

Chapter 2

On the Moments of Consistent and Fluid Heavy Bodies Floating in Fluids

Archimedes discussed very precisely and famously bodies sitting in liquid. Later, by another approach, Galileo and Stevin demonstrated the same point, since the truth can be confirmed in countless ways. Myself I devised and presented new demonstrations of these propositions, proceeding in a very different way, not for the pleasure of changing but because they lead to points which must be explained later. But at first some hypotheses must be put forwards.

Supposition I

I firstly suppose that any body, either dense or fluid, among those which compose the terrestrial globe, is heavy and exerts the force or effort of its gravity even if it is a fluid whether homogeneous or not. This, however, will be confirmed in due place by the most obvious reasons and by experiments.

Supposition II

Secondly, I suppose that the force or effort by which fluids tend to unite with the terrestrial sphere is exerted along vertical lines. This results from the fact that any heavy body, by its natural instinct, strives to arrive to the centre of the earth by the shortest way. Consequently, the direction of this movement or compressive effort occurs along radii of the earth. These are perpendicular to the horizon which surrounds the earth spherically. Consequently, it is obvious that the movement or compressive effort of all the parts of a fluid is carried out along perpendicular lines.

Supposition III

Thirdly, it is impossible for any heavy body to move in a spontaneous and natural movement when it cannot approach the centre of the earth. This is obvious, since all the earthy parts in so far as they are heavy bodies, tend to arrive to the centre of the earth by their normal instinct. Their longing desire cannot be satisfied without a movement. Consequently, when the end ceases to be, the means must stop necessarily, i.e. when a heavy body cannot approach the centre of the earth more than previously, it will not move at all. Consequently, these bodies are immobile. If they moved, they would have to recede from the centre of the earth or to circle sideways. In the former instance this would result in an action contrary to the natural instinct of heavy bodies, which is impossible. In the latter instance a vain and frustrating action would be carried out: the heavy body indeed would acquire nothing more since it could not arrive closer to the centre of the earth by hypothesis. Actually, an action of nature carried out by chance and without any goal is absurd and inconceivable. Consequently, bodies unable to come closer to the centre of the earth cannot possibly move at all. Therefore, they must remain immobile in the position in which they were.

Supposition IV

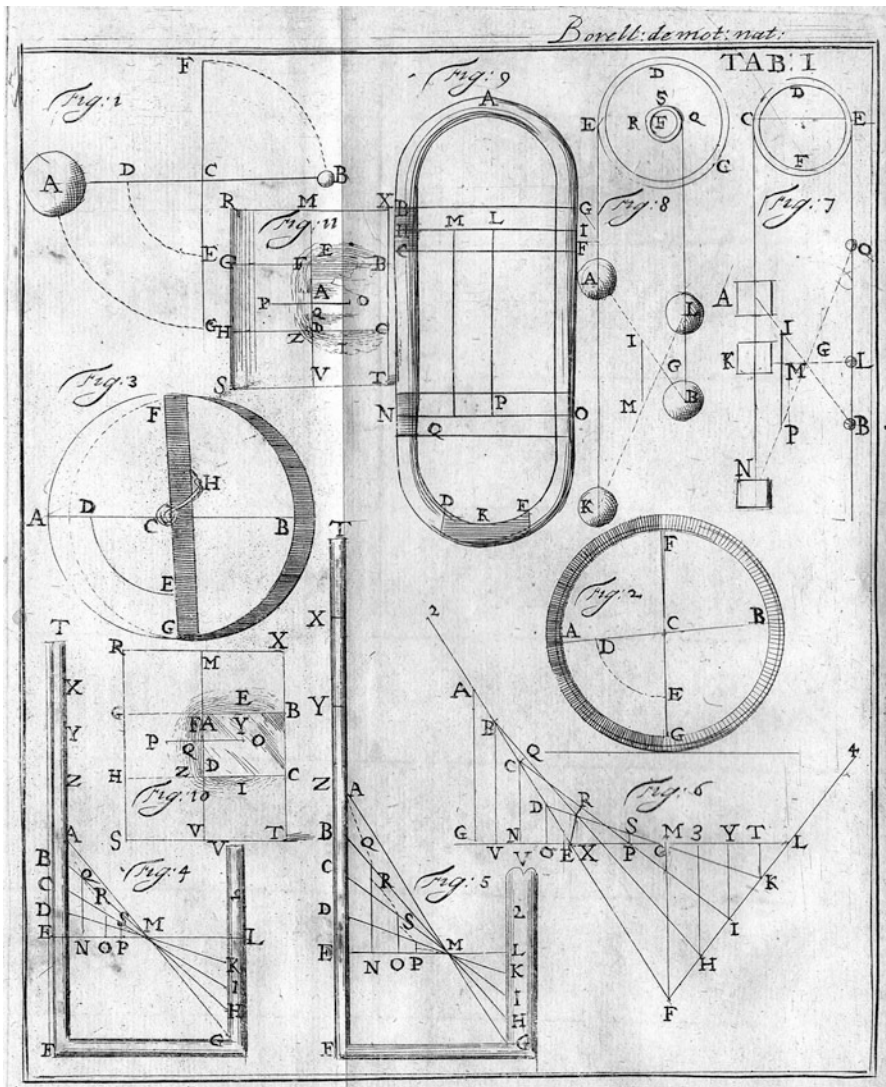
Moreover, Archimedes supposed as a first principle known by itself that the less compressed parts of a consistent fluid which are continuous in the same horizontal plane must be expelled vertically upwards by the more compressed parts of the fluid. This principle actually, although very true, presents, however, with some obscurity since it is not evident why the parts of the fluid can be compressed more or less. Neither is it clear how a natural downwards action must result in an opposite upwards action of another part of the fluid, i.e. a retrogression from the centre of the earth. Consequently, it is worth showing clearly the truth of the mentioned operation and deducing it from better known and more obvious principles.

Proposition I

When a heavy body is not suspended by its centre of gravity, a part of it rises because the whole heavy body falls.

Let a heavy body AB large or made of two parts be located at the extremities of a horizontal balance AB (Table 2.1, Fig. 1). Their common centre of gravity is D. The balance is supported at point C at a distance from the centre of gravity D. I claim that the opposite part B will rise over the arc BF because the whole heavy body AB

Table 2.1



approaches the centre of the earth more than previously. The two heavy bodies A and B exert their gravity and effort of compression at their common centre of gravity D. This centre of gravity D is remote from the stable support C. Consequently, the whole will behave like a horizontal pendulum suspended from point C whereas all the weight is applied at the centre D at the end of the rope or line CD. But the nature of a pendulum is such that it tends to move downwards over the arc DE of a quadrant about the fixed centre C down to the lowest point E which is closer to the centre of the

earth than in the horizontal position D. All this necessary and natural operation obviously results from the descent of all the body. The pendulum CD cannot possibly be brought to its lowest position CE without the rigid balance becoming perpendicular such as GCF. This cannot be achieved unless the less heavy part B of the balance rises upwards over the arc BF. Consequently, the fall and descent of all the heavy body AB from its high position D down to its lowest position E is the actual and regular cause of the ascent of the body B over the arc BF. Q.E.D.

Consequently, it appears that the simple fall or descent of a heavy body is the actual and regular cause of the upwards displacement and ascent of a part of it. This occurs whenever all the heavy body is supported from an actual or imaginary point of the balance so that the movement of all the parts does not take place along parallel and vertical straight lines but over the circular lines described by a pendulum. In such circular movement, the ascent of the less heavy part B over the arc BF is as necessary and natural as the fall and descent of all the heavy body over the arc DE down to the lowest point E. Although the ascent of the part B is commonly considered as a violent movement, nevertheless if one considers the rotation and the due situation of the heavy body, in so far as this situation is natural and acquired and produced by natural instinct, it is indeed impossible perfectly to attain this due situation without ascent of the part B. Actually, who will the end must also be willing the means which are necessary to attain the end. It is thus logical to infer that the less heavy body is impelled upwards towards F by a natural force. It must, therefore, be admitted that the ascent over BF is natural or rather that the violence of the said upwards movement must be included in a natural operation. Whatever happens, it is sufficient for us that this operation is necessary and cannot occur in another way. Actually, let the others call it either natural or violent, at will.

Proposition II

The same point is verified in fluids contained in a circular siphon.

In order to have two bodies at the extremities of a balance it is not always necessary that the heavy bodies A and B be attached to a rigid and consistent beam such as ACB (Table 2.1, Fig. 2). A circular pipe AGBF can be conceived. When filled with water or another fluid liquid, its right part FAG is heavier than the remaining part of the fluid GBF. If the fluid FAG is quicksilver and FBG simple water, a balance will also be constituted. The centre of gravity of the two liquids will not lie on the vertical diameter FCG but outside it between C and A, i.e. at some point D. The centre of all the quantity of fluid will be C and all the fluid is suspended there. Two opposite movements will occur about C: a descent of the fluid A and an ascent of the opposite fluid B. Since the common centre of gravity D of the two fluids is at a distance from the centre of suspension C, a pendulum results which carries out a circular movement over the arc DE.

Proposition III

Presentation of an instrument in which perpetual motion seems to be carried out, and detection of its shortcoming and insufficiency.

Here, in a short and not completely superfluous digression, I will indicate the impossibility of perpetual motion in a machine which seems to have such likeliness that anybody would swear that movement can be easily continued with such an instrument. My excellent friend Clement, the seventh disciple of Galileo, explained to me in the past this speculation and the structure of this instrument. By looking at winding-drums, or at the wheels by which small boats are pulled at Pisa and in Belgium from one canal to another by the force of one man alone who rotates the wheel by climbing on its inside, as spits are rotated by dogs in a winding-drum in kitchens, he thought that a winding-drum could be made the left half side of which could always be occupied by a liquid heavier than that in the right half. As shown in the diagram (Table 2.1, Fig. 3), the copper winding-drum AFBG is formed by a curved cylindrical surface of copper and two circular parallel flat sheets, perfectly smooth and welded to the cylindrical surface. In the cavity of the drum there is a flat sheet FCG which is used as a partition. Half the cylinder FCGA is filled with water or quicksilver, the other half BFCG with oil or air. The partition FCG is welded to the axis HC inside the drum and can be rotated about the fixed axle C by way of a crank H so as to fit exactly the internal surfaces of the two flat bases and the concave surface of the drum. The rotated partition must be tight like a cap and it must prevent the water or mercury contained in the half cylinder FAG from leaking, whereas the other space GBF remains full of air or oil. The drum is suspended on the axle C resting on two supports so as to be able freely to rotate in a vertical plane. If the hand retained perpetually the crank H and the partition FCG attached to it in a vertical position, we would in this instance unquestionably have a perpetual imaginary balance ACB with equal arms, said my friend, and this balance would be compressed by unequal weights. The arm CA would carry the weight of a half cylinder of mercury or water whereas the opposite arm CB would be compressed by a lighter weight of oil or air. The centre of gravity of these unequal weights would always lie at a point D between C and A. Consequently, the balance AB should always tip to the side A or rather it would constitute a horizontal pendulum CD suspended at the centre C. Therefore, the pendulum should descend over the arc DE. Since the heavier liquid FAG cannot flow downwards, impeded by the partition FCG retained in a vertical position by the hand, all the half cylinder of mercury compressing and climbing the concave surface of the drum AG which is rotatable would impel it. This should rotate downwards from A towards G since it is not impeded by any binding. Consequently, the drum would always be able to rotate from A towards G since the cause of such rotation would always persist, i.e. the pendulum CD would always be maintained in a horizontal position and thus would always compress and climb the surface AG of the winding-drum. Therefore, it seems that such an artifice can result in perpetual movement of the winding-drum.

As I said, this seems so likely that none of the many friends to whom I explained the artifice suspected its deceit. Nevertheless, although I never cared bringing this artifice into practice, I do not hesitate asserting that perpetuity of movement cannot be achieved this way. Indeed I cannot believe that heavy bodies can ever move spontaneously when not even a javelin is able of descending more than previously and cannot approach the centre of the earth. Since the common centre of gravity D of the two fluids is always retained in the same horizontal plane ABCD, it seems impossible to me that the wheel or the winding-drum AGBF rotates in the direction of A towards G. Although the common centre of gravity D is at a distance from the fixed centre of rotation C and thus constitutes a horizontal pendulum, I say that it is retained and suspended by the force of the hand which prevents the partition FG from being rotated by the force of the weight acting at the centre D. It is as if a pendulum CD supported by a subjacent hand could not move downwards over the arc DE. Although, in our instance, the pendulum CD is not something continuous and bound to the centre C, when exerting its effort over the arc DE, it nevertheless behaves exactly as if it had been bound to the centre C. The one who prevents the descent of the heavy body D which can move only over the arc DE necessarily prevents its motive action of displacement. Therefore, it cannot impel the fluid FAG since this is absolutely immobile, nor rotate the drum. One can in no way conceive that a projectile be impelled by a body which is absolutely immobile. Indeed an impeller must be provided with an impetus and movement to be able to impress a degree of impetus into a projectile. Since the quicksilver FAG is absolutely inert and deprived of movement, it appears completely impossible that it succeeds in impressing any degree of impetus into the projected drum. Therefore, the winding-drum is not displaced. Not only a rotatory movement cannot be continued perpetually by this artifice but the artifice cannot even start a movement. After this digression let us return to our matter.

Proposition IV

The centre of gravity of a fluid descending in pipes or in a siphon with two straight vertical arms descends over a curved route along a parabolic line.

A siphon TFGV comprises two pipes TF and GV parallel to each other, vertical and perpendicular to the basis FG (Table 2.1, Fig. 4). They have the same width. The capacity of the portion TF of the cylinder above the horizontal drawn across V, TA in the first instance and TC in the second, is equal to the capacity GV which in this instance is divided into a certain number of equal parts at X, Y, Z, I, L, 2. The points A, B, C, D, E are the centres of gravity of the cylinders TF, XF, YF, ZF, and AF or CF. Similarly, H, I, K, L are the centres of gravity of the cylinders GI, GL, G2, GV. Since the centres of gravity A and B divide into two equal parts the cylinders TF and XF, $TF/XF = AF/BF$. By converting the proportion and permuting: $TF/AF = TX/AB$. Therefore, $AB = TX/2$ and HG is half the cylinder IG. At first water rises up to T and is depressed in the right pipe down to G. Then it

rises up to I and lowers from T to X. The two straight lines AG and BH intersect at M and point M is on the horizontal line EL. Therefore, since the two cylinders of water AB and HG are equal, being half the equal cylinders TX and IG, the ratio of the heights AB/HG is equal to the ratio of the bases of the cylinders H/A. For the same reason, AE/LG is equal to the ratio of the bases H/A. Therefore, the ratio of the heights AE/LG is equal to AB/HG and the two straight lines AE and GL are perpendicular to the horizontal FG or EL and are thus parallel. Because of the similarity of the triangles, $AM/MG = BM/MH = EM/ML$. Therefore, the straight lines AG, BH and EL intersect each other at the same point M. The ratio of the sum of the masses of water (XBF+GHI)/IHG is equal to the ratio of the lengths HB/BQ. After division, the ratio of the masses of water XBF/GHI is equal to the ratio of the distances HQ/QB. Therefore, point Q is the centre of gravity of the water XBF+GHI. When water attained its upper point T and the pipe GLV was completely empty, the centre of gravity of all the water TAF, remaining at the midpoint A, acted as if the cylinder was suspended from point A. After the water is depressed down to Y and raised up to L in the opposite pipe, its centre of gravity is found at point R. Finally, when the water is depressed down to A in the first instance and down to Y in the second and raised up to V, the centre of gravity of the water which is horizontal is exactly at the centre of the suspension M because the ratio of the bases V/A or the ratio of the cylinder of water GLV to the equally high cylinder AEF in the first instance or to CEF in the second is equal to the inverse ratio of the distances EM/ML. It must be shown that the points A, Q, R, S, M are on the same parabolic line (Table 2.1, Fig. 5). Since the mass of water TX is equal to the mass of water GHI, the sum of the masses XBF+GHI is equal to the mass of water TAF. But the ratio of the masses of water (XBF+GHI)/GHI was equal to the ratio of the lines HB/BQ or, after QN is drawn parallel to AE, to LE/EN. Thus, the ratio FAT/TX and the ratio of FA/2 to AB/2 are equal to LE/EN. But actually EA/AF = MA/AG = ME/EL. Thus, by arranging the equation, EA/AB = ME/EN. After conversion of the ratio, EA/EB = EM/MN = EB/NQ. Consequently the three continuous proportionals EA, EB and NQ are in the same ratio as EM/MN. Thus, $(EM)^2/(MN)^2 = AE/NQ$. Therefore, the points A and Q are on a parabola the vertex of which is M. Consequently, while the water in the siphon descends to its equilibrium, its centre of gravity moves over a parabola. Q.E.D.

Proposition V

The conditions are the same. If the pipes of the siphon of equal width form an angle inclined to the horizon, the same is demonstrated.

An inverted siphon of the same length forms an angle so that its half arms AF and FL are equally inclined to the horizon EL (Table 2.1, Fig. 6). An isosceles triangle EFL is thus formed. The part EA of the higher arm is equal to FL and to FE. Again I claim that the water in all the arm F2, half of which is AF, while flowing upwards through the pipe FL4 and descending through 2A, has also its centre of gravity

moving along a parabola. Equal parts are divided at points A, B, C, D, E and F, H, I, K, L. The centres of gravity are as previously. Perpendiculars AG, BV, CN, DO, FM, H3, etc. are drawn as well as connecting lines DK, CI, BH. In the isosceles triangle the angles L and E are equal. O and T are right angles. The hypotenuses DE, KL are equal. Thus, in the similar triangles DOE and KTL, the sides DO and KT are equal. The straight line OE is equal to TL. Adding to each of them TE which is common gives LE = OT which, like DK, is bisected at point Z since the sides DO and TK are equal and parallel. The other straight lines NY and CI are also equal to the previous ones and are bisected at point P, and so on. Since the pipes and the volumes of water which they contain AB and FH are equal, BFH is equal to AF. Then $HB/BQ = BHF/FH = FA/AB$. Therefore, the halves of the numerators will be in the same ratio to the same denominators: $EA/AB = XB/BQ$. After conversion of the ratio, $EA/EB = AG/BV = GE/EV = 2GM/2MN = BX/XQ = VX/XN = BV/QN$. Consequently, there are three continuous proportionals AG, BV and QN in the same ratio as MG/MN. Therefore, the ratio of the squares $(MG)^2/(MN)^2$ is equal to the ratio of the lengths AG/QN. Thus, the two points A and Q are on a parabola.

If the arms of the siphon are vertical, whether they have the same diameter or not, the common centre of gravity of the fluid in its descent always describes a parabola. If the arms of the siphon are inclined equally to the horizon, the centre of gravity in its descent describes a parabola whenever the arms have the same diameter.

Corollary I

If, in a siphon forming an angle, one arm is dilated and the other is very narrow, the centre of gravity in its descent describes a pseudo-hyperbola.

Corollary II

If one of the arms is vertical and the other is inclined, the common centre of gravity in its descent describes a pseudo-ellipse.

After these premises, another property of the balance or of the siphon must be mentioned in which the centre of gravity moves, not over an oblique or a curve, but vertically. For the understanding of this, the following must be presented.

Proposition VI

Two unequal weights are suspended from a rope turning round a pulley. While one of them rises, their common centre of gravity descends vertically.

A heavier weight A and a lighter one B are attached at the extremities of a rope ADB which is supposed to be weightless, and turns round the pulley CDE (Table 2.1, Fig. 7). This is rotatable about a fixed axle F. The ropes AC and BE compress the pulley vertically and are tangent to its periphery at the opposite extremities C and E of its horizontal diameter or balance CE. The ropes CA and EB thus are parallel. A straight line AB is drawn between them. Its midpoint is G. The ratio of the weights A/B is equal to the ratio of the distances BI/IA. Obviously (according to mechanics) point I is the common centre of gravity of the two attached weights A and B. The rope does not alter this proportion since it is supposed to be weightless. Then the lesser weight B rises up to L and the heavier weight A descends down to K. I claim that the common centre of gravity of both descends about the centre of the balance or about the stable support G, in a straight and vertical movement. The connecting straight line KL is drawn. Since the rope ADB remains the same and is equal to KDL, after the common part ADL is removed, the descent AK is equal to the ascent BL. Therefore, in the similar triangles, since the homogeneous sides AK and BL are parallel: $AK/BL = AG/GB = KML/M$. The sides AK and BL are equal. Therefore, the straight connecting lines AB and KL intersect each other at G, their midpoint. The same happens when the weights are moved to N and O. Point G thus is the centre or the fixed support of the balance AB in whatever way it turns. Finally a straight line IP is drawn through I. This line is parallel to the ropes and intersects the balances KL and NO at points M and P. At points I, M and P the balances are divided in the same inverse ratio of the opposite weights. Thus these points are the centres of gravity of these balances with the suspended weights. Therefore, even if the lesser weight B rises through BLO, the two weights A and B suspended at their common centre of gravity I descend about the firm centre G at the end of the pendulum GI, not in a circular movement, but vertically from I through M and P. Q.E.D.

Proposition VII

The same is demonstrated for the case when the weights turn about concentric pulleys of unequal diameters.

Let a pulley CDE rotate about an axle F together with another concentric and smaller pulley RSQ (Table 2.1, Fig. 8). A weight B is attached to the rope SQB and a weight A to the other rope DEA. These ropes are weightless. As previously, it will be shown that the ropes EA and BQ are parallel. A connecting straight line AB is drawn. The ratio of the weights A/B is equal to the inverse ratio of the distances BI/IA. Point I appears to be the common centre of gravity of the weights A and B since the ropes are supposed to be weightless. Through rotation of the pulley, the weight B rises up to L and the opposite weight A descends down to K. A connecting straight line KL is drawn, which intersects the straight line AB at G. I claim that the common centre of gravity I of the two weights A and B moves vertically straight downwards about the stable centre of the balance G. As a result of the rotation of

the pulley, the extremity A of the rope descends by as much as the rope is unwound from the wheel CDE and the weight B rises by as much as the rope BQS is wound about the wheel QSR. The two wheels are connected concentrically and rotate together about the fixed axle F. Thus the ratio of the descent AK to the ascent BL is equal to the ratio of the circumference CDE to the circumference RSQ or to the ratio of their radii FE/FQ. The triangles AGK and BGL are similar since their sides AK and BL are parallel. Thus, $AG/GB = KG/GL = AK/BL$. Consequently, the same fixed point G divides two balances AB and KL in the same ratio as the ratio of the displacements of their extremities. Thus, according to mechanics, point G is the centre and firm support of the two balances AB and KL. Through I a straight line IM is drawn parallel to the ropes and thus vertical. It intersects KL at M. Obviously, the two balances are divided at I and M in the inverse ratio of the hanging weights. Points I and M are thus the centres of gravity of the balances. Therefore, although the weight B rises through BL, the common centre of gravity I of the weights A and B is suspended about the firm centre G and descends at the end of the rope GI in a straight movement vertically downwards through IM. Q.E.D.

These mechanical speculations will contribute greatly to the understanding of the movement of bodies in fluids for the explanation of which the following must be considered at first.

Proposition VIII

Reason why a fluid moves in a siphon continuous and closed on itself.

A siphon ABDG is closed on itself (Table 2.1, Fig. 9). Its lateral arms BN and GD are straight, parallel, vertical and of equal size. It contains a drop of mercury BC which in narrow pipes remains collected in the same position. The remainder of the cavity of the tube BAGDC is filled with water. From points B and C and from the centre of gravity H of the drop of mercury, three straight horizontal lines BG, HI and CF are drawn. L is the midpoint of HI. Two heavy bodies, the mercury BC and the water GF, are suspended in the same imaginary balance HI. These two bodies are agitated by opposite movements and are suspended from this horizontal balance. The upper and lower parts of water neither impede nor enhance the action of these bodies. The water AB is balanced by its counterpart AG since they are homogeneous and of the same height. The lowest parts of the water CD and FE also balance each other. The weight of the water FG at the same horizontal level, opposes the compressive action of the mercury CB. The ratio of the weight of the mercury BC to the gravity of the water FG is equal to the inverse ratio of the distances IM/MH. Point M thus is the centre of gravity of the two bodies BC and GF. The imaginary balance HI is supported at point L on the vertical straight line LK drawn from the lowest point of the pipe where the balance and the quantities of liquids are divided into two equal parts. Consequently, a pendulum LM is formed. Therefore, according to the laws of mechanics, the balance tips with descent of the body BC and ascent of the water FG. This occurs because the common centre of

gravity M must necessarily move down, according to the nature of a pendulum. But this movement of the centre of gravity M is not circular. It is straight and vertical along the line MQ, as occurs for a pulley. This operation proceeds as follows. At first the cylinder of mercury CB moves downwards displacing its centre of gravity H to N. It is again compared with another cylinder of water equal to FG from the considered area and the centre of gravity of which is O. This creates a new horizontal balance NO intersected by the straight lines LP and MQ parallel to ENGO, at P and Q. The centre of this balance is P. Again the parts of water balanced on both sides above and below neither enhance nor impede the two equal bodies, one of mercury N and one of water O, which are compared with each other in the same horizontal balance. They are divided by the parallel lines HN, MQ and IO in the same ratios. The centre of gravity Q of these bodies is moved to point Q. Thus it has moved downwards along the straight vertical line MQ. This descent will continue until the mercurial body is brought to the lowest position of the pipe DE, when its centre of gravity H attains exactly the lowest point K of the pipe.

Do not say that the straight horizontal balance HI which is perpetually renewed is a fiction. Actually, the two cylinders CB and GF are supported by the plane of the subjacent water CF which is mobile since it yields to the descent of the mercury CB and the surface F rises at the same time and at the same velocity about the midpoint of the balance. Consequently, the two bodies BC and GF, while they both compress the fluid balance subjected to their weights and are forced to move together at an equal velocity in opposite displacements, necessarily constitute a balance. They exert the energy of all their compression at its centre of gravity. However, it is true that this balance does not tip but is continuously renewed in its horizontal position since the raised water no longer acts against the pressure of the mercury CB because it is balanced by the water raised above the mercury CB on the opposite side.

Proposition IX

A body heavier than water when immersed in water, while sinking, forms with an equal mass of adjacent liquid a balance with equal arms. Its centre of gravity, while continuously descending, raises the lighter adjacent water and always renews a horizontal balance.

A vessel RSTX is full of water (Table 2.1, Fig. 10). A prism of marble ABCD is put in its depths. Its horizontal bases AB and CD are prolonged to G and H. The plane AD is prolonged vertically upwards and downwards to M and V. We thus have here an oblong siphon contorted on itself as was presented in the previous proposition since the water BMGHVC surrounds the prism above, laterally and below. The prism AC cannot move downwards without the subjacent water CID being expelled, flowing laterally towards P and upwards up to the place left by the stony prism at E. The two parts MT and MS thus are like the two lateral arms of the siphon. They join mutually in the common side MV. The two upper portions of water XA and MG balance each other since they are homogeneous, have the same

specific weight and the same height. The two subjacent portions of water CV and DS also balance each other. Thus only the two adjacent bodies must be compared, the stone BD and the water AH, which are comprised between the horizontal planes BG and HC. Both are supported by the plane of the subjacent water HC which is neither firm nor impervious, but is easily movable and yielding. Consequently, the two bodies BD and AH are suspended as if they were carried by and leaning on the balance HC. The mobile centre of this balance would be the midpoint D dividing the balance HC into two halves. The centre of gravity O of the stone BD is connected to the centre of gravity P of the water AH by a straight line. This is divided at Y in the inverse ratio of the weights of the bodies. Y is thus the common centre of gravity of the stone BD and the water AH. The balance PO is divided into two halves by the plane MV at Q. A pendulum QY appears extending horizontally towards O as a result of the excess of the gravity of the stone over the specific weight of the water. Consequently, the balance must necessarily tip and the stone BD thus sinks. During this descent, the subjacent water expelled from I flows upwards over a curved route ZF up to E. Again a horizontal balance is formed by comparing the stone BD with the water at its side in a lower position. Thus, the imaginary horizontal balance is divided in the same ratio. The pendulum is flexed downwards by a force equal to that previously exerted. The centre of gravity sinks vertically until the stone touches the bottom of the vessel.

Proposition X

The same phenomenon occurs but reverted, when the immersed body is less heavy than the adjacent water.

If a prism BD is made of wood and its specific weight is less than that of the water AH, the conditions remain the same except that the common centre of gravity Y lies between Q and P (Table 2.1, Fig. 11). Therefore, the composite body made of water and wood exerts a force, impelling the centre of gravity Y downwards and hereby the subjacent water HDVS is compressed more violently. Because of its continuity and natural consistency which does not yield to pressure, this is necessarily impelled towards I. It thus exerts a force expelling upwards the surface DC of the wood. While rising, the wood must expel the incumbent water E which is displaced from E through FZ towards I in a transverse and oblique movement. Thus the wood is expelled upwards by the circular movement of the ambient water. But the mechanical reason of this action results from the imaginary horizontal balance PO perpetually tipping about the centre O downwards on the side of the centre of gravity Q and upwards on the side O. Above all it must be noticed that this imaginary horizontal balance is renewed again and again while the wood is rising and is compared with other lateral prisms of water which are successively intercepted between the two horizontal planes GB and HC. Thus, the wood immersed in the water never rests until it is brought up to the upper surface of the water and some part of it emerges.

Corollary

This confirms the truth of the principle of Archimedes: the nature of a consistent fluid requires that the more compressed of its parts lying at the same level impel vertically upwards the less compressed elements.

Because of its consistency, the subjacent water HCTS is not condensed and is mobile since it is fluid. Thus, a flexible balance is formed. Its subjacent part HV is more compressed than DT (because the part GD of water is heavier than the wood AC). Consequently, the fluid balance HDC must tip, lowering HD and raising DC. Therefore, all the water HSVD depressed downwards impels the water DVTC upwards.

Proposition XI

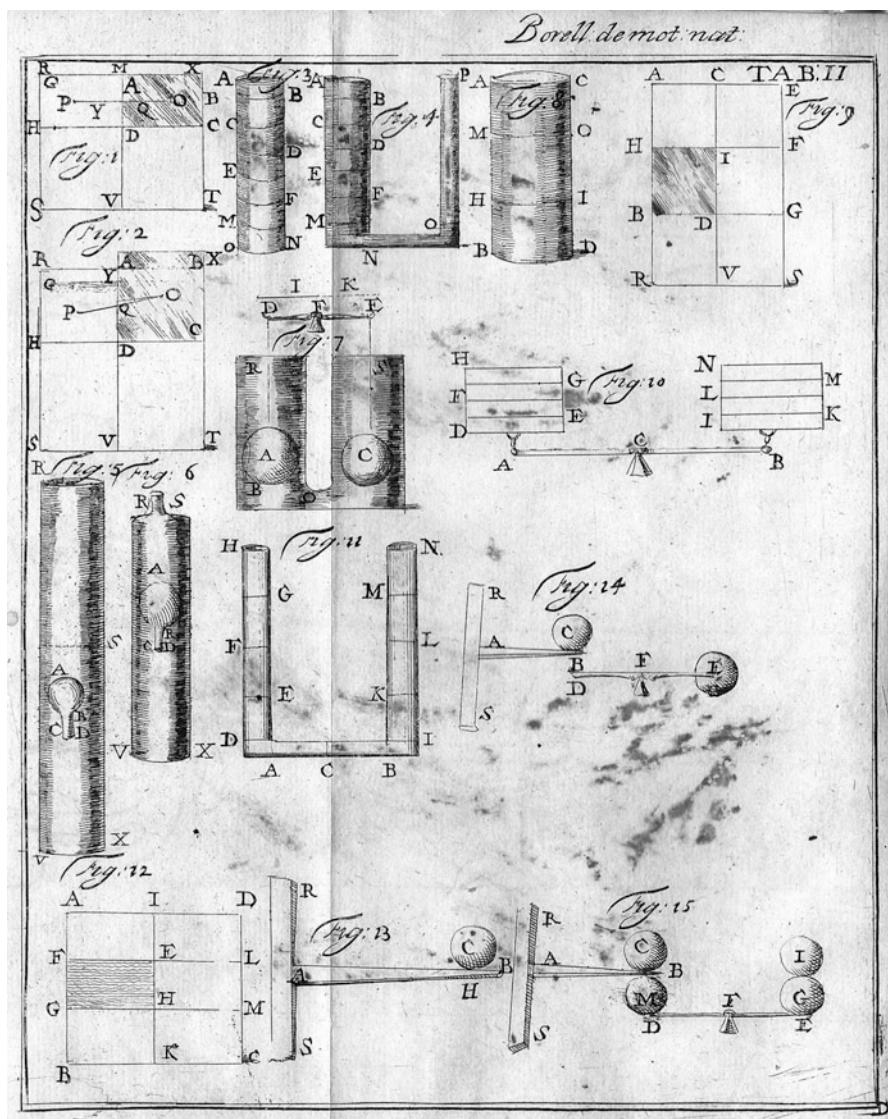
If a solid body is laid on water, the descent of the common centre of gravity does not occur vertically but in a curved movement over a parabola.

In the process of the mentioned operation the variation of the centre of gravity and the mechanics of the operation are remarkable.

In the same vessel a wooden prism ABCD has been brought up to the surface RX of the water (Table 2.2, Fig. 1). Two prisms, one of wood BD and one of water AH present in the same subjacent horizontal plane HC are considered. They form an imaginary balance PO mobile about its support Q. The centre of gravity of the two bodies is on the water side Y between the centre Q and the extremity of the arm P. Consequently, the balance must tip to the side Y and its extremity O must rise with the wood up to the surface RX of the water. Consequently, the upper portion of the wood is raised above the surface of the water as appears in Fig. 2. Then the prism of water GD is progressively reduced, the more so the more the wooden prism emerges. In this ascent, while the adjacent prism of water decreases, its weight which previously exceeded the gravity of the wood BD, after its continuous diminution, is finally reduced exactly to the magnitude of the weight of the wooden cylinder BD (Table 2.2, Fig. 2). Then their centres of gravity are joined by the straight line PO which is divided into two halves at point Q. The centre and its support are there. The ratio of the weight of the wood BD to the equal weight of the water GD is equal to the inverse ratio of the distances PQ/QO and the common centre of gravity Y thus lies exactly at the centre or support Q of the balance. Consequently, the weights balance each other and the scales are at rest. The wooden prism BD does not rise further nor does it sink downwards again unless it accidentally acquires some impetus.

Thus, when the wood BD begins emerging above the surface of the water RX, the common centre of gravity Y rises more and more in an oblique and curved movement until it coincides with the fulcrum Q of the balance PO displaced upwards. Similarly, when the water rises in one of the arms of the siphon, its centre

Table 2.2



of gravity moves downwards over a curved parabolic line, as said above. A siphon with unequal arms thus must be imagined. When the upper base AB of the wood arrives at the surface of the water, the excess of the specific weight of the water AH over the weight of the wood BD acts as if another fluid of the same specific weight as the wood BD and bigger in mass was acting on the base HD. Unquestionably this fluid of a lesser specific weight than the water AH would be raised higher.

Its absolute weight would be equal to the weight of fresh water AH. Therefore, this fluid by descending from its higher position would raise the depressed wood BD exactly as would occur in the siphon described above.

With this theory, it is easy to resolve and demonstrate all the propositions which are demonstrated by Archimedes in his first book on the floating bodies.

Proposition XII

When a solid rises or sinks in a fluid, there is no linear balance and the centre of gravity is not at one point. The balance usually is at the surface, its support is a line about the centre of the figure and the common gravity is also exerted in a line.

I will only point out that the point which is usually and vulgarly called common centre of gravity is not always used in the mechanical operation thus described. The composite balance made of a solid and the ambient fluid is not always linear but sometimes forms a surface in which not only the support but even the place where the common gravity is exerted is usually a line sometimes straight, sometimes curved and often made of several straight segments. A solid prism or cylinder is immersed vertically into water. While the prism sinks, all the water surrounding it is raised or, if the prism rises, the water is lowered. The prism must be considered with a ring or rather a tube of fluid surrounding it. This forms a flat balance the support of which is a line at the limit of the immersed cylinder and the ambient fluid. The place where the common gravity is exerted is not a point but is also a line drawn in this horizontal plane. But for the sake of facility, a sector must be imagined in this plane, from the centre of the superficial balance on the axis of the cylinder up to the surface of the ambient water, which moves together with the cylinder in the opposite direction. Or one must rather imagine a radius, not indivisible, but physical. This can be used as a peculiar balance with its support and centre of gravity. All the superficial balance actually is made of several and countless small radiating balances. This is enough for a general introduction. More will be explained below.

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