Chapter 2
Scenarios, Notation and Stability Conditions for Adaptive Predictive Control

2.1 Introduction

This chapter further develops the description of adaptive predictive control concepts expounded in the previous chapter, defining different scenarios where they can be applied, introducing notation that defines a mathematical language for analysis, and determining the design conditions that guarantee the desired performance of the controller.

After defining the scenarios, we will describe the process equation mathematically and the two functions that the Adaptive Predictive (AP) model carries out. Firstly, we will consider what is called the ideal case without pure time delays in the process, then consider the existence of pure time delays and later what is called the *real case*, which considers hypotheses that define the actual operating context for industrial control applications. Secondly, we will define the control objective from the perspective of stability, introducing the concepts of global and asymptotic stability for the adaptive predictive controller.

Finally, we will state a Conjecture that establishes conditions for the design of both the Driver Block and the Adaptive Mechanism that, when verified, guarantees the desired stability and performance for the adaptive predictive controller.

2.2 Scenarios for Design and Analysis

In this section, without entering into the issue of notation, different scenarios will be introduced which can be considered when addressing the problem of the design of an adaptive predictive controller in relation to the process and its operating environment.

To define these scenarios within a theoretical framework enabling this design and its systematic analysis, it is necessary to use a mathematical model to describe the process dynamic and its interaction with the environment. This model plays a relevant role in this context. In effect, the control system is conceived so that the real process behaves in accordance with certain specifications and during the control

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system design phase, considers the model as "equivalent" to the process and at least on paper, ensures that the model under control behaves in the desired way.

It is important to point out that the process model and the Adaptive Predictive (AP) model are different entities. While it has been stated that the AP model is primarily a tool describing the process, it is also the tool that the control system uses to predict the process response and to calculate the control action. In this and subsequent chapters of the book, we will consider linear models in discrete time like those described in Chap. 1 (Sect. 1.6), for both the process and the AP model. They do not have to agree and in general they do not. Moreover, the consideration that a process is described by a model assumes the introduction of certain hypotheses which may be more or less realistic, depending on the difficulties of the process and its interaction with the operating environment. In this section we define design scenarios for adaptive predictive control systems paying attention to various hypotheses, and progressively approaching the context of industrial processes.

The problem of the synthesis of an adaptive predictive control system can initially be approached in a theoretical manner for an *ideal case*, based on the following hypotheses:

(a) The process is described by linear equations with constant parameters.

(b) The equations of the process and the model have the same order.

(c) There exist no measurement noises, or unmeasurable disturbances acting on the process.

However, if we want to guarantee that the adaptive predictive control system works satisfactorily in an industrial environment, the synthesis problem must be approached using hypotheses that agree with such an environment. These hypotheses can be the following:

(a_1) The process is described by linear equations, but with time varying parameters.

(b_1) The process and model equations may have different orders.

(c_1) There exist measurement noises and unmeasurable disturbances randomly acting on the process.

The hypotheses given above are useful for defining the various scenarios or *real cases* that we will consider in this book, and are as follows:

- **The real case with no difference in structure**. In this case hypotheses (a) and (b) of the ideal case will be maintained, but hypothesis (c) will be substituted by hypothesis (c_1).

- **The real case with difference in structure**. In this case only hypothesis (a) of the ideal case will be maintained, while (b) and (c) will be substituted by hypotheses (b_1) and (c_1).

- **The real case with time varying parameters**. This case takes into account hypotheses (a_1), (b_1) and (c_1). The first hypothesis accounts for the basically non-linear and variable nature of the industrial process. Hence, when describing it through linear equations, parametric changes will occur due to any kind of variation in the conditions of the operation environment.

2.3 Process and AP Model in the Ideal Case

As was presented in Chap. 1 and described in Fig. 2.1, the adaptive predictive control scheme resulted from the combination of a predictive controller and an adaptive system. This combination, as has already been explained, does not consist of a simple juxtaposition of both systems, but exploits the benefits derived from their interaction. In effect, the knowledge of the process dynamic acquired by the adaptive model, by means of the adaptation mechanism, is used by the predictive model of the predictive controller to calculate the control signal.

We can better understand this combination when we consider that both systems share the same model, which is adjusted periodically by the adaptation mechanism and which carries out two different functions, one in the predictive control scheme, the other in the adaptive system. This shared model is given the name *adaptive predictive (AP) model*, alluding to the two functions it performs.

In this section we will consider the ideal scenario based on two simple examples, one without and the other with pure time delays, to give a mathematical description of the process and of the two AP model functions. Also, we will introduce notation that will permit us to generalize these descriptions and which will be used throughout the development and proofs presented in this book.

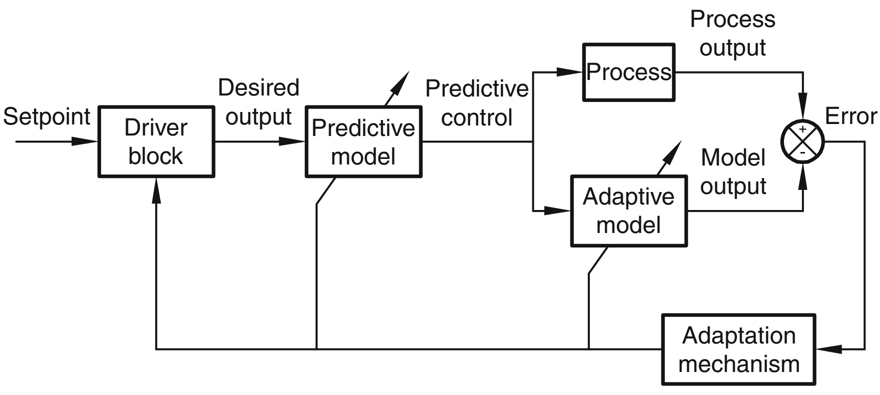

Fig. 2.1 Adaptive predictive control scheme

2.3.1 Example of a Process without Pure Delays

In this first example we will consider that the process shown in Fig. 2.1 can be described by the following transfer function in z:

$$T(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}}\,. \tag{2.1}$$

It can be observed that the process does not contain delays, except for the inherent delay due to discretization. There are no noise or perturbations acting on the process, and its dynamic behaviour is governed by the following difference equation:

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + b_1 u(k-1) + b_2 u(k-2), \tag{2.2}$$

where u and y represent the input and output variables respectively.

One of the two functions carried out by the AP model, as can be seen in Fig. 2.1, is to generate the output of the adaptive model. This model output is an estimation of the process output that can be generated at every instant k in two different ways, represented by the following equations for the example under consideration:

$$\begin{aligned}\hat{y}(k|k-1) = {} & \hat{a}_1(k-1)y(k-1) + \hat{a}_2(k-1)y(k-2) \\ & + \hat{b}_1(k-1)u(k-1) + \hat{b}_2(k-1)u(k-2),\end{aligned} \tag{2.3}$$

$$\hat{y}(k|k) = \hat{a}_1(k)y(k-1) + \hat{a}_2(k)y(k-2) + \hat{b}_1(k)u(k-1) + \hat{b}_2(k)u(k-2). \tag{2.4}$$

The term $\hat{y}(k|k-1)$ represents the estimation of the process output at the instant k, which is calculated according to (2.3) using the parameter values of the AP model $(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$ given by the adaptation mechanism at the instant $k-1$. On the other hand, $\hat{y}(k|k)$ indicates the estimation of the output variable at the instant k but using all the information available up to instant k, particularly the most recent AP model parameters adjusted at the instant k. The estimation achieved by the Eq. (2.3) is called the *a priori* estimation of the process output as it is calculated based on the information received at the previous instant $k-1$. The calculation carried out by the Eq. (2.4) is called the *a posteriori* estimation. In both cases, if the AP model parameters were equal to those of the process, we would have an exact estimation of the process output, which justifies the selection of (2.3) and (2.4) to describe the estimating function of the AP model.

The type of notation used in the previous equations will be the standard for the rest of the book. In general, for a generic variable whose measured value at instant k is represented by $v(k)$, its estimated value at instant k_f, using only the information available up to instant k_i, will be expressed as $\hat{v}(k_f|k_i)$. This information will consist of information about the process input and output variables and the AP model parameters. On the other hand, to refer to a generic parameter of the form $p(k)$, we will indicate that it is a parameter of the model that describes the process, while the notation $\hat{p}(k)$ will represent the estimation of this parameter as calculated by the adaptive mechanism at the indicated instant k.

The other function of the AP model is to calculate the predictive control action. For this calculation, the AP model calculates the control action to be applied at each instant k, so that the predicted future process output equals a desired value. In the example we are considering, where there are no pure time delays, the prediction for instant $k+1$ is achieved in the following way:

$$\hat{y}(k+1|k) = \hat{a}_1(k)y(k) + \hat{a}_2(k)y(k-1) + \hat{b}_1(k)u(k) + \hat{b}_2(k)u(k-1). \qquad (2.5)$$

This equation provides the process output prediction for the instant $k+1$, $\hat{y}(k+1|k)$, using both the value of the AP model parameters and the inputs/outputs of the process all updated at instant k. Note that the equation uses the same notation introduced in (2.3) and (2.4) since the prediction of the process output at instant $k+1$ is still an estimation of the output at instant $k+1$, using all the acquired knowledge of the process dynamic up to instant k.

Making the predicted output in (2.5) equal to a desired value expressed in the form $y_d(k+1)$, we obtain the following calculation of the adaptive predictive control signal:

$$u(k) = \frac{y_d(k+1) - \hat{a}_1(k)y(k) - \hat{a}_2(k)y(k-1) - \hat{b}_2(k)u(k-1)}{\hat{b}_1(k)}. \qquad (2.6)$$

In order to generalise the concepts expressed previously in this example and to simplify the writing of the corresponding equations, we will introduce the following additional notation:

$$\begin{aligned}
\theta^T &= [a_1, a_2, b_1, b_2], \\
\hat{\theta}(k)^T &= [\hat{a}_1(k), \hat{a}_2(k), \hat{b}_1(k), \hat{b}_2(k)], \\
\hat{\theta}_0(k)^T &= [\hat{a}_1(k), \hat{a}_2(k), \hat{b}_2(k)]; \qquad \hat{\theta}_1(k) = \hat{b}_1(k), \\
\phi(k-1)^T &= [y(k-1), y(k-2), u(k-1), u(k-2)], \\
\phi(k)^T &= [y(k), y(k-1), u(k), u(k-1)], \\
\phi_0(k)^T &= [y(k), y(k-1), u(k-1)]\,,
\end{aligned} \qquad (2.7)$$

where T indicates the transposed vector.

In accordance with this notation, the parameters of the process are grouped into a vector θ and similarly, the parameters of the AP model are grouped in a vector $\hat{\theta}(k)$. In the same way, we define a vector ϕ that contains the values of the input and output variables in the equations at different time instants.

Using the previous notation, the Eqs. (2.2)–(2.6) can be expressed in the following form:

$$\begin{aligned}
y(k) &= \theta^T \phi(k-1), \\
\hat{y}(k|k-1) &= \hat{\theta}(k-1)^T \phi(k-1), \\
\hat{y}(k|k) &= \hat{\theta}(k)^T \phi(k-1), \\
\hat{y}(k+1|k) &= \hat{\theta}(k)^T \phi(k) = \hat{\theta}_0(k)^T \phi_0(k) + \hat{\theta}_1(k) u(k), \\
u(k) &= \frac{y_d(k+1) - \hat{\theta}_0(k)^T \phi_0(k)}{\hat{\theta}_1(k)}.
\end{aligned} \tag{2.8}$$

The new notation greatly simplifies the equations to express different relationships between the input and output variables which are seen to be scalar products of parameter and input-output vectors. Moreover, these expressions are more general than the equivalent ones in (2.2)–(2.6) since its expression would not change even if the process and the adaptive models were of a higher order than considered in this case. It would simply be necessary to increase the dimensions of the vectors in accordance with the appropriate order.

2.3.2 Example of Process with Pure Time Delays

In this second example, we will consider the process seen in Fig. 2.1 responds to the following transfer function in z:

$$T(z) = z^{-1} \frac{b_1 z^{-1}}{1 - a_1 z^{-1}} . \tag{2.9}$$

The process is of first order and has, in addition to a discretization delay, a pure time delay of one control period. Neither noise nor perturbations acting on the process are considered, and its dynamic behaviour is described by the following difference equation:

$$y(k) = a_1 y(k-1) + b_1 u(k-2). \tag{2.10}$$

In this case, for the mathematical expression of the two functions carried out by the AP model, there exist two alternatives that we will now consider.

2.3.2.1 Alternative 1

The a priori and a posteriori estimations of the process output at the instant k (first function of the AP model) can be expressed in the following form:

$$\begin{aligned}
\hat{y}(k|k-1) &= \hat{a}_1(k-1) y(k-1) + \hat{b}_1(k-1) u(k-2), \\
\hat{y}(k|k) &= \hat{a}_1(k) y(k-1) + \hat{b}_1(k) u(k-2) .
\end{aligned} \tag{2.11}$$

In both cases it can be seen that due to the existence of a pure time delay, the calculated estimation at the instant k does not depend on the control action applied to the process at the instant $k-1$, but on the action $u(k-2)$ applied at $k-2$.

The function of the AP model prediction can be described by means of the equation

$$\hat{y}(k+1|k) = \hat{a}_1(k)y(k) + \hat{b}_1(k)u(k-1)\,, \tag{2.12}$$

where, due to a pure time delay, it can be seen that the predicted output at $k+1$ does not depend on the control action at k, but on the control action at $k-1$, and this is the reason the control action $u(k)$ cannot be calculated based on this equation. In order that the output prediction of the process be a function of the control signal $u(k)$, we must consider this output prediction at the instant $k+2$ which will depend on the output at instant $k+1$. Since we do not have the measurement of the process output at instant $k+1$, we can substitute it for the estimation of this measurement given by the Eq. (2.12). Hence, the prediction in question can be expressed in the form

$$\begin{aligned} \hat{y}(k+2|k) &= \hat{a}_1(k)\hat{y}(k+1|k) + \hat{b}_1(k)u(k) \\ &= \hat{a}_1(k)^2\, y(k) + \hat{b}_1(k)u(k) + \hat{a}_1(k)\hat{b}_1(k)u(k-1), \end{aligned} \tag{2.13}$$

where the control action $u(k)$ appears explicitly.

Substituting the predicted process output at the instant $k+2$ into (2.13) for the corresponding desired output $y_d(k+2)$, we obtain

$$u(k) = \frac{y_d(k+2) - \hat{a}_1(k)^2 y(k) - \hat{a}_1(k)\hat{b}_1(k)u(k-1)}{\hat{b}_1(k)}, \tag{2.14}$$

which represents the calculation of the adaptive predictive control signal for this alternative.

2.3.2.2 Alternative 2

The second alternative for defining the two functions of the AP model considers the recursive substitution of the value of $y(k-1)$, obtained using the process model (2.10), into the Eq. (2.10). The following equation is derived and used to describe the dynamic behaviour of the process:

$$\begin{aligned} y(k) &= a_1[a_1 y(k-2) + b_1 u(k-3)] + b_1 u(k-2) \\ &= a_1^2 y(k-2) + b_1 u(k-2) + a_1 b_1 u(k-3). \end{aligned} \tag{2.15}$$

In this equation the value of the process output measurement at the instant k is a function of the inputs and outputs of the process measured at the instant $k-2$ and at previous instants. Given that, in the context of adaptive predictive control, the parameters of the process are unknown, we can rewrite the Eq. (2.15) in the form

$$y(k) = a_1^* y(k-2) + b_1^* u(k-2) + b_2^* u(k-3). \tag{2.16}$$

Based on this method of representing the process dynamics, the first function of the AP model of estimating the process output can be defined by means of the following equations:

$$\hat{y}(k|k-1) = \hat{a}_1(k-1)y(k-2) + \hat{b}_1(k-1)u(k-2) + \hat{b}_2(k-1)u(k-3), \tag{2.17}$$

$$\hat{y}(k|k) = \hat{a}_1(k)y(k-2) + \hat{b}_1(k)u(k-2) + \hat{b}_2(k)u(k-3), \tag{2.18}$$

where (2.17) generates the a priori estimation and (2.18) the a posteriori estimation, and the parameters of the AP model are an estimation of the parameters of the process Eq. (2.16).

The function of the AP model prediction can now be defined by means of the equation:

$$\hat{y}(k+2|k) = \hat{a}_1(k)y(k) + \hat{b}_1(k)u(k) + \hat{b}_2(k)u(k-1), \tag{2.19}$$

where the predicted process output for the instant $k+2$ is explicitly a function of the predictive control signal $u(k)$. Substituting the predicted process output at instant $k+2$ into (2.19) for the corresponding desired output $y_d(k+2)$, we obtain the control signal

$$u(k) = \frac{y_d(k+2) - \hat{a}_1(k)y(k) - \hat{b}_2(k)u(k-1)}{\hat{b}_1(k)}, \tag{2.20}$$

that represents the calculation of the adaptive predictive control action for this alternative.

The two alternatives used to deal with this example are valid. Nevertheless, for the sake of simplicity, we will use in this book the second alternative when processes with pure time delays are considered.

As in the first example, and with the objective of facilitating the generalization of the results obtained and simplifying the writing of the corresponding equations, we will introduce the following notation for this example:

$$\begin{aligned}
\theta^T &= [a_1^*, b_1^*, b_2^*], \\
\hat{\theta}(k)^T &= [\hat{a}_1(k), \hat{b}_1(k), \hat{b}_2(k)], \\
\hat{\theta}_0(k)^T &= [\hat{a}_1(k), \hat{b}_2(k)]; \qquad \hat{\theta}_1(k) = \hat{b}_1(k), \\
\phi(k)^T &= [y(k), u(k), u(k-1)], \\
\phi_0(k)^T &= [y(k), u(k-1)]\,,
\end{aligned} \tag{2.21}$$

where θ and $\hat{\theta}(k)$ are the vectors of the process parameters and of the AP model respectively, and $\phi(k)$ is the vector of the inputs and outputs. Using this notation the Eqs. (2.16)–(2.20) can now be written in the following form

$$
\begin{aligned}
y(k) &= \theta^T \phi(k-d), \\
\hat{y}(k|k-1) &= \hat{\theta}(k-1)^T \phi(k-d), \\
\hat{y}(k|k) &= \hat{\theta}(k)^T \phi(k-d), \\
\hat{y}(k+d|k) &= \hat{\theta}(k)^T \phi(k) = \hat{\theta}_0(k)^T \phi_0(k) + \hat{\theta}_1(k) u(k), \\
u(k) &= \frac{y_d(k+d) - \hat{\theta}_0(k)^T \phi_0(k)}{\hat{\theta}_1(k)},
\end{aligned}
\tag{2.22}
$$

where the integer d represents the sum of the discretization delay and the pure time delay of the process. In the example that we are considering the pure time delay has been 1 and therefore we have $d = 2$.

These equations represent a simplified form of the process dynamic and the functions of the AP model in this example. However, they are general in the sense that they serve in any situation where the process and the AP model are of higher order than the example under consideration, and the process has any number of pure delays.

2.4 General Description of the Real Case

2.4.1 Description of the Process

For the real case with time varying parameters, let us consider a single-input, single-output process where the relation between the inputs and outputs, using notation similar to that used in the preceding section, may be given by

$$
\begin{aligned}
y_a(k) = &\sum_{i=1}^{n} a_i(k) y_a(k-i) + \sum_{i=1}^{m} b_i(k) u_a(k-r-i) \\
&+ \sum_{i=1}^{p} c_i(k) w_a(k-r_1-i) + \xi(k),
\end{aligned}
\tag{2.23}
$$

where y_a, u_a and w_a are the actual, present or previous values of the process output, input and measurable disturbance respectively; r and r_1 represent the pure time delays related to the process input and measurable disturbances respectively; $\xi(k)$ represents the effect of the unmeasured disturbances on the process output at instant k; and a_i, b_i and c_i are the process parameters which, in the context of adaptive systems, are generally unknown and time variant.

Using Eq. (2.23) to express the outputs y_a at instants $k-1, \ldots, k-r$ and recursively substituting the results into the right-hand side of (2.23), the above process equation may be written in the following form

$$
\begin{aligned}
y_a(k) = &\sum_{i=1}^{n} a_i^*(k) y_a(k-r-i) + \sum_{i=1}^{m+r} b_i^*(k) u_a(k-r-i) \\
&+ \sum_{i=1}^{p+r} c_i^*(k) w_a(k-r_1-i) + \xi(k),
\end{aligned} \tag{2.24}
$$

where parameters a_i^*, b_i^* y c_i^* are easily obtained from a_i, b_i y c_i.

In order to use a more manageable notation, the Eq. (2.24) will be expressed in the form

$$
y_a(k) = \theta(k)^T \phi_a(k-d) + \xi(k), \tag{2.25}
$$

where

$$
\begin{aligned}
\phi_a(k-d)^T = &[y_a(k-d), y_a(k-d-1), \ldots, y_a(k-d-n+1), \\
&u_a(k-d), u_a(k-d-1), \ldots, u_a(k-2d-m+2), \\
&w_a(k-d_1), w_a(k-d_1-1), \ldots, w_a(k-p-d-d_1+2)]
\end{aligned}
$$

and

$$
\begin{aligned}
\theta(k)^T = &[a_1^*(k), a_2^*(k), \ldots, a_n^*(k),\ b_1^*(k), b_2^*(k), \ldots, b_{m+r}^*(k), \\
&c_1^*(k), c_2^*(k), \ldots, c_{p+r}^*(k)],
\end{aligned}
$$

since $\phi_a(k-d)$ and $\theta(k)$ are the input/output and the parameters of the process vectors at instants $k-d$ and k respectively, we define them in a manner similar to that used in the ideal case with delays. The integer d represents the time delay related to the process input and includes the discretization delay plus the pure delay, that is, $d = r + 1$; d_1 is the equivalent for the measurable disturbances, $d_1 = r_1 + 1$.

The measured values of the process variables differ from their actual values due to measurement errors, noise, etc., as expressed by the following equations:

$$
\begin{aligned}
y(k) &= y_a(k) + n_y(k), \\
u(k) &= u_a(k) + n_u(k), \\
w(k) &= w_a(k) + n_w(k),
\end{aligned} \tag{2.26}
$$

where, for each variable, subscript a denotes its actual value and n denotes the additional corrupting signal.

As a result, the corresponding measured vector ϕ becomes

$$
\phi(k) = \phi_a(k) + n_\phi(k). \tag{2.27}
$$

This vector is known as the *input/output (I/O) vector* or *regression vector*. Substituting (2.26) and (2.27) into process Eq. (2.25) we obtain

$$y(k) = \theta(k)^T \phi(k-d) + \Delta(k),$$
$$\Delta(k) = n_y(k) - \theta(k)^T n_\phi(k-d) + \xi(k). \tag{2.28}$$

$\Delta(k)$ can be referred to as the *perturbation signal* and represents the effect of the unmeasured perturbations and measurement noise acting on the process. The previous assumptions about the relation between the inputs and outputs of the process come from hypotheses (a_1) and (c_1) of the real case, in which the existence of measurement noise and unmeasured perturbations are considered and where the process parameters may vary with time.

The process Eq. (2.28) may also be written in the form:

$$y(k) = \theta_o(k)^T \phi_o(k-d) + \theta_1(k)u(k-d) + \Delta(k), \tag{2.29}$$

where $\theta_1(k)$ is the single parameter included in vector $\theta(k)$ in the process Eq. (2.28) that multiplies the control signal in the inner product at instant $k-d$, $u(k-d)$. $\theta_o(k)$ and $\phi_o(k-d)$ result after the exclusion of the parameter $\theta_1(k)$ and the control signal $u(k-d)$ from the parameter vector $\theta(k)$ and the I/O vector $\phi(k-d)$ respectively.

The parameter vector $\theta_o(k)$ and the parameter $\theta_1(k)$ are always assumed to be bounded, while the absolute value of $\theta_1(k)$ is assumed to be greater than a certain positive constant, that is, $|\theta_1(k)| > \nu > 0$.

2.4.2 Description of the AP Model Functions

The simplified notations used in this section for the mathematical description of the AP model functions have been introduced in the adaptive predictive control scheme shown in Fig. 2.2.

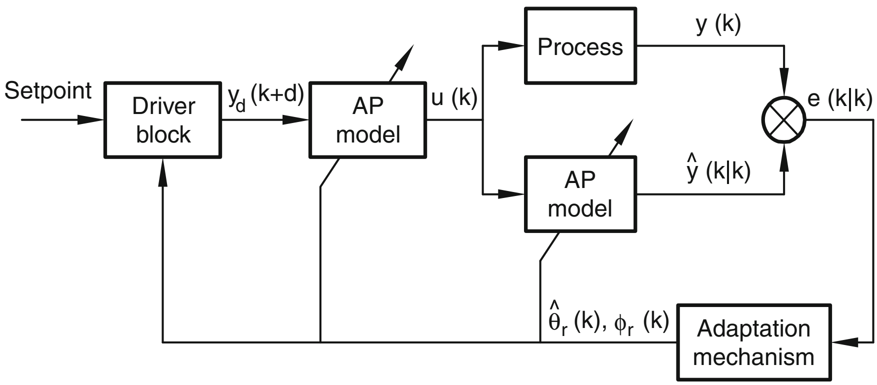

Fig. 2.2 Adaptive predictive control scheme

In the adaptive system, the AP model provides an estimation of the process output at instant k using the model parameters also estimated at instant k, which will be denoted by $\hat{\theta}_r(k)$, and the control signals and process outputs already applied or measured at previous instants, which are included in the I/O vector denoted by $\phi_r(k-d)$. This estimation is expressed in the form

$$\hat{y}(k|k) = \hat{\theta}_r(k)^T \phi_r(k-d), \tag{2.30}$$

where

$$\begin{aligned}\phi_r(k-d)^T = [&y(k-d), y(k-d-1), \ldots, y(k-d-n_r+1),\\ &u(k-d), u(k-d-1), \ldots, u(k-d-m_r+1),\\ &w(k-d_1), w(k-d_1-1), \ldots, w(k-d_1-p_r+1)]\end{aligned}$$

and

$$\begin{aligned}\hat{\theta}_r(k) = [&\hat{a}_1(k), \hat{a}_2(k), \ldots, \hat{a}_{n_r}(k),\ \hat{b}_1(k), \hat{b}_2(k), \ldots, \hat{b}_{m_r}(k),\\ &\hat{c}_1(k), \hat{c}_2(k), \ldots, , \hat{c}_{p_r}(k)].\end{aligned}$$

The dimensions of ϕ_r and θ_r are usually less than or equal to the dimensions of ϕ and θ previously considered in process equation (2.28). Thus, $\phi_r(k-d)$ contains a subset of the most recent process inputs and outputs included in $\phi(k-d)$. These assumptions account for hypothesis (b_1) of the real case, in which it was considered that the process and model equations had different orders.

The a posteriori and a priori estimation errors are given by the equations

$$e(k|k) = y(k) - \hat{y}(k|k) = y(k) - \hat{\theta}_r(k)^T \phi_r(k-d) \tag{2.31}$$

$$e(k|k-1) = y(k) - \hat{y}(k|k-1) = y(k) - \hat{\theta}_r(k-1)^T \phi_r(k-d). \tag{2.32}$$

The predictive function of the AP model can now be defined by means of the expression

$$\hat{y}(k+d|k) = \hat{\theta}_r(k)^T \phi_r(k). \tag{2.33}$$

When $r > r_1$, some terms within $\phi_r(k)$ related to the disturbance w will not have been measured at instant k yet and, since they will be generally unknown, their value will be taken as equal to the latest measured value. For the sake of simplicity we will assume in the following that $r \leq r_1$.

Applying the principle of predictive control, that is to say, substituting the predicted value for the desired at $k+d$, $y_d(k+d)$, we obtain the expression

$$y_d(k+d) = \hat{\theta}_r(k)^T \phi_r(k). \tag{2.34}$$

This equation may also be written in the form

$$y_d(k+d) = \hat{\theta}_{ro}(k)^T \phi_{ro}(k) + \hat{\theta}_1(k)u(k), \tag{2.35}$$

where $\hat{\theta}_{ro}(k)$ and $\phi_{ro}(k)$ result from excluding the parameter $\hat{\theta}_1(k)$ and the control signal $u(k)$, respectively, from $\hat{\theta}_r(k)$ and $\phi_r(k)$.

The predictive control law can be written from (2.35) in the form

$$u(k) = \frac{y_d(k+d) - \hat{\theta}_{ro}(k)^T \phi_{ro}(k)}{\hat{\theta}_1(k)}. \tag{2.36}$$

Clearly, the adaptation mechanism must always guarantee that the parameter $\hat{\theta}_1(k)$ is not zero for any instant k.

The difference between the process output and the desired output is defined as the *control* or *tracking error*

$$\varepsilon(k) = y(k) - y_d(k). \tag{2.37}$$

which will play an important role in characterizing the performance of adaptive predictive controllers, as is considered in the following section.

2.5 Control Objectives

Recalling the basic concepts introduced in the previous chapter, if the process dynamic is known, the application of predictive control allows us to guide the process output by means of suitably selected trajectories. The precision of the guide is limited only by the level of noise and perturbations, defined in the previous section as the perturbation signal, acting on the process. In this context, the control objective can be determined by the design objective of the driver block, which generates the desired trajectory for the process output, and which can be defined conceptually in the two following points:

- The desired trajectory must drive the process output towards the setpoint value, as set by the operator, in accordance with the desired dynamic. As a consequence, the desired trajectory remains bounded, as long as changes in the setpoint value introduced by the operator also remain bounded, which from now on, will always be considered the case.
- The desired trajectory must be *physically realizable*, that is, the sequence of control signals capable of producing a process output that follows the desired trajectory must also be bounded.

Nevertheless, it is difficult in industrial practice to have a precise knowledge of the process dynamic. Even though we might, on occasion, be able to obtain some knowledge of the process, given its varying dynamic, it could evolve at any time, and that does happen frequently in the industrial domain. The objective in adding an adaptive system to the predictive controller is precisely to achieve the satisfactory results that predictive control could obtain if the process dynamic were known in the variable operating environment under consideration.

Consequently, the objectives we should expect to reach with the application of adaptive predictive control can be conceptually defined as follows:

1. After a certain period of adaptation, the process output must follow a desired trajectory with a tracking error which must always remain bounded in the real case and must tend towards zero in the ideal case.
2. The desired trajectory must respond to the desired dynamic, be bounded and physically realizable.

The first point concerns the design of the adaptation mechanism, whereas the second point summarizes the two design objectives of the driver block noted previously. In fact, the boundedness of the control sequence is imposed, in practice, by the limitations of the actuators which determine the control action applied to the process. Also, the boundedness of the desired output is naturally associated with a limited variation range of the sensors which measure the process variables.

These boundedness conditions do not, in practice, pose limitations on the ability to control, since any variable that due to its nature would evolve in an unbounded way, could always be controlled by means of an associated incremental or derivative variable evolving within a certain limited variation range.

In short, the control objectives we have presented can be summarized by saying that the ultimate aim of adaptive predictive control is to make the process output follow a desired, bounded and physically realizable trajectory.

2.6 Design from the Perspective of Stability

In this section we transfer the above intuitive control objectives to a mathematical setting in terms of stability. Thus we will have a framework with which to work out all the subsequent formulation involved in the design of AP controllers in this and the following chapters of this book.

The stability theory results, presented in this book and in previous literature, are in fact related to the context of a control loop, which has usually been referred so far in the stability analysis as control system. In the following, we will keep this denomination in the stability analysis for the control loop, since it can properly be considered as a simple case of a control system and it is in agreement with previous literature. On the other hand, stability of control loops must imply stability of the control system including them.

The stability perspective of the adaptive system design was mentioned in Sect. 1.10. Within the adaptive system operation, the adaptation mechanism must adjust the parameters of the AP model in order to make this model produce an output that is as close to the process output as possible when both receive the same input. Thus it is reasonable to characterize the performance of the adaptive system by the difference between the process output and the AP model output. In our case, this difference is represented by the a posteriori estimation error $e(k|k)$.

In the ideal case considered in Sect. 2.2, under hypotheses (a)–(c), the result we should expect from a good solution to the problem of synthesis of the adaptation mechanism is that the error $e(k|k) \to 0$ as $k \to \infty$ from any initial condition. If we obtain this result, associating the estimation error $e(k|k)$ with the state of the adaptive system with an equilibrium state at zero, we can say that the adaptive system is globally asymptotically stable, as defined classically in Appendix A.

In the real cases considered in Sect. 2.2 it is not realistic to require the estimation error to tend asymptotically to zero since, for example, simply the measurement noise could cause this error to deviate from zero, even in the hypothetical case that this value had been reached. Therefore, the result to be expected from a good solution to the problem of designing the adaptation mechanism is that from any initial condition, the error should become bounded after a certain sampling instant k_f and the corresponding bound should be the smallest possible taking into account the level of noise, disturbances of all types, and parametric changes acting on the process. This result can be expressed mathematically in the form

$$|e(k|k)| < \bar{M} \text{ for all } k \geq k_f.$$

In the real cases, we may also associate the error $e(k|k)$ with the state of the adaptive system, but now considering the disturbances and noise as exogenous inputs. Then we can interpret the above boundedness condition of this error in terms of the stability concepts outlined in Appendix A by saying that the adaptive system is externally stable.

When considering the basic adaptive predictive control loop or control system, we would like to relate the desired performance to the corresponding stability concepts considered above for the adaptive system. Thus we can assume that the tracking error $\varepsilon(k)$, which represents the difference between the driving desired output and the process output, is related to the state of the adaptive predictive control system (APCS) and, if the adaptive system has been designed to be stable, the tracking error should satisfy the stability properties previously considered for the estimation error $e(k|k)$.

However, this result is not sufficient to cover the desired performance objectives for APCS as stated in the previous section. In fact, the AP controller has to generate a control signal within conditions imposed in practice; mainly that it must be bounded. As will be analyzed in Chap. 3, in order to make the process output follow certain trajectories, it may be necessary to apply unbounded control signals. In such cases, even with stable estimation and tracking errors, we would obtain an undesirable APCS performance. In the same way, the process output driven by APCS will have to evolve bounded and within the range limits of the sensors. The same consideration may also be extended to the different kinds of perturbation. All these requirements on the input/output signals can be included in a single condition by stating that the input/output vector $\phi(k)$ must be bounded. This condition has necessarily to be included for satisfactory performance of APCS.

Taking the preceding considerations into account, we may now state the following definition of *global stability* for APCS that corresponds to the desired performance.

Definition 2.1 An adaptive predictive control system is said to be *globally stable* if the following conditions are satisfied:

(1) $|\varepsilon(k)| \leq M < +\infty \qquad \forall k \geq k_f > 0.$

(2) $\|\phi(k)\| \leq \Omega < +\infty \qquad \forall k \geq k_f > 0.$

$\|\cdot\|$ denotes the Euclidean norm.

The above definition corresponds to the stability result that may be expected in the real cases. For the ideal case, the expected result will correspond to the following definition.

Definition 2.2 An adaptive predictive control system is said to be *globally asymptotically stable* if the following conditions are satisfied:

(1) $\varepsilon(k) \to 0$ as $k \to \infty$.

(2) $\|\phi(k)\| \leq \Omega < +\infty \qquad \forall k \geq k_f > 0.$

The methodological and theoretical developments that follow in this book are aimed at the design of adaptive predictive control systems that verify the stability results expressed in the prior definitions, and particularly the design of the driver block and the adaptation mechanism.

2.7 Stability Conditions

This section establishes the principles of adaptive predictive control system design, which we will provide in this book by means of a conjecture. This conjecture establishes conditions for the driver block and the adaptation mechanism which guarantee the global stability of the adaptive predictive control system in the sense of Definition 2.1 (for the real case) and Definition 2.2 (for the ideal case), and as a consequence, the achievement of the objectives in desired operating performance for the control system.

The conjecture considers the description of the process and the two functions of the AP model described for the real case in Sect. 2.4, of which the ideal case is a particular case.

Conjecture 2.1 If the driver block verifies that the desired output $y_d(k + r + 1)$ is

(1) bounded, and
(2) physically realizable,

and, for certain values M and k_f, the adaptive system (or adaptive mechanism) satisfies the conditions

(a) $\hat{\theta}_r(k) = \hat{\theta}_r(k-d), \quad \forall k \geq k_f > 0,$ and
(b) $|e(k|k)| \leq M < \infty, \quad \forall k \geq k_f > 0,$

then the adaptive predictive control system will fulfil the following properties:

(I) $|\varepsilon(k)| = |y(k) - y_d(k)| \leq M < \infty, \quad \forall k \geq k_f > 0,$ and
(II) $\|\phi(k)\| \leq \Omega < \infty, \quad \forall k \geq k_f > 0.$

Proof The Eq. (2.34), which defines the function of the AP model making the predicted output equal to the desired output at instant $k+d$, can be written for instant k in the form:

$$y_d(k) = \hat{\theta}_r(k-d)^T \phi_r(k-d). \tag{2.38}$$

Comparing the Eqs. (2.30) and (2.38), it is obvious that, if condition a holds, we can deduce:

$$y_d(k) = \hat{y}(k|k), \quad \forall k \geq k_f > 0,$$

and as a consequence:

$$\varepsilon(k) = e(k|k), \quad \forall k \geq k_f > 0.$$

Based on this result and condition b of the Conjecture, the property I is directly deduced. Also, if condition 1 is satisfied, based on property I previously proven, it can be derived that $y(k)$ will be bounded for all $k \geq k_f$. Additionally, if condition 2 is satisfied, also based on property I, we can also derive that $u(k)$ will be bounded for all $k \geq k_f - d$. As a result, property II is proven, thus concluding the demonstration of this conjecture. □

It may be arguable to use the term conjecture when, as proven, it is a mathematical result. However, previous literature used this nomenclature because it established a sound stability conclusion from premises that were not yet proven. Under this scheme, conditions 1, 2, a and b are used as the guidelines for the design of the driver block and the adaptive mechanism in order to satisfy the stability objective.

In relation to the design of the driver block, verifying condition 1 of the desired output boundedness is simple and in principle only requires the boundedness of the corresponding setpoints. To verify condition 2, that of physical realizability, we must first take into account that the noise and non-measurable perturbations, that is to say, the perturbations vector, is continuously acting on the real process, and secondly, the effect of the dynamics of the process in question.

The consideration of the perturbation vector acting on the process originated the the first design of the driver block within the framework of the so called "basic predictive control strategy". The limitations of the basic strategy led to the consideration of the process dynamic in the design of the driver block and further, to what we will refer to as the "extended predictive control strategy". Both predictive control strategies, and their corresponding driver block designs, are the topic of Part II.

With regards to the design of the adaptive system or the adaptation mechanism, condition a defines a form of convergence of the AP model parameters, while condition b formulates the stability of the adaptive system as we have considered in the previous section. Conditions a and b represent a change in philosophy or perspective regarding control system design, as will be explained next.

The design of control systems has traditionally been based on the knowledge of the equations representing the process dynamic behaviour. Control methodologies have used so far this knowledge, at least theoretically, to determine the control laws to apply to processes. It is for this reason that process identification techniques have been one of the most important areas of focus in control theory research.

Therefore, it should be noted that conditions a and b established by the Conjecture for achieving desired control performance, definitely do not require identification of the process parameters by means of the AP model parameters. In effect, condition a does not require that the parameters of the AP model be equal to or tend towards the process parameters, but only that these parameters converge to certain values that do not have to be equal to those of the process. Additionally, condition b requires the stability of the adaptive system, as indicated previously.

Hence, the difficulty in resolving the identification problem, which is particularly well known in the case of industrial processes, is avoided or substituted, as suggested by the Conjecture, by the verification of less demanding or easier to attain conditions.

Part III will present solutions illustrating the synthesis problem of the adaptation mechanism, in the various contexts of the ideal case, the real case without structural differences and the real case with structural differences. It will be demonstrated that the proposed adaptive systems verify the conditions of convergence and stability of the Conjecture in the case where the series of values $\{\|\phi_r(k)\|\}$ is bounded. Obviously, the boundedness of the input/output vector cannot be guaranteed by the adaptive system, since the control signal is produced by the adaptive predictive controller. Thus, it is only the adaptive predictive control system that can guarantee the condition of boundedness. Nevertheless, it will be shown that the proposed adaptive systems have properties which come close to satisfying the conditions of the Conjecture and these are inherent to the adaptive system itself, independently of whether the series $\{\|\phi_r(k)\|\}$ is bounded or not. Part IV will combine these properties with those of physical realizability, boundedness of the desired output, and the principle of predictive control to formally prove the boundedness of the series $\{\|\phi_r(k)\|\}$, and consequently, achieve the desired performance objective of the control system.

In the real case where the process parameters vary with time, condition a of the Conjecture cannot be attained when the variation of the process parameters is permanent. However, Chap. 6 in Part III will consider the synthesis of the adaptation mechanism for this real case and prove a form of convergence of the AP model parameters that, within the reasonable restrictions of an industrial environment, may be sufficient to achieve the desired practical objectives of the control system performance as illustrated in the application chapters of this book.

2.8 From Conceptual Knowledge to a Profound Understanding

This chapter completes Part I, in which the conceptual and intuitive knowledge of the material covered in this book were presented. The basic concepts were presented in Chap. 1 and in this chapter we provided a mathematical language that enabled a basic level of analysis. Hence, we have deduced the conditions that must be verified for the technological realization of the concepts, in order to achieve the desired results in their practical application.

The technological realization of the adaptive predictive expert control (ADEX) concepts, explained in this Part, must

1. Define all the information processing relating to the operation of the control system, verifying the previously stated stability conditions.
2. Ensure the computing support to enable the calculations associated to the required information processing.
3. Define the system operation in the appropriate environment that will enable the necessary information flows to take place, including real time capture of the process signals and the application to the process of the control signals generated.

In this book, Parts II, III, V and VI define and analyse a design for the different blocks that enable the functioning of adaptive predictive expert control. Part II, therefore, focused on the analysis and design of the driver block and its operation together with the Predictive Model, included in the Control Block. Similarly, Part III is focused on the analysis and design of the Adaptation Mechanism, and Part VI presents a design for the Expert System, also included in the Control Block.

Also, Part VI presents the design and instructions for the using the software platform ADEX COP (an acronym of ADEX "Control and Optimization Platform") version 1. This platform enables the integration of ADEX controllers into the control logic of currently available commercial systems, hence guaranteeing the programming support required for the calculations that must be made by the controllers and, at the same time, the appropriate operating environment. Additionally, Part VI presents the design and application of the ADEX COP version 2 platform and the ADEX controller module.

In accordance with the learning focus of this book, the profound knowledge of the technology must be acquired by means of practical experimentation. In this sense, Parts V and VI present the student with examples of predictive control applications without adaptation, adaptive predictive control or adaptive predictive expert control to various real processes. Additionally, Parts I, II and III present exercises that allow the student to simulate processes by means of programming, and apply the technological knowledge which is the objective of these units.

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