

ERRATUM ELECTRICITY DERIVATIVES

The Chapter 4, section 4.1 contains several mistakes.

Formula (4.7), page 68, is incorrect. The volatility σ_a should be replaced by the volatility σ_M .

The equivalent volatility in formula (4.9), page 70, omits to differentiate the different correlation between assets.

The numerical illustration of Kirk's formula presented in Figures (4.1) is erroneous as well as the illustration of the deltas in Figure (4.2).

We provide hereafter a correction of the pages 66-70. We took advantage of the corrections of those mistakes to correct also several typos.

We would like to thanks Marie Bernhart and Emmanuel Gobet for pointing out those mistakes.

4.1 Spreads

The most commonly used spread options in the electricity markets are fuel-spread options and locational spread options. A fuel-spread option consists of an owner with the right to buy electricity and sell a fuel (either gas, coal, or oil). And the holder of a locational spread has the right to buy power from one zone and to sell it to another at the cost of its transportation. The fuel-spread options are embedded in thermal power plants. A power plant owner who wants to hedge the generation value just has to sell the fuel spread at different future maturities. As section 2.3 shows, the payoff of the fuel-spread options that are of some interest to power plants are of the form:

$$\left(S_T^e - hS_T^f - gS_T^c - K\right)^+$$

with S_T^e the price of electricity, S_T^f the price of fuel, S_T^c the price of carbon at time T , and K the start-up cost. The parameters h and g are respectively the heat rate of the power plant and its emission factor. So, there are the three assets involved in this payoff. It might be that more assets are involved when the power is quoted in euros while gas is quoted in pounds and coal in US dollars. In this situation, the fuel-spread option includes five assets (three prices and two exchange rates).

It is not possible to neglect the strike price induced by the start-up cost. Here is an example with the order of magnitude of the costs and the prices:

- For a coal-fired plant, the fixed start-up cost is $\approx 50,000$ €. Running 12 hours per day with a capacity of 500 MW leads to a $K = 8$ €/MWh ($50.000/(500 \times 12)$). The dark spread at the time this book is written is around 4 €/MWh and it was 30 €/MWh at its recent peak.
- For a gas-fired plant, the fixed start-up cost is $\approx 15,000$ €. Running 6 hours per day with a capacity of 500 MW leads to a $K = 5$ €/MWh. The crack spread at the time this book is written is -15€/MWh.

The situation is not better for the locational spread options. In Europe for instance, the transportation cost for generation is around 2 €/MWh, and the spread between France and Germany is around 4 €/MWh.

Thus, the managers of generation assets are interested in the valuation and hedging of options whose payoff can be summarised as:

$$p(T, S_0^a, S_0^b, K) = \left(S_T^a - S_T^b - K\right)^+ \quad (4.1)$$

with S^a and S^b the price of the two assets, K the strike, and T the maturity.

Since Margrabe's paper [129] on the options to exchange one asset against the other, important studies have been written on spread options. Despite the intense research activity in this field, there is little hope to expect closed-formed formulas for spread options in situations with a non-zero strike price. Nor is there much hope

for more than two assets and for asset prices following a more general dynamic than a geometric Brownian motion.

I begin here by reporting some of the most commonly used formulas for spread options, which are Margrabe's formulas and their derivatives. These formulas provide a first approximation of the value of the spread options and can be used as benchmarks for the more complex models. These formulas can be justified by asymptotic expansions when time or volatility go to zero (see Landon [120]). I then turn to the valuation formulas in the models which are closer to the observed dynamic of the commodities and which are applicable to electricity. Further, I say a word on the papers which provide the valuations of the spread options for specific electricity price models.

Black & Scholes assets model. I assume $dS^i = S^i [rdt + \sigma_i dW^i]$, $i = a, b$ with $dW^a dW^b = \rho dt$. When $K = 0$, the value of the spread option whose payoff is given by (4.1) is:

$$p = e^{-rT} [S_0^a N(d_1) - S_0^b N(d_2)] \quad (4.2)$$

with

$$d_1 = \frac{\log(S_0^a/S_0^b) + \frac{\sigma^2 T}{2}}{\sigma \sqrt{T}}, \quad d_2 = d_1 - \sigma \sqrt{T},$$

and

$$\sigma^2 = \sigma_a^2 - 2\rho\sigma_a\sigma_b + \sigma_b^2. \quad (4.3)$$

Margrabe's formula is an application of the Black & Scholes formula for the asset S^a/S^b and the equivalent volatility (4.3). Margrabe's formula ceases to be true for a non-zero strike price. Only approximations are available. Kirk (1995) and Eydeland & Wolyniec [83] are the most commonly cited. Another approximation formula can be found in the technical report by Bjerksund and Stensland (2006) [34]. To avoid the multiplication of formulas, I limit the chapter to three of them.

Kirk's formula (1995) reads as

$$\hat{p}^K = e^{-rT} [S_0^a N(d_1^K) - (S_0^b + K) N(d_2^K)] \quad (4.4)$$

with

$$d_1^K = \frac{\log(S_0^a/(S_0^b + K)) + \frac{\sigma_K^2 T}{2}}{\sigma_K \sqrt{T}}, \quad d_2^K = d_1^K - \sigma_K \sqrt{T},$$

and the equivalent volatility is:

$$\sigma_K^2 = \sigma_a^2 - 2\rho\sigma_a\sigma_b \frac{S_0^b}{S_0^b + K} + \sigma_b^2 \left(\frac{S_0^b}{S_0^b + K} \right)^2. \quad (4.5)$$

Kirk's formula consists of using Margrabe's formula with a strike equal to $S_0^b + K$ and of changing the volatility for S^b accordingly. Here, the reader should note the non-symmetric role of the two assets in the equivalent volatility σ_K . This is not the case in the variant proposed by Eydeland & Wolyniec's formula [83, pp 345-346]:

$$\hat{p}^E = e^{-rT} \left[S_0^a N(d_1^E) - (S_0^b + K) N(d_2^E) \right] \quad (4.6)$$

with

$$d_1^E = \frac{\log(S_0^a / (S_0^b + K)) + \frac{\sigma_E^2 T}{2}}{\sigma_E \sqrt{T}}, \quad d_2^E = d_1^E - \sigma_E \sqrt{T},$$

and

$$\sigma_E^2 = \sigma_a^2 - 2\rho\sigma_a\sigma_b \frac{S_0^a}{S_0^b + K} + \sigma_b^2 \left(\frac{S_0^a}{S_0^b + K} \right)^2.$$

In this last formula, it is possible to exchange an asset b at a lower price S_0^b for a higher strike K and still have the same value for the spread option.

Another basic and yet efficient way to approximate the value of the spread options is to approximate the density of the spread at maturity with a Gaussian distribution and by matching its two first moments. This is the method proposed in Carmona & Durrleman ([55, prop. 4.1]). This method leads to the following approximation:

$$\hat{p}^M = (m - Ke^{-rT}) N(d_1^M) + \sigma_M N'(d_1^M) \quad (4.7)$$

with

$$d_1^M = \frac{m - Ke^{-rT}}{\sigma_M}, \quad m = (S_0^a - S_0^b) e^{-rT}$$

and

$$\sigma_M^2 = e^{-2rT} \left[(S_0^a)^2 (e^{\sigma_a^2 T} - 1) + (S_0^b)^2 (e^{\sigma_b^2 T} - 1) - 2S_0^a S_0^b (e^{\rho\sigma_a\sigma_b T} - 1) \right]. \quad (4.8)$$

A systematic analysis of the relative precision of these different methods is too long for this review. But, it is possible to quickly illustrate them with some numerical computations and at the same time give the reader some reasoning behind the behaviour reflected by the value of the spread options. Specifically, I take a null interest rate, a maturity of one year, and a reference volatility of 10% for asset a and 15% for asset b . Further, I choose S_0^b and K such that $S_0^b + K = S_0^a$ so that the initial spread value is always zero when K changes. I consider only the case with a strong correlation of either 80% or -80% . The resulting value for the spread option for the different approximation formulas are plotted on Figure 4.1.

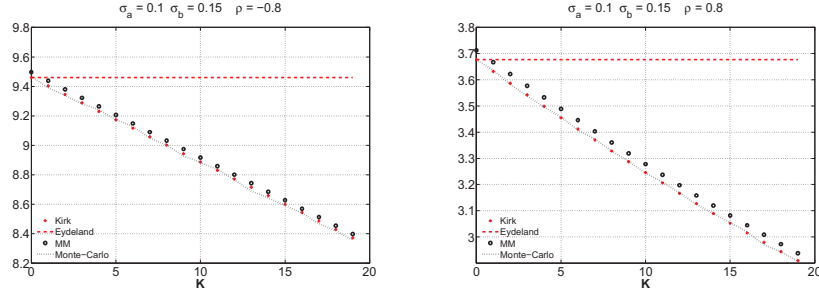


Fig. 4.1 Different approximation formulas for the value of a spread option: Kirk, Eydeland & Wolinieck, Moment Matching (MM) and a Monte-Carlo estimation (MC). Left with strong negative correlation. Right with strong positive correlation

Several comments can be made about Figure 4.1. First, the value of the option spread can be significantly positive although the initial spread is zero. The value decreases when the correlation between the assets increases: the stronger the correlation the less probable the spread will increase. Regarding the approximation formulas, Eydeland & Wolinieck's formula provides a constant value in this situation because, since we choose $S_b + K = S_a$, their equivalent volatility is constant. Kirk's method as well as the moment matching method provide a precise and constant estimation of the value of the option for both positive and negative correlation, when compared to the Monte-Carlo approximation. In our illustration, the moment matching method tends to slightly overestimate the value of the spread option for positive correlation. In contrast, Kirk's formula is a good analytical approximation obtained at the cost of a small modification of Margrabe's initial formula. Note in particular that Kirk's formula preserves the exact value of the spread option when $K = 0$.

Regarding the Greeks, I illustrate the effect of the volatility of the asset a on the sensitivity of the spread option value w.r.t. a . However, I change the conditions of the computations because when the strike is zero, the volatility of the assets has the same effect. Thus, as before, I take a null interest rate and an one-year maturity option but now, $S_0^a = 100$, $S_0^b = 80$, and $K = 10$. At time zero, the spread has a positive value. Sensitivities are computed using finite difference. Figure 4.2 presents the deltas obtained for the different approximation formulas above. For both types of correlations, the approximations remain good for the volatilities in a reasonable range. For high volatility, the moment matching method ceases to provide a reliable estimate.

In the case where more than two assets are taken into account but where the dynamic of their prices are still driven by the geometric Brownian motions, several developments are done. The most basic one is the extension of Kirk's formula to three assets by Alos, Eydeland & Laurence (2011) [6]. Considering that the asset prices S^a, S^b , and S^c follow the geometric Brownian motions with the correlations ρ_{ab}, ρ_{ac} , and ρ_{bc} , the value of the call option with payoff $(S_T^a - S_T^b - S_T^c - K)^+$ is

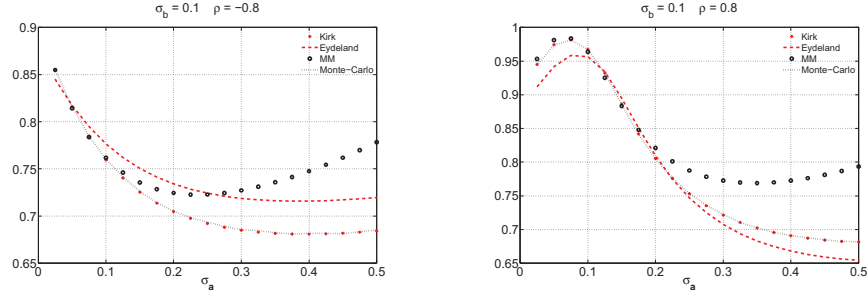


Fig. 4.2 Delta w.r.t. asset price a for the different approximation formulas for the values of the spread options: Kirk, Eydeland & Woliniec, Moment Matching (MM) and a Monte-Carlo estimation (MC). Left with strong negative correlation. Right with strong positive correlation

approximated for small maturities T by

$$\hat{p}^L = e^{-rT} \left[S_0^a N(d_1^L) - (S_0^b + S_0^c + K) N(d_2^L) \right] \quad (4.9)$$

with

$$d_1^L = \frac{\log(S_0^a / (S_0^b + S_0^c + K)) + \frac{\sigma_L^2 T}{2}}{\sigma_L \sqrt{T}} \quad d_2^L = d_1^L - \sigma_L \sqrt{T}$$

and

$$\sigma_L^2 = \sigma_a^2 + \sigma_b^2 \pi_b^2 + \sigma_c^2 \pi_c^2 - 2\rho_{ab} \sigma_a \sigma_b \pi_b - 2\rho_{ac} \sigma_a \sigma_c \pi_c + 2\rho_{bc} \sigma_b \sigma_c \pi_b \pi_c,$$

where $\pi_b = S_0^b / (S_0^b + S_0^c + K)$ and $\pi_c = S_0^c / (S_0^b + S_0^c + K)$.

The numerical illustrations reported by the authors show that relation (4.9) can provide precise results even for the spread options with a maturity of one year.

The creation of approximation formulas for more general spread options (more assets and more complex price dynamics) is still an active field of research. An example is provided by Landon's PhD thesis [120, chap. 8] which provides formulas for general spread options with n assets based on asymptotic expansions.

Beyond Black & Scholes. Several authors propose valuation formulas for the spread options in mean-reversion models. Some formulas are provided in Tsitakis et al. [157] for two-asset spread options with an Ornstein-Uhlenbeck dynamic and a zero strike price. A more general situation is addressed in Hikspoors & Jaimungal (2007) [99] where two-factor mean-reversion models are used for each asset. The closed-form formulas are provided up to a specific change of measure to preserve the mean-reversion in the risk-neutral measure.

Another approach is to directly model the spread dynamic. Dempster et al. [72] develop this approach with co-integration, or in Benth & Benth [28] and Cartea & González-Pedraz [61] with a mean-reverting jump-diffusion process. With this simplification, the problem is brought back to the valuation of a call option on an asset following a mean-reverting process, problems for which the closed-form solutions

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