Chapter 2
Theoretical Development

The ε-*NTU* method was formally introduced in a 1942 unpublished paper by London and Seban (A generalization of the methods of heat exchangers analysis. Stanford University, Stanford, California, 1980). Later on, Kays and London in 1952 used extensively the procedure in their well-known book "Compact Heat Exchangers", Kays and London (Compact heat exchangers. McGraw Hill, New York, 1998), where data for different geometries and flow arrangements can be found. From that time on, applications of the ε-*NTU* method have been growing to the point that in the present this could be considered as the most accepted heat exchangers design and analysis procedure.

2.1 Fundamentals of the ε-*NTU* Method

The ε-*NTU* method was formally introduced in a 1942 unpublished paper by London and Seban (1980). Later on, Kays and London in 1952 used extensively the procedure in their well-known book "Compact Heat Exchangers", Kays and London (1998), where data for different geometries and flow arrangements can be found. From that time on, applications of the ε-*NTU* method have been growing to the point that in the present this could be considered as the most accepted heat exchangers design and analysis procedure.

In the ε-*NTU* method the heat exchanger effectiveness, ε, plays a central role, though its concept is relatively simple. The idea of effectiveness of the heat exchanger has to do with the energy conservation, since it is defined as the ratio between the actual rate of heat transfer and the maximum rate of heat transfer for given inlet temperatures of the hot and cold streams. It is defined as:

$$\varepsilon = \frac{q}{q_{\max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{\min}\left(T_{h,i} - T_{c,i}\right)} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{\min}\left(T_{h,i} - T_{c,i}\right)} \tag{2.1}$$

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The rate of heat transfer between the hot and cold streams is given by the numerators of Eq. (2.1):

$$q = C_h\left(T_{h,i} - T_{h,o}\right) = C_c\left(T_{c,o} - T_{c,i}\right) \tag{2.2}$$

Indices "h" and "c" stand for hot and cold whereas "i" and "o" refer to the inlet and outlet sections. Note that the maximum rate of heat transfer is the one that would be obtained in a counter-current heat exchanger with the same inlet temperatures of the hot and cold fluids when the fluid with the lower heat capacity, $C_{\min}$, would attain the same temperature as the inlet one of the hot (or cold) fluid.

The actual heat transfer rate can be written as:

$$q = \varepsilon q_{\max} = \varepsilon C_{\min}\left(T_{h,i} - T_{c,i}\right) \tag{2.3}$$

It is interesting to note that when one of the streams is changing phase at constant pressure, the heat capacity numerically tends to infinity. Thus the other stream is the $C_{\min}$ one. In general, it is possible to express the thermal effectiveness as a function of the number of transfer units *NTU*, the heat capacity rate ratio, C^*, and the flow arrangement of the heat exchanger,

$$\varepsilon = \Phi\left(NTU,\ C^*,\ \textit{flow arrangement}\right) \tag{2.4}$$

The dimensionless parameters *NTU* and C^* are defined as:

$$NTU = \frac{UA}{C_{\min}} \tag{2.5}$$

and

$$C^* = \frac{C_{\min}}{C_{\max}} \tag{2.6}$$

U and *A* stand for the overall heat transfer coefficient and the heat transfer surface area. According to its definition, Eq. (2.6), the heat capacity ratio is a number less or equal to one. According to Shah and Sekuliç (2003), the exchanger heat transfer rate can be written in terms of the mean temperature difference between the hot and cold streams as:

$$q = UA\Delta T_m \tag{2.7}$$

Thus, combining Eqs. (2.3) and (2.7), an alternate expression for the thermal effectiveness of the heat exchanger can be obtained.

$$\varepsilon = \left(\frac{UA}{C_{\min}}\right)\left(\frac{\Delta T_m}{\Delta T_{\max}}\right) = NTU\left(\frac{\Delta T_m}{\Delta T_{\max}}\right) \tag{2.8}$$

Note that ΔT_m is the effective mean temperature difference also known as the "mean temperature driving potential", Shah and Sekuliç (2003). Note also that ΔT_m is related to the log men temperature difference, *LMTD*, by the following expression:

$$\Delta T_m = F(LMTD) \tag{2.9}$$

F is the well-known correction factor of the *LMTD* procedure.

According to Shah and Sekuliç (2003), *NTU* could be considered as a dimensionless size or, in other words, the thermal size of the heat exchanger. Thus it is a design parameter. Another point of view regarding the physical meaning of this dimensionless parameter could be devised in terms of a temperature difference ratio. In fact, the following expression results by introducing Eqs. (2.2) and (2.7) into Eq. (2.5):

$$NTU = \frac{q/\Delta T_m}{q/\Delta T_{\max}} = \frac{\Delta T_{\max}}{\Delta T_m} \tag{2.10}$$

Note that

$$NTU = \begin{cases} \frac{\Delta T_c}{\Delta T_m} \to C_{\min} = C_c \\ \frac{\Delta T_h}{\Delta T_m} \to C_{\min} = C_h \end{cases}$$

Thus, *NTU* could be considered as the ratio between the maximum temperature variation and the mean temperature difference between the streams in the heat exchanger. High values of *NTU* would correspond, for example, to low temperature difference between the hot and cold streams. Figure 2.1 illustrates operational conditions corresponding to high and low values of *NTU*. Note that in the first case the value of the mean temperature difference is relatively high, corresponding to low *NTU*, whereas in the second is low, high *NTU*.

The functional relation expressed by Eq. (2.4) could have been obtained by dimensional analysis, the Buckinham's π theorem. This approach just proves that the effectiveness could be expressed as a function of *NTU*, C^*, and the particular geometry of the heat exchanger. However, the correlation that relates these dimensionless parameters is yet to be determined. This will be done for the counter flow heat exchanger of Fig. 2.1 assuming that the heat capacity of the hot fluid is lower than the cold one, corresponding to the right plot. To start, let us consider the expression for the log mean temperature difference:

$$LMTD = \frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ln\left(\frac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}\right)}$$

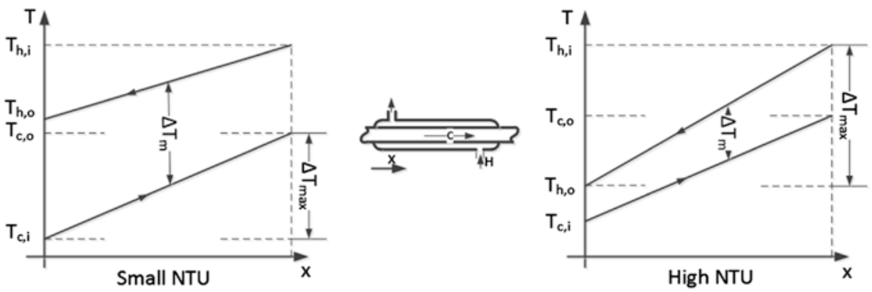

Fig. 2.1 Plots of temperatures of the hot and cold fluids along a counter flow heat exchanger illustrating conditions of high and low *NTU*

The left and right hand sides of Eq. (2.7) can be transformed into the following expressions:

$$\frac{q}{UA} = \frac{\left(T_{h,i} - T_{h,o}\right) - \left(T_{c,o} - T_{c,i}\right)}{\ln\left(\frac{T_{h,i}-T_{c,o}}{T_{h,o}-T_{c,i}}\right)} = \frac{\frac{q}{C_h} - \frac{q}{C_c}}{\ln\left(\frac{T_{h,i}-T_{c,o}}{T_{h,o}-T_{c,i}}\right)}$$

Combining the left and the right hand side and canceling out q, results:

$$\frac{\frac{UA}{C_h}\left(1 - \frac{C_h}{C_c}\right)}{\ln\left(\frac{T_{h,i}-T_{c,o}}{T_{h,o}-T_{c,i}}\right)} = 1 \Rightarrow \frac{T_{h,o} - T_{c,i}}{T_{h,i} - T_{c,o}} = \exp\left[-\frac{UA}{C_h}\left(1 - \frac{C_h}{C_c}\right)\right]$$

By summing and subtracting $T_{h,i}$ in the numerator of the right hand side and $T_{c,i}$ in the denominator of the left hand side of the above equation and introducing the definition equation of the thermal effectiveness, Eq. (2.1), remembering that $C_{\min} = C_h$, results:

$$\frac{-\varepsilon + 1}{1 - \varepsilon C^*} = \exp\left[-NTU\left(1 - C^*\right)\right]$$

Finally,

$$\varepsilon = \frac{1 - \exp\left[-NTU(1 - C^*)\right]}{1 - C^* \exp\left[-NTU(1 - C^*)\right]} \tag{2.11}$$

Equation (2.11) was obtained for a geometry corresponding to the counter flow heat exchanger of Fig. 2.1. Expressions for other geometries can be found in heat transfer text books and in the general literature. Data for advanced geometries will be presented herein as case studies applications of the code HETE.

The functional relation, Eq. (2.4), is applied for rating the heat exchanger by determining the thermal effectiveness for a given geometry. In designing, ε-NTU correlations are used in sizing the heat exchanger based on the inlet and outlet temperatures of the streams. In this case explicit NTU correlations are needed in terms of ε and C^* for a given geometry.

Table 2.1 presents a summary of correlations for one-pass cross-flow configurations (ESDU 86018 1991; Stevens et al. 1957; Bacliç and Heggs 1985). Details regarding the derivation of these relationships can be found in the aforementioned publications and others cited herein. Correlations for unmixed-unmixed flow arrangements deserve some comments. Equation (2.16) (Table 2.1), proposed by Mason (1955), valid for an infinite number of tube rows, was the one used as reference by Navarro and Cabezas-Gómez (2005) for comparison purposes with the numerical results from the HETE code. In addition, this correlation is one of several suggested by Bacliç and Hegss (1985) for computing the effectiveness for this flow arrangement. Equation (2.17), also for an infinite number of rows, has been obtained by curve fitting numerical data. According to DiGiovanni and Webb (1989) its origin is uncertain, though it appears in a footnote on page 483 of the book by Eckert (1959). Navarro and Cabezas-Gómez (2005) claim that the application of this correlation can

Table 2.1 ε-*NTU* relationships for one pass cross-flow configurations with one or more rows

N_r	C_{min} side	Relation	Equations
1	A	$\varepsilon_A = 1 - e^{-(1-e^{NTU_A \cdot C_A^*})/C_A^*}$	(2.12a)
	B	$\varepsilon_B = \frac{1}{C_B^*}\left[1 - e^{-C_B^*(1-e^{NTU_B})}\right]$	(2.12b)
2	A	$\varepsilon_A = 1 - e^{-2K/C_A^*}\left(1 + \frac{K^2}{C_A^*}\right),\ K = 1 - e^{-NTU_A \cdot C_A^*/2}$	(2.13a)
	B	$\varepsilon_B = \frac{1}{C_B^*}\left[1 - e^{-2KC_B^*}\left(1 + K^2 C_B^*\right)\right],\ K = 1 - e^{-NTU_B/2}$	(2.13b)
3	A	$\varepsilon_A = 1 - e^{-3K/C_A^*}\left(1 + \frac{K^2(3-K)}{C_A^*} + \frac{3K^4}{2(C_A^*)^2}\right),\ K = 1 - e^{-NTU_A \cdot C_A^*/3}$	(2.14a)
	B	$\varepsilon_B = \frac{1}{C_B^*}\left[1 - e^{-3KC_B^*}\left(1 + K^2(3-K) \cdot C_B^* + \frac{3K^4(C_B^*)^2}{2}\right)\right],\ K = 1 - e^{-NTU_B/3}$	(2.14b)
4	A	$\varepsilon_A = 1 - e^{-4K/C_A^*}\left(1 + \frac{K^2(6-4K+K^2)}{C_A^*} + \frac{4K^4(2-K)}{(C_A^*)^2} + \frac{8K^6}{3(C_A^*)^3}\right),\ K = 1 - e^{-NTU_A \cdot C_A^*/4}$	(2.15a)
	B	$\varepsilon_B = \frac{1}{C_B^*}\left[1 - e^{-4KC_B^*}\left(1 + K^2(6-4K+K^2)C_B^* + 4K^4(2-K)\left(C_B^*\right)^2 + \frac{8K^6(C_B^*)^3}{3}\right)\right],\ K = 1 - e^{-NTU_B/4}$	(2.15b)
∞	Both fluids unmixed	$\varepsilon = \frac{1}{C^* NTU}\sum_{n=0}^{\infty}\left\{\left[1 - e^{-NTU}\sum_{m=0}^{n}\frac{(NTU)^m}{m!}\right]\left[1 - e^{-C^* NTU}\sum_{m=0}^{n}\frac{(C^* NTU)^m}{m!}\right]\right\}$	(2.16)
∞	Both fluids unmixed	$\varepsilon = 1 - e^{\left[NTU^{0.22}\left(e^{-C^* NTU^{0.78}} - 1\right)/C^*\right]}$	(2.17)
∞	Both fluids unmixed	$\varepsilon = 1 - e^{-NTU} - e^{[-(1+C^*)NTU]}\sum_{n=1}^{\infty} C^{*n} P_n(NTU),\ P_n(y) = \frac{1}{(n+1)!}\sum_{j=1}^{n}\frac{(n+1-j)}{j!}y^{n+j}$	(2.18)

Equations (2.12)–(2.15) from Kays and London (1998), ESDU 86018 (1991), Stevens et al. (1957); Eq. (2.15) from Stevens et al. (1957) and Bacliç and Heggs (1985); Eq. (2.16) from ESDU 86018 (1991); Eq. (2.18) from Shah and Sekuliç (2003)
Fluid A mixed, Fluid B unmixed, $C_A^* = 1/C_B^*$, $\varepsilon_B = \varepsilon_A C_A^*$, $NTU_B = NTU_A C_A^*$, $C_A^* = C_A/C_B$

result in deviations of the order of 4 % for certain values of NTU and C^* when compared with more accurate solutions such as the HETE code and Eq. (2.16). Shah and Sekuliç (2003) suggest the use of Eq. (2.18) for the same flow arrangement.

Table 2.2 presents correlations for different multi-pass parallel and counter-cross-flow arrangements with widespread applications, Kays and London (1998), Taborek (1983), ESDU 86018 (1991), and Cabezas-Gómez et al. (2007). Equations (2.22) and (2.26) can be found in heat transfer and specialized heat exchanger textbooks such as the ones by Incropera et al. (2008) and Nellis and Klein (2009).

The development of the HETE code and its numerical solution will be the subject of the text that follows.

2.2 Differential Governing Equations

The numerical procedure of the HETE code is based on a mathematical model whose governing equations will be presented in this section. Based on the procedure by Kays and London (1998) (Appendix C), the governing equations consist of the Energy Conservation applied to a one tube row cross-flow heat exchanger with one fluid mixed and other unmixed. The procedure presented herein can be found in previous publications as in Navarro and Cabezas-Gómez (2005, 2007) and Cabezas-Gómez et al. (2007). The governing equations of the HETE code are based on the following basic assumptions, which can also be found in the books by Kays and London (1998) and Shah and Sekuliç (2003): (1) the heat exchanger operates under steady state conditions; (2) heat losses to the surroundings are neglected, that is, the heat exchanger is assumed to be adiabatic; (3) there are no thermal energy sources or sinks in the heat exchanger walls or fluids; (4) the tube side fluid is perfectly mixed in the tube cross section, its temperature varying linearly along the tube axis; (5) the external fluid is unmixed, i.e., there are fins in the external side; (6) heat transfer coefficients and transport properties of the fluids and heat exchanger walls are constant; (7) axial heat transfer in the solid walls and fluids are neglected; (8) there is no phase change in both streams; (9) for analysis purposes, the in-tube fluid is assumed the hot fluid though governing equations could be applied otherwise, that is, the in-tube being the cold one, by just interchanging the subscripts “c” and “h”.

Figure 2.2 illustrates the temperature distribution of both fluids along the transversal and longitudinal direction with respect to the in-tube fluid for a one pass cross-flow mixed-unmixed heat exchanger having one tube row with one circuit. Along the differential strip of length “dx” shown in Fig. 2.2, the mass flow rate of the external fluid (cold one) is small and since the heat transfer rate is also small, one might expect that the in-tube fluid temperature remains essentially constant, as suggested in the figure. An energy balance in the differential strip length for the hot and cold fluids can be written as:

$$\delta q = -C_h dT_h \tag{2.27}$$

$$\delta q = (dC_c)(\Delta T_c) \tag{2.28}$$

Table 2.2 ε-*NTU* relationships for multi-pass parallel and counter cross-flow configurations

N_r	C_{min} side	Relation	Equation
Multi-pass parallel cross-flow			
2	A	$\varepsilon_A = \left(1 - \frac{K}{2}\right)\left(1 - e^{-2K/C_A^*}\right),\ K = 1 - e^{-NTU_A \cdot C_A^*/2}$	(2.19a)
	B	$\varepsilon_B = \frac{1}{C_B^*}\left(1 - \frac{K}{2}\right)\left(1 - e^{-2KC_B^*}\right),\ K = 1 - e^{-NTU_B/2}$	(2.19b)
3	A	$\varepsilon_A = 1 - \left(1 - \frac{K}{2}\right)^2 e^{-3K/C_A^*} - K\left(1 - \frac{K}{4} + \frac{K}{C_A^*}\left(1 - \frac{K}{2}\right)\right)e^{-K/C_A^*},\ K = 1 - e^{-NTU_A \cdot C_A^*/3}$	(2.20a)
	B	$\varepsilon_B = \frac{1}{C_B^*}\left[1 - \left(1 - \frac{K}{2}\right)^2 e^{-3KC_B^*} - K\left(1 - \frac{K}{4} + KC_B^*\left(1 - \frac{K}{2}\right)\right)e^{-KC_B^*}\right],\ K = 1 - e^{-NTU_B/3}$	(2.20b)
4	A	$\varepsilon_A = 1 - \frac{K}{2}\left(1 - \frac{K}{2} + \frac{K^2}{4}\right) - K\left(1 - \frac{K}{2}\right)\left(1 + 2\frac{K}{C_A^*}\left(1 - \frac{K}{2}\right)\right)e^{-2K/C_A^*} - \left(1 - \frac{K}{2}\right)^3 e^{-4K/C_A^*},$ $K = 1 - e^{-NTU_A \cdot C_A^*/4}$	(2.21a)
	B	$\varepsilon_B = \frac{1}{C_B^*}\left[1 - \frac{K}{2}\left(1 - \frac{K}{2} + \frac{K^2}{4}\right) - K\left(1 - \frac{K}{2}\right)\left(1 + 2KC_B^*\left(1 - \frac{K}{2}\right)\right)e^{-2KC_B^*} - \left(1 - \frac{K}{2}\right)^3 e^{-4KC_B^*}\right],$ $K = 1 - e^{-NTU_B/4}$	(2.21b)
∞	Parallel flow A or B	$\varepsilon = \frac{1 - e^{-NTU(1+C^*)}}{1 + C^*}$	(2.22)
Multi-pass counter cross-flow			
2	A	$\varepsilon_A = 1 - \left(\frac{K}{2} + \left(1 - \frac{K}{2}\right)e^{2K/C_A^*}\right)^{-1},\ K = 1 - e^{-NTU_A \cdot C_A^*/2}$	(2.23a)
	B	$\varepsilon_B = \frac{1}{C_B^*}\left[1 - \left(\frac{K}{2} + \left(1 - \frac{K}{2}\right)e^{2KC_B^*}\right)^{-1}\right],\ K = 1 - e^{-NTU_B/2}$	(2.23b)
3	A	$\varepsilon_A = 1 - \left(\left(1 - \frac{K}{2}\right)^2 e^{3K/C_A^*} + \left(K\left(1 - \frac{K}{4}\right) - \left(1 - \frac{K}{2}\right)\frac{K^2}{C_A^*}\right)e^{K/C_A^*}\right)^{-1},\ K = 1 - e^{-NTU_A \cdot C_A^*/3}$	(2.24a)
	B	$\varepsilon_B = \frac{1}{C_B^*}\left[1 - \left(\left(1 - \frac{K}{2}\right)^2 e^{3KC_B^*} + \left(K\left(1 - \frac{K}{4}\right) - \left(1 - \frac{K}{2}\right)K^2 C_B^*\right)e^{KC_B^*}\right)^{-1}\right],\ K = 1 - e^{-NTU_B/3}$	(2.24b)

(continued)

Table 2.2 (continued)

N_r	C_{min} side	Relation	Equation
4	A	$\varepsilon_A = 1 - \left(\frac{K}{2}\left(1 - \frac{K}{2} + \frac{K^2}{4}\right) + K\left(1 - \frac{K}{2}\right)\left(1 - 2\frac{K}{C_A^*}\left(1 - \frac{K}{2}\right)\right)e^{2K/C_A^*} + \left(1 - \frac{K}{2}\right)^3 e^{4K/C_A^*}\right)^{-1}$, $K = 1 - e^{-NTU_A \cdot C_A^*/4}$	(2.25a)
	B	$\varepsilon_B = \frac{1}{C_B^*}\left[1 - \left(\frac{K}{2}\left(1 - \frac{K}{2} + \frac{K^2}{4}\right) + K\left(1 - \frac{K}{2}\right)\left(1 - 2KC_B^*\left(1 - \frac{K}{2}\right)\right)e^{2KC_B^*} + \left(1 - \frac{K}{2}\right)^3 e^{4KC_B^*}\right)^{-1}\right]$, $K = 1 - e^{-NTU_B/4}$	(2.25b)
∞	Counterflow A or B	$\varepsilon = \frac{1 - e^{-NTU(1-C^*)}}{1 - C^* e^{-NTU(1-C^*)}}$	(2.26)

Equations (2.20) and (2.24) from Kays and London (1998); Eqs. (2.19) from ESDU 86018 (1991); Eqs. (2.21)–(2.23) from Taborek (1983) and ESDU 86018 (1991)

Fluid A mixed, Fluid B unmixed, $C_A^* = 1/C_B^*$, $\varepsilon_B = \varepsilon_A\, C_A^*$, $NTU_B = NTU_A\, C_A^*$, $C_A^* = C_A/C_B$

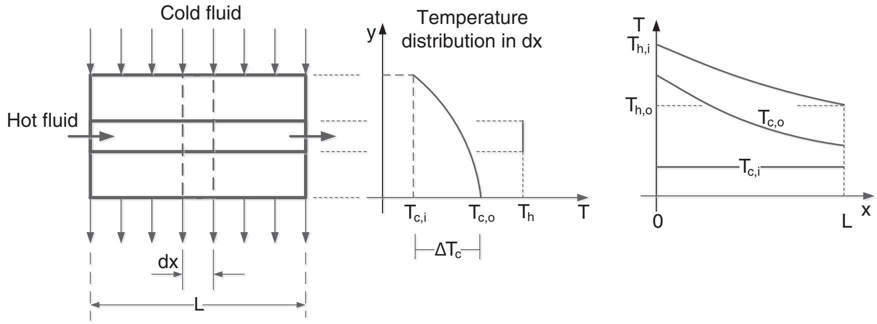

Fig. 2.2 Temperature variation along the transversal and longitudinal directions in a one pass cross-flow mixed-unmixed heat exchanger

Note that ΔT_c is the temperature variation of the cold fluid in the differential strip, that is, $\Delta T_c = T_{c,o} - T_{c,i}$, where the subscripts stand for the outlet and inlet temperatures of the external fluid (cold one) in the strip.

Given the fact that the mass flow rate of the cold fluid in the differential strip is small, it can be concluded that the heat capacity ratio for the strip heat exchanger is given by the following expression:

$$dC^* = \frac{dC_c}{C_h} \to 0 \tag{2.29}$$

This is a differential heat capacity ratio, which tends to zero since the mass flow rate of the cold fluid also tends to zero. Physically this result is equivalent as considering the differential strip heat exchanger a condenser, since the temperature of the hot fluid remains essentially constant, as shown in Fig. 2.2. The "local thermal effectiveness" of the strip exchanger, Γ, can thus be determined from Eq. (2.11) which assumes the following expression for the heat capacity ratio tending to zero:

$$\Gamma = \frac{\Delta T_c}{T_h - T_{c,i}} = 1 - \exp\left[-\frac{UdA}{dC_c}\right] \tag{2.30}$$

Assuming that both the frontal (approaching) area of the cold fluid side, A_{fr}, and the surface heat transfer area, A, are uniform over the heat exchanger width, the following expressions can be written:

$$\frac{dC_c}{dA_{fr}} = \frac{C_c}{A_{fr}} = \text{constant} \tag{2.31}$$

$$\frac{dC_c}{dA} = \frac{C_c}{A} = \text{constant} \tag{2.32}$$

Thus, introducing Eq. (2.32) into Eq. (2.30), the following local thermal effectiveness expression results, valid over the width L of the heat exchanger:

$$\Gamma = 1 - e^{-\frac{UA}{C_c}} = \text{constant} \tag{2.33}$$

The combination of Eqs. (2.27), (2.28), and (2.30) along with Eq. (2.31), leads to the following general equation:

$$\frac{dT_h}{T_h - T_{c,i}} = -\Gamma dC^* = -\Gamma \left(\frac{C_c}{C_h}\right)\left(\frac{dA_{fr}}{A_{fr}}\right) \tag{2.34}$$

Note that the values of the physical parameters C_c, C_h and A_{fr} are physical and geometric characteristics of the heat exchanger and as such considered as being constant. In addition, the set of equations developed so far are valid for a one pass cross-flow heat exchanger with one fluid mixed and another unmixed. This set of equations could be analytically integrated and the resulting *ε-NTU* correlation would be one of Eqs. (2.12a) or (2.12b) depending on which of the fluids is mixed or unmixed.

Cross-flow heat exchangers for engineering applications are commonly characterized by complex flow arrangements with several circuits and rows (see Chap. 6 for several examples). The previous set of equations Eqs. (2.27)–(2.34) does not have a trivial solution for those complex configurations since the conditions under which Eq. (2.34) has been derived are no longer valid due to: (1) application of Eqs. (2.32) and (2.33) is questionable in those cases; (2) the cold fluid inlet temperature at each tube row is not uniform as in Fig. 2.2. An adequate and accurate solution for heat exchanger complex geometries can be sought through a numerical computational procedure as the HETE code. This procedure is based on a set of algebraic equations, amenable to a numerical solution, whose development and analysis will be dealt with in the next section.

2.3 Finite Elements Governing Equations of the HETE Code

A differential point of view of the heat exchanger was considered in the last section. Though solutions of the differential set of equations for simple flow arrangements have been found, more complex flow arrangements are not amenable to simple solutions. In those cases, numerical solutions of a discretized set of equations could be the answer. This is the case of the HETE code whose finite elements governing equations will be developed in the present section.

The element in the HETE code is a finite volume heat exchanger involving a finite length of the in-tube fluid with its corresponding external fluid counterpart, as illustrated in Fig. 2.3. The governing equations of Sect. 2.2 are based on the assumption that the cold fluid mass flow rate is small in comparison with the hot fluid one, as suggested by Eq. (2.29). This is not the case for the element of Fig. 2.3. Thus the hot fluid temperature variation along the element cannot be neglected. Instead, it is assumed that it varies linearly so that its average temperature can be written as:

$$T_h^e = 0.5\left(T_{h,i}^e + T_{h,o}^e\right) \tag{2.35}$$

The superscript "*e*" has been introduced to designate an element whose identification, characterized by its space location, will be designated by the subscripts (i, j, k).

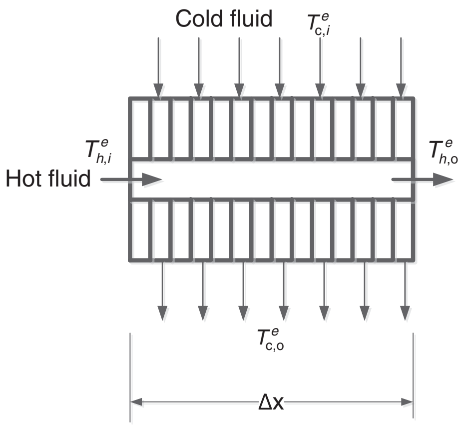

Fig. 2.3 Illustration of an element with associated inlet and outlet temperatures

By integrating the energy conservation for the hot fluid along the element, Eq. (2.27), the following expression results:

$$q^e = -C_h^e(T_{h,o}^e - T_{h,i}^e) \tag{2.36}$$

The integration of the energy conservation applied to the cold fluid, Eq. (2.28), followed by the introduction of Eq. (2.31), results in the following expression:

$$q^e = \Delta T_c^e \int_e dC_c = \Delta T_c^e \int_e \frac{C_c}{A_{fr}} dA_{fr} = \Delta T_c^e C_c^e \tag{2.37}$$

As previously observed, C_c and A_{fr} are the heat capacity of the cold fluid and the frontal area of the heat exchanger. At his point it is important to stress that, since the inlet as well as the outlet temperatures of the cold stream might vary along width of the element, the cold fluid variation along the element, $\Delta T_c^e = T_{c,o}^e - T_{c,i}^e$, is determined in terms of the average inlet and outlet temperatures of the cold fluid, that is, $T_{c,i}^e$ and $T_{c,o}^e$.

The closure of the set of algebraic equations requires an additional equation which is the one of the thermal effectiveness, given by integration of Eq. (2.30) over the total area of the element. For that purpose, Eq. (2.32) must be taken into account in such a way to introduce the relation $dA^e/dC_c^e = A^e/C_c^e$ into Eq. (2.30) with the following result:

$$\Gamma^e = \frac{\Delta T_c^e}{(T_h^e - T_{c,i}^e)} = 1 - e^{-\frac{UA^e}{C_c^e}} \tag{2.38}$$

Note that the temperature of the hot fluid is the average over the element instead of the inlet one. The right hand side of the equation above is formally identical to that of Eq. (2.33) though in this case the area and the heat capacity refer to the element. The result expressed by Eq. (2.38) is physically equivalent to admit that the size of the element is small. The smaller the element the more accurate will be the solution of the set of governing equations of the element. This aspect will be addressed further on under the discussion of numerical results from the HETE code.

The above set of governing equations for each element is a closed system with four equations, Eqs. (2.35)–(2.38), and four unknowns namely $T^e_{h,o}$, $T^e_{c,o}$, q^e, and Γ^e. The other known parameters involve the element inlet temperatures, $T^e_{h,i}$ and $T^e_{c,i}$, the heat transfer area of the element, the overall heat transfer coefficient, and the heat capacities of both fluids over the element. The numerical solution extended to the whole heat exchanger requires an iterative procedure which will be introduced in the following chapter.

An alternate approach to the solution of the set of governing equations would be through the reduction of the number of unknowns (and equations) by rearranging the four available equations in such a way to obtain explicit expressions for the element outlet temperatures of the hot and cold streams which assume the following expressions:

$$T^e_{c,o} = \frac{\Lambda^e + 2(1-\Gamma^e)}{2+\Lambda^e} T^e_{c,i} + \frac{2\Gamma^e}{2+\Lambda^e} T^e_{h,i} \tag{2.39}$$

$$T^e_{h,o} = \frac{2-\Lambda^e}{2+\Lambda^e} T^e_{h,i} + \frac{2\Lambda^e}{2+\Lambda^e} T^e_{c,i} \tag{2.40}$$

The parameter Λ^e is given by the following expression

$$\Lambda^e = C^e_c \Gamma^e / C^e_h \tag{2.41}$$

Note that the outlet temperatures can be directly determined from Eqs. (2.39) and (2.40) since the element thermal effectiveness can be determined from Eq. (2.38). The explicit expressions for the element outlet temperatures have been implemented into the HETE code, as will be shown in the next chapter.

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