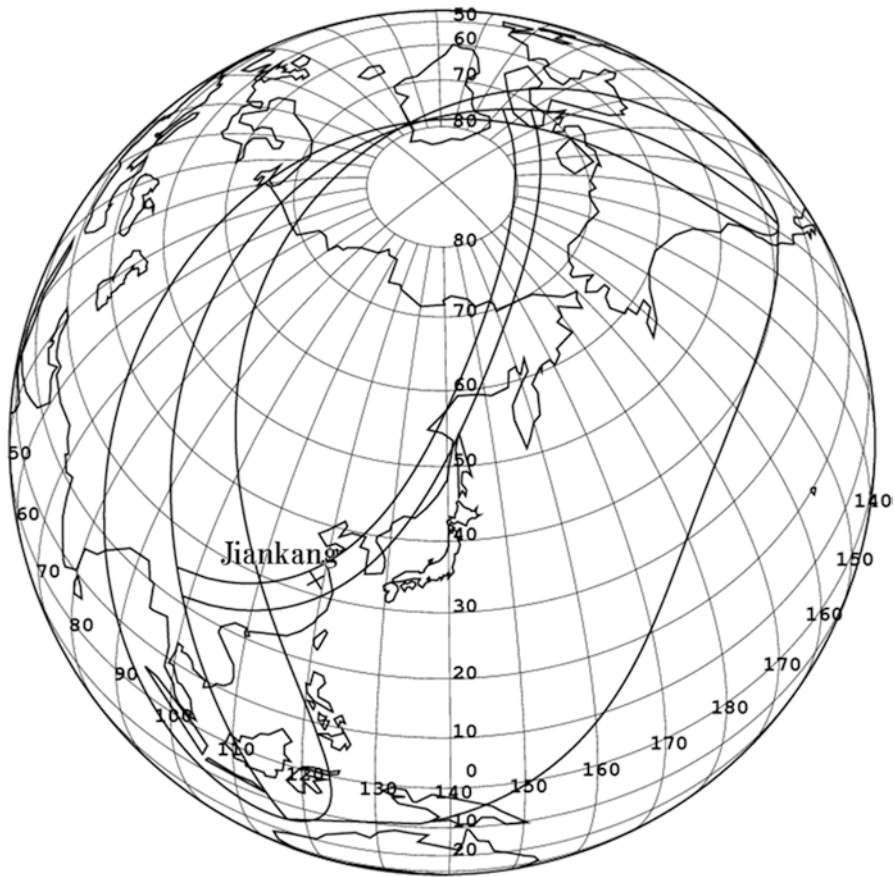


**Fig. 4** Lunar occultation of Saturn on 22 August 513. The explanation for Fig. 3 also applies to this figure

Assuming the current value for the lunar tidal acceleration, we obtain  $\Delta T$  values of  $3,573 \text{ s} < \Delta T < 5,195 \text{ s}$  from the 516 eclipse in Jiankang, and  $3,469 \text{ s} < \Delta T < 5,093 \text{ s}$  from the 522 eclipse also in Jiankang, and hence as a common range we obtain

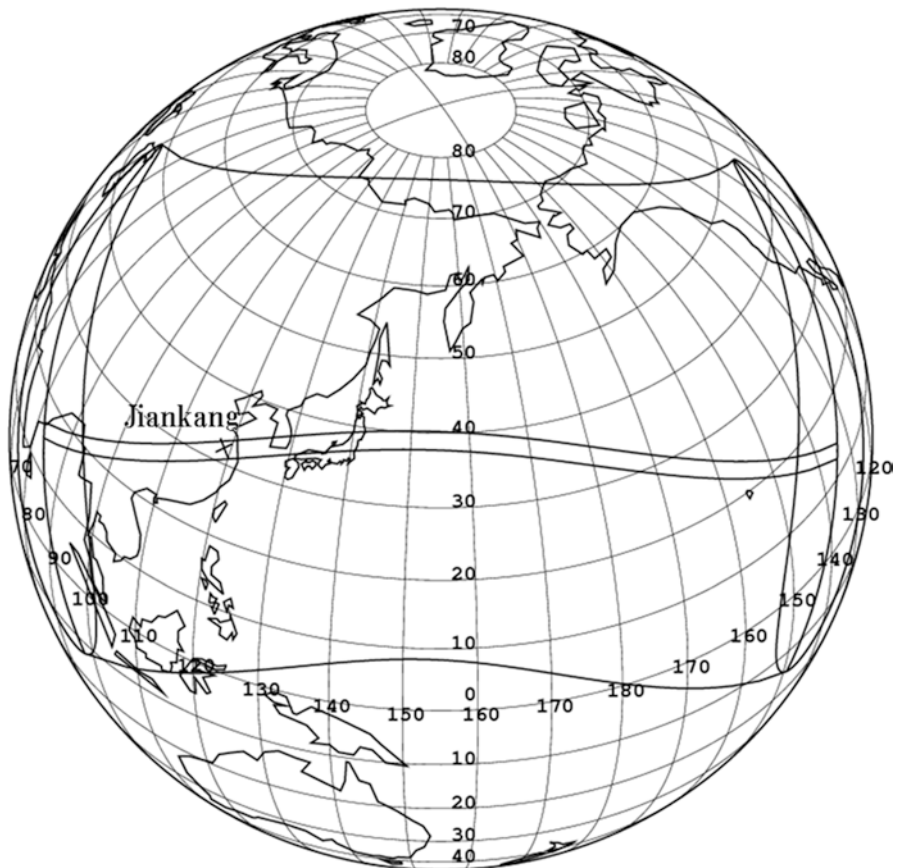
$$3,573 \text{ s} < \Delta T < 5,093 \text{ s}$$

for the epoch around the year 520. On the other hand Stephenson (1997) gave  $\Delta T = 5,500 \text{ s}$  for the epoch around the year 520. However, this value was mainly determined by the results obtained from three eclipses: the total solar eclipse on 10 August 454 in Jiankang (Stephenson 1997: 242–243; the resulting  $\Delta T$  range was  $6,130 \text{ s} < \Delta T < 7,900 \text{ s}$ ); the rising eclipsed Moon on 24 May 514 in Luoyang



**Fig. 5** Annular solar eclipse on 18 April 516. Those within the *narrow band* saw the annular solar eclipse. The *cross* indicates the location of the capital, Jiankang, of Liang at the time when the observations were assumed to have been made. The figure was drawn assuming  $\Delta T = 4,500$  sec and the current lunar tidal acceleration

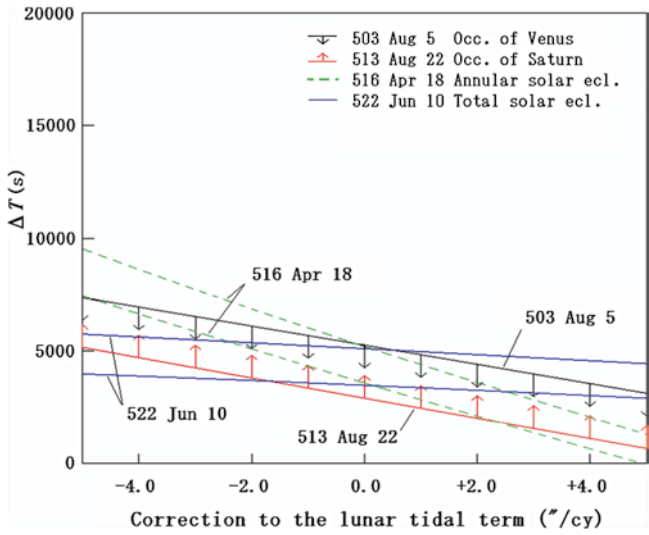
(Stephenson 1997: 320–321; the resulting  $\Delta T$  range was  $-250 \text{ s} < \Delta T < 5,500 \text{ s}$ ); and the total solar eclipse on 5 August 761 in Chang’an (Stephenson 1997: 247–248; the resulting  $\Delta T$  range was  $1,720 \text{ s} < \Delta T < 3,290 \text{ s}$ ). A part of Stephenson’s Fig. 14.4 is reproduced here in Fig. 8 and our result is plotted on it. The solid line is the cubic spline curve adopted by Stephenson to represent  $\Delta T$ . From the figure we can see that although our result misses the  $\Delta T$  curve he drew, it does not contradict any of the records used by him. In fact if we assume that  $\Delta T$  decreased a little more rapidly between the years 454 and 516, we can draw a line for  $\Delta T$  which satisfies all the data used by Stephenson and our new result.



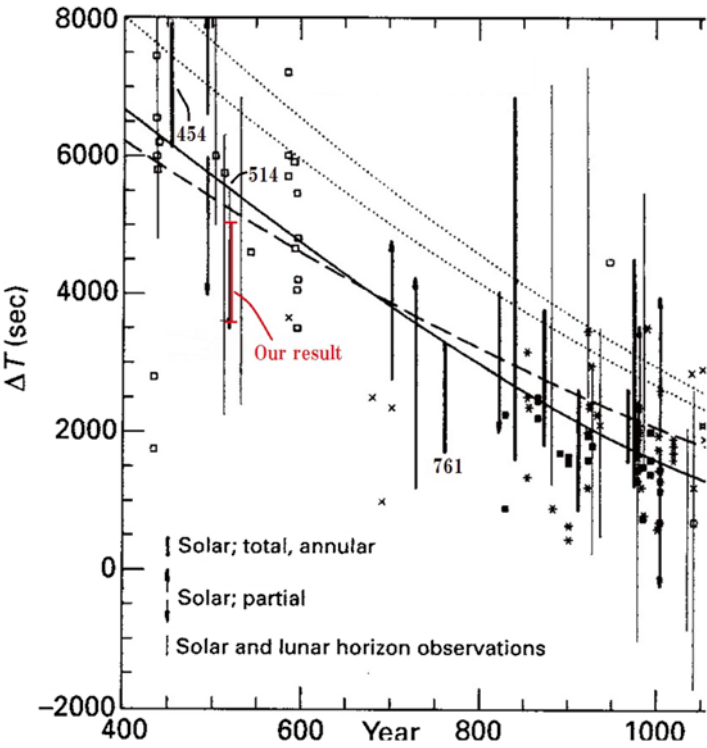
**Fig. 6** Total solar eclipse on 10 June 522. Those within the *narrow band* saw the total solar eclipse. For the cross, the  $\Delta T$  value and the lunar tidal acceleration the same explanation applies as in Fig. 5

## 5 Conclusion

We have been investigating  $\Delta T$  and lunar tidal acceleration by depicting possible areas of them on a diagram using contemporaneous plural eclipses and occultations. We have shown that the records of solar eclipses between 198 and 181 BC and of solar eclipses and lunar occultations between AD 503 and 522 give a value for the lunar tidal acceleration that is consistent with the current one. We also obtained reliable ranges of  $\Delta T$  from these records.



**Fig. 7** Diagram showing the possible area of the parameter values from the Chinese events between the years 503 and 522



**Fig. 8** Reproduction of a part of Fig. 14.4 by Stephenson (1997), with the addition of our result

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