Preface

Credo ut intelligam.[1]

—St Anselm of Canterbury, Proslogion, 1

This is a book on quantum forces. More specifically, it is about predicting certain kinds of forces that arise between macroscopic bodies, for which we require quantum electrodynamics. More precisely, it is about the problem of predicting the size and nature of these quantum-mechanical forces when they arise within inhomogeneous media, in which the optical properties of a material are continuously changing as a function of position. However, in attempting to apply existing theory to such cases—and the general presumption is that it ought to be applicable—we uncover a number of *surprises*. In a sense, the basic thrust of this thesis is not so much to persuade its readers of certain answers as to convince them of unresolved questions. Some precise solutions to new problems are to be found within these pages, each of them interesting in its own right, but they are representative of a rather small class of Casimir problems involving inhomogeneous media that can be solved at present.

I hope this provocative paragraph will prove sufficiently intriguing to induce the turning of a few more pages, as I mean to avoid disclosing too many 'plot spoilers'.[2] I should warn the reader that it is also my preference to introduce formal physical theory in much the same way as I have learned it when left to my own devices—that is, as and when it is needed. While certain things are assumed to be familiar (some basic electrodynamics and some quantum mechanics, for instance), I occasionally invoke ideas or results beyond the common core of a Bachelor degree in Physics that are not 'fleshed out' until a subsequent section, when I think I can get away with doing so. For the more axiomatically minded, this may cause some minor abrasion. Nevertheless, to insist on understanding everything before we will say anything *about* it, do anything

[1] Translated: I believe in order that I may understand.

[2] For a summary of the conclusions, see the Outlook.

with it, or invest some kind of belief *in* it, is a limit in which nothing may ever be said or done again; the heat-death of the universe is the state that requires the most information, but the prospects for doing Physics at this point seem rather bleak.

Outline of Chapters

This thesis is divided up as follows. In Part I, the basic foundations of Casimir Physics are introduced to the reader, consisting of Casimir's groundbreaking thought-experiment, the prediction of the original Casimir force, a discussion of dispersion forces in general, select experimental verifications of Casimir phenomena, and the generalisation of the Casimir Effect to arbitrary geometries and more realistic materials using the basic framework of macroscopic quantum electrodynamics.

In Chap. 1, we consider Casimir's original thought-experiment, involving two perfect mirrors in a vacuum at zero temperature. Casimir's prediction of a finite, attractive force is recovered, following a discussion of how to regularise (and then renormalise) the infinite zero-point energy of the electromagnetic field, whose 'quantum fluctuations' are claimed to be the cause of it. The Casimir force, in fact, is part of a family of dispersion forces that Casimir studied, ubiquitous in nature, and we take a few moments here to survey the broader field, touching on both van der Waals and Casimir-Polder phenomena. The chapter is concluded by briefly reviewing some of the important experimental verifications of the Casimir effect.

Chapter 2 presents an outline of some of the basic elements of the theory of macroscopic quantum electrodynamics (macro-QED), affording a formal basis for ideas alluded to in Chap. 1, and deriving some of the principal results that will be used throughout this discussion, including the Lifshitz formula for the force between two separated dielectric half-spaces, and the ground-state energy of a quantum field. A new argument is offered for the correct form of the Casimir-Lifshitz stress tensor. The discussion, however, remains phenomenologically driven: the additional lengths we would be obliged to go to in presenting a properly *canonical* theory of macro-QED are not deemed to pay sufficient dividends for our restricted purposes, and may be studied elsewhere.

With the basics of macro-QED in place, Chap. 3 offers a brief excursus on the subject of the disputed *nature* of the Casimir force, which has been described, on the one hand, as an effect resulting from the alteration of the zero-point electromagnetic energy, by the imposition of external boundary conditions, and, on the other hand, as simply a giant van der Waals force between the metal plates. A different perspective is put forward, in which the Casimir force is seen to arise from the fluctuations of a polariton field involving the coupled, quantised system of dielectric material and electromagnetic fields. In its ground state, the system cannot be separated into material or electromagnetic quanta, which arguably splits the debate about the nature of the force straight down the middle.

In Parts II and III, we consider the case of the Casimir force in inhomogeneous media. Four peer-reviewed calculations are presented, resulting from this research, the first two focussed on the general problem of extending present theory to the case of macroscopic bodies in which the optical properties are varying continuously (the *surprises*), and the second two targeting exceptional cases of inhomogeneous media where the Casimir force can be determined exactly using ideas from transformation optics, leading to some interesting questions (the *conundrums*).

In the beginning, we considered the case of the Casimir force between two parallel plates separated by empty space, and subsequently saw that the force could also be calculated for the case in which the cavity is filled with a liquid medium. Chapter 4 asks the simple question of what happens to the force when the medium between the plates is *inhomogeneous*; that is, when the refractive index profile of the interposing liquid varies continuously as a function of position. A calculation is presented, based on Casimir's original approach to the cavity problem, but introducing the simple modification of a spatially dependent permittivity between the plates. A finite prediction for the Casimir force appears to be possible using a simple mode summation.

Chapter 5 reconsiders the same problem, only this time using the more sophisticated apparatus of Lifshitz theory, in which the detailed dispersive behaviour of the medium can be incorporated. The stress tensor inside the medium is determined using a piece-wise approximation, which is then taken to the continuum limit. However, the predicted force is now surprisingly illusive, and the possible need for incorporating additional information about the system, and perhaps more of the microphysical properties of the liquid, is discussed.

Noting the pathological nature of the Casimir-Lifshitz stress in an inhomogeneous medium, Chap. 6 considers the possibility of introducing an idealised inhomogeneous medium in the chamber that effectively modifies the size of an empty cavity. In this case, the Casimir force should be predictable and finite. The apparent contradiction with the previous chapter (Chap. 5) is explored and resolved, and in doing so we are able to determine an exact expression for the Casimir force for the case of a '*C*-slice' that (theoretically) reduces quantum stiction between attractive surfaces.[3]

In Chap. 7, we consider the case of the Casimir stress in Maxwell's fisheye—another inhomogeneous metamaterial with some remarkable properties. The stress tensor is infinite everywhere throughout the medium. However, a simple alternative regularisation is motivated, resulting in a finite stress tensor, leading to some perplexing questions about the nature of regularisation and our current understanding of the Casimir force.

Our journey ends with a survey of what we have learned in the *Outlook*, some suggestions for how to take this work further forward, and the proposal of a new experiment. I hope you enjoy the ride.

Rehovot, February 2014 Dr. William M.R. Simpson

[3] The phenomenon of quantum stiction leads to technological difficulties for micro and nano-electromechanical devices.

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