Chapter 2
Methods of Morphological Design (Synthesis)

Abstract This chapter (Partially based on: (i) Levin MS (2009) Towards morphological system design. In: Proc. of IEEE 7th Int. Conf. on Industrial Informatics INDIN-2009, Cardiff, UK, pp 95–100 (ii) Levin MS (2012) Morphological methods for design of modular systems (a survey). Electronic preprint, p 20, Jan. 9, 2012. http://arxiv.org/abs/1201.1712 [cs.SE]) addresses combinatorial morphological approaches to design of a modular system including the following: basic morphological analysis, multicriteria version of morphological analysis with the usage of closeness of a composite solution to ideal point, multicriteria version of morphological analysis with selection of Pareto-efficient composite solutions, hierarchical morphological multicriteria design, etc. A numerical example for a GSM communication system illustrates the application of the approaches.

2.1 Introduction

Morphological analysis (MA) was firstly suggested by F. Zwicky in 1943 for design of aerospace systems. Morphological analysis is a well-known general powerful method to synthesis of modular systems (i.e., composition) in various domains (e.g., [48, 516, 628, 636, 894, 895, 1146]). MA is based on *divide and conquer* technique. A hierarchical structure of the designed system is a basis for usage of the method. The following basic partitioning techniques can be used to obtain the required hierarchical system model: (a) partitioning by system component/parts, (b) partitioning by system functions, (c) partitioning by system properties/attributes, and (d) integrated techniques. In this chapter, system hierarchy of system components (parts, subsystems) is considered as a basic one. Many years the usage of morphological analysis in system design was very limited by the reason that the method leads to a very large combinatorial domain of possible solutions. On the other hand, contemporary computer systems can solve very complex computational problems and hierarchical system models can be used as a basis for partitioning/decomposition solving frameworks.

Recent trends in the study, usage, and modification/extension of morphological analysis may be considered as the following:

M.S. Levin, *Modular System Design and Evaluation*,
Decision Engineering, DOI 10.1007/978-3-319-09876-0_2

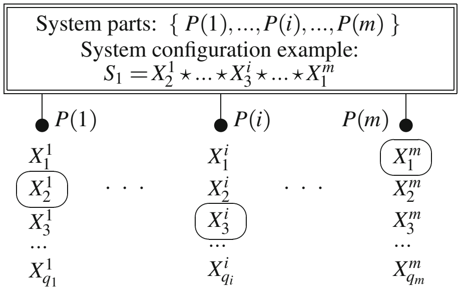


Fig. 2.1 System configuration problem (selection) [642]

(1) hierarchical systems modeling,
(2) optimization models,
(3) multicriteria decision making, and
(4) taking into account uncertainty (i.e., probabilistic and/or fuzzy estimates).

Generally, morphological system design approaches are targeted to design of system configuration as a selection of alternatives for systems parts (e.g., [642]). Figure 2.1 illustrates this problem. Here, a composite (modular) system consists of m system parts: $\{P(1), ..., P(i), ..., P(m)\}$. For each system part (i.e., $\forall i$, $i = \overline{1, m}$) there are corresponding alternatives (i.e., design alternatives DAs) $\{X^i_1, X^i_2, ..., X^i_{q_i}\}$, where q_i is the number of alternatives for part i. Thus, the problem is:

Select an alternative for each system part while taking into account some local and/or global objectives/preferences and constraints.

Evidently, the objective/prereferences and constraints are based on (correspond to) quality of the selected alternatives and quality of compatibility among the selected alternatives. In [642] (Chap. 5), some other system configuration problems are described as well (e.g., reconfiguration, selection and allocation).

Our basic list of morphological design approaches consists of the following:

(1) the basic version of morphological analysis (by F. Zwicky) (MA) (e.g., [85, 129, 516, 894, 1146]);
(2) the modification of morphological analysis as searching for an admissible (by compatibility) element combination (one representative from each morphological class, i.e., a set of alternatives for system part/component) that is the closest to a combination consisting of the best elements (at each morphological class) (e.g., [48, 290, 599]);
(3) modification of morphological analysis via reducing to linear programming (MA and linear programming) [568];
(4) modification of morphological analysis via reducing to multiple choice problem (MCP) [370, 541, 743] or multicriteria multiple choice problem (e.g., [691, 983]);

Table 2.1 Description of approaches

Method	Scale for DAs	Scale for IC	Quality of decision	Some sources
1. Morphological analysis (MA)	None	{0, 1}	Admissibility	[516, 894, 1146]
2. Closeness to ideal point	None	{0, 1}	"Distance" to ideal point	[48, 290, 599]
3. MA & linear programming	Quantitative	{0, 1}	Additive function	[568]
4. Multiple choice problem or its multicriteria version	Quantitative	None	Additive function or multicriteria description	[370, 691, 983]
5. Quadratic assignment problem (QAP)	Quantitative	Quantitative	Additive function	[160, 177, 642]
6. Pareto-based MA	None	{0, 1}	Multicriteria description	[310, 361]
7. HMMD	Quantitative and ordinal, mapping to ordinal	Ordinal	Point at poset based on multiset	[626, 628, 636]
8. HMMD & interval multiset estimates	Poset based on interval multiset	Ordinal	Point at poset based on interval multiset	[655, 661, 668]

(5) modification of morphological analysis via reducing to quadratic assignment problem (QAP) (e.g., [160, 177, 628, 642]);
(6) the multicriteria modification of morphological analysis as follows (Pareto-based MA): (a) searching for all admissible (by compatibility) elements combinations (one representative from each morphological class), (b) evaluation of the found combinations upon a set of criteria, and (c) selection of the Pareto-efficient solutions (e.g., [310, 361]);
(7) hierarchical morphological multicriteria design (HMMD) approach [626, 628, 636]; and
(8) a new version of hierarchical morphological multicriteria design approach based on the usage of interval multiset estimates for DAs [655, 661, 668] (Chap. 3).

Table 2.1 contains some properties of the approaches above.

In addition, it is reasonable to point out that MA-based methods are successfully used in digital image processing: structural analysis of images, object detection and identification in images (e.g., [867, 868, 1054, 1055, 1056, 1057]).

2.2 Morphological Design Approaches

2.2.1 Morphological Analysis

The MA approach consists of the following stages:

Stage 1. Building a system structure as a set of system parts/components.

Stage 2. Generation of design alternatives (DAs) for each system part (i.e., a morphological class).

Stage 3. Binary assessment of compatibility for each DAs pair (one DA from one morphological class, other DA from another morphological class). Value of compatibility 1 corresponds to compatibility of two corresponding DAs, value 0 corresponds to incompatibility.

Stage 4. Generation of all admissible compositions (one DA for each system part) while taking into account compatibility for each two DAs in each obtained composition.

The method above is an enumerative one. Figure 2.2 illustrates MA (binary compatibility estimates are depicted in Table 2.2).

Here, the following morphological classes are examined: (a) morphological class 1: $\{X^1_1, X^1_2, X^1_3, X^1_4, X^1_5\}$, (b) morphological class i: $\{X^i_1, X^i_2, X^i_3, X^i_4, X^i_5\}$,

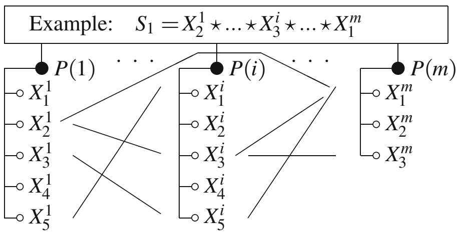


Fig. 2.2 Illustration for MA [643]

Table 2.2 Binary compatibility [643]

	X^i_1	X^i_2	X^i_3	X^i_4	X^i_5	X^m_1	X^m_2	X^m_3
X^1_1	0	0	0	0	0	0	0	0
X^1_2	0	0	1	0	0	1	0	0
X^1_3	0	0	0	0	1	0	0	0
X^1_4	0	0	0	0	0	0	0	0
X^1_5	1	0	0	0	0	0	0	0
X^i_1						0	0	0
X^i_2						0	0	0
X^i_3						1	0	1
X^i_4						0	0	0
X^i_5						1	0	0

and (c) morphological class m: $\{X_1^m, X_2^m, X_3^m\}$. Further, a simplified case is considered for three system parts (and corresponding morphological classes). The resultant (admissible) solution (composition or composite design alternative) is: $S_1 = X_2^1 \star \cdots \star X_3^i \star \cdots \star X_1^m$.

2.2.2 Method of Closeness to Ideal Point

First, modification of MA as method of closeness to ideal point was suggested (e.g., [48, 290]). Illustration for method of closeness to ideal point is shown in Fig. 2.3 (binary compatibility estimates are contained in Table 2.3).

Here, for each system part (from the corresponding morphological class) the best design alternatives (as an ideal) is selected (e.g., by expert judgment). In the illustrative example (Fig. 2.3), the ideal design alternatives are: X_1^1, X_3^i, and X_3^m. Thus, the ideal point (i.e., solution) is: $S_o = X_1^1 \star \cdots \star X_3^i \star \cdots \star X_3^m$. Unfortunately, this solution S_o is inadmissible (by compatibility). Admissible solutions are the following: $S_1 = X_2^1 \star \cdots \star X_3^i \star \cdots \star X_1^m$ and $S_2 = X_5^1 \star \cdots \star X_3^i \star \cdots \star X_3^m$.

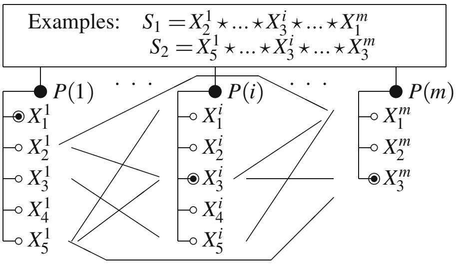


Fig. 2.3 Illustration for MA with ideal point [643]

Table 2.3 Binary compatibility [643]

	X_1^i	X_2^i	X_3^i	X_4^i	X_5^i	X_1^m	X_2^m	X_3^m
X_1^1	0	0	0	0	0	0	0	0
X_2^1	0	0	1	0	0	1	0	0
X_3^1	0	0	0	0	1	0	0	0
X_4^1	0	0	0	0	0	0	0	0
X_5^1	1	0	1	0	0	0	0	1
X_1^i						0	0	0
X_3^i						0	0	0
X_3^i						1	0	1
X_4^i						0	0	0
X_5^i						1	0	0

Let $\rho(S', S'')$ be a proximity (e.g., by elements) for two composite design alternatives $S', S'' \in \{S\}$. Then it is reasonable to search for the following solution $S^* \in \{S^a\} \subseteq \{S\}$ ($\{S^a\}$ is a set of admissible solutions): $S^* = Arg \ \min_{S \in \{S^a\}} \ \rho(S, S_o)$. Clearly, in the illustrative example solution $S_2 = X_5^1 \star \cdots \star X_3^i \star \cdots \star X_3^m$ is more close to ideal solution S_o (i.e., $\rho(S_2, S_o) \preceq \rho(S_1, S_o)$). Generally, various versions of proximity (as real functions, vectors, etc.) can by examined (e.g., [48, 290]).

2.2.3 Pareto-Based Morphological Approach

An integrated method (MA and multicriteria decision making, an enumerative method) was suggested as follows (e.g., [310, 361]):

Stage 1. Usage of basic MA to get a set of admissible compositions.

Stage 2. Generation of criteria for evaluation of the admissible compositions.

Stage 3. Evaluation of admissible compositions upon criteria and selection of Pareto-efficient solutions.

Figure 2.4 illustrates Pareto-based MA. Concurrently, binary compatibility estimates are depicted in Table 2.4. Here, admissible solutions are the following: $S_1 = X_2^1 \star \cdots \star X_3^i \star \cdots \star X_1^m$, $S_2 = X_5^1 \star \cdots \star X_3^i \star \cdots \star X_3^m$, and $S_3 = X_5^1 \star \cdots \star X_5^i \star \cdots \star X_3^m$. Further, the solutions have to be evaluated upon criteria and Pareto-efficient solution(s) will be selected.

2.2.4 Linear Programming

In [568], morphological analysis is reduced to linear programming. Here, constraints imposed on the solution are reduced to a set of inequalities of Boolean variables and quality criterion for the solution as an additive function is used. A solving process may be based on a heuristic or on a enumerative method.

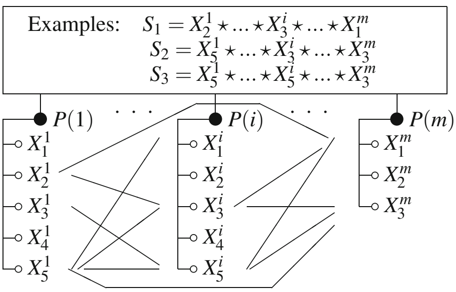


Fig. 2.4 Illustration for Pareto-based MA [643]

Table 2.4 Binary compatibility [643]

	X_1^i	X_2^i	X_3^i	X_4^i	X_5^i	X_1^m	X_2^m	X_3^m
X_1^1	0	0	0	0	0	0	0	0
X_2^1	0	0	1	0	0	1	0	0
X_3^1	0	0	0	0	1	0	0	0
X_4^1	0	0	0	0	0	0	0	0
X_5^1	1	0	1	0	1	0	0	1
X_1^i						0	0	0
X_2^i						0	0	0
X_3^i						1	0	1
X_4^i						0	0	0
X_5^i						1	0	1

2.2.5 Multiple Choice Problem

The basic knapsack problem is (e.g., [370, 541, 743]):

$$\max \sum_{i=1}^{m} c_i x_i \quad s.t. \ \sum_{i=1}^{m} a_i x_i \le b, \ x_i \in \{0, 1\}, \quad i = \overline{1, m},$$

where $x_i = 1$ if item i is selected, c_i is a value (utility) for item i, and a_i is a weight of item i (or resource required). Often nonnegative coefficients are assumed. The problem is NP-hard [370, 743] and can be solved by enumerative methods (e.g., Branch-and-Bound, dynamic programming), approximation schemes with a limited relative error (FPTAS) (e.g., [541, 743]). In the case of multiple choice problem (e.g., [541, 743]), the items are divided into groups and we select element(s) from each group while taking into account a total resource constraint (or constraints). Here, each element has two indices: (i, j), where i corresponds to number of group and j corresponds to number of item in the group. In the case of multicriteria description of items (i.e., vector estimate), each element (i.e., (i, j)) has vector profit $\overline{c_{i,j}} = (c^1_{i,j}, \ldots, c^{\xi}_{i,j}, \ldots, c^r_{i,j})$ and multicriteria multiple choice problem is:

$$\max \sum_{i=1}^{m} \sum_{j=1}^{q_i} c^{\xi}_{ij} x_{ij}, \quad \forall \xi = \overline{1, r}$$

$$s.t. \ \sum_{i=1}^{m} \sum_{j=1}^{q_i} a_{ij} x_{ij} \le b, \quad \sum_{j=1}^{q_i} x_{ij} = 1 \quad \forall i = \overline{1, m}, \quad x_{ij} \in \{0, 1\}.$$

For this problem formulation it is reasonable to search for Pareto-efficient solutions. This design approach was used for design and redesign/improvement of applied systems (software, hardware, communication) [647, 691, 983]. Here, the following solving schemes can be used: (i) enumerative algorithms (e.g., Branch-and-Bound, dynamic programming), (ii) heuristic based on preliminary multicriteria ranking of elements to get their priorities and step-by-step packing the knapsack (i.e., greedy approach), (iii) multicriteria ranking of elements to get their ordinal priorities and usage of approximation solving scheme (as for knapsack problem) based on discrete space of system excellence (as later in HMMD).

2.2.6 Assignment/Allocation Problems

Assignment/allocation problems are widely used in many domains (e.g., [177, 370, 832]). Simple assignment problem involves nonnegative correspondence matrix $\Upsilon = ||c_{i,j}||$ $(i = \overline{1,n},\ j = \overline{1,n})$ where $c_{i,j}$ is a profit ('utility') to assign element i to position j. The problem is (e.g., [370]):

Find assignment $\pi = (\pi(1), \dots, \pi(i), \dots, \pi(n))$ *of elements* i $(i = \overline{1,n})$ *to positions* $\pi(i)$, *which corresponds to a total effectiveness:* $\sum_{i=1}^{n} c_{i,\pi(i)} \rightarrow \max$.

A more complicated well-known model as quadratic assignment problem (QAP) includes interconnection between elements of different groups (each group corresponds to a certain position) (e.g., [177, 832]). Let a nonnegative value $d(i, j_1, k, j_2)$ be a profit of compatibility between item j_1 in group J_i and item j_2 in group J_k. Also, this value of compatibility is added to the objective function. QAP may be considered as a version of MA. Thus, QAP can be formulated as follows:

$$\max \sum_{i=1}^{m} \sum_{j=1}^{q_i} c_{i,j} x_{i,j} + \sum_{l<k} \sum_{j_1=1}^{q_l} \sum_{j_2=1}^{q_k} d(l, j_1, k, j_2)\, x_{l,j_1} x_{k,j_2}, \quad l = \overline{1,m}, \quad k = \overline{1,m};$$

$$s.t. \ \sum_{i=1}^{m} \sum_{j=1}^{q_i} a_{i,j}\, x_{i,j} \le b, \quad \sum_{j=1}^{q_i} x_{i,j} \le 1 \ \forall i = \overline{1,m}, \quad x_{i,j} \in \{0, 1\}.$$

QAP is NP-hard. Enumerative methods (e.g., Branch-and-Bound) or heuristics (e.g., greedy algorithms, tabu search, genetic algorithms) are usually used for the problem. In the case of multicriteria assignment problem the objective function is transformed into a vector function, i.e., $c_{i,j} \Rightarrow \overline{c_{i,j}} = (c^1_{i,j}, \dots, c^{\xi}_{i,j}, \dots, c^r_{i,j})$ and the vector objective function is, for example:

$$\left(\sum_{i=1}^{m} \sum_{j=1}^{n} c^1_{i,j} x_{i,j}, \dots, \sum_{i=1}^{m} \sum_{j=1}^{n} c^{\xi}_{i,j} x_{i,j}, \dots, \sum_{i=1}^{m} \sum_{j=1}^{n} c^r_{i,j} x_{i,j} \right).$$

Here, Pareto-efficient solutions are searched for. Analogically, QAP can be transformed into a multcriteria QAP.

2.2.7 Hierarchical Morphological Multicriteria Design (HMMD)

Hierarchical Morphological Multicriteria Design (HMMD) method was suggested by Mark Sh. Levin (e.g., [621, 626, 628, 636, 642]). The assumptions of HMMD are the following: (a) a tree-like structure of the system; (b) a composite estimate for system quality that integrates components (subsystems, parts) qualities and qualities of interconnections IC (compatibility) across subsystems; (c) monotonic criteria for the system and its components; and (d) quality of system components and IC are evaluated on the basis of coordinated ordinal scales. The designations are: (1) design alternatives (DAs) for leaf nodes of the model; (2) priorities of DAs ($\iota = \overline{1, l}$; 1 corresponds to the best one); (3) ordinal compatibility for each pair of DAs ($w = \overline{1, \nu}$; ν corresponds to the best level). The basic phases of HMMD are:

Phase 1. Design of the tree-like system model (a preliminary phase).
Phase 2. Generating DAs for leaf nodes of the system model.
Phase 3. Hierarchical selection and composing of DAs into composite DAs for the corresponding higher level of the system hierarchy (morphological clique problem).
Phase 4. Analysis and improvement of the resultant composite DAs (decisions).

Further, morphological clique problem is described. System S consists of m parts (components): $\{P(1), \dots, P(i), \dots, P(m)\}$ (Fig. 2.1). For each system part, a set of DAs is generated. The problem is:

Find composite design alternative $S = S(1) \star \dots \star S(i) \star \dots \star S(m)$ *of* DAs (*one representative design alternative* $S(i)$ *for each system component/part* $P(i)$, $i = \overline{1, m}$) *with non-zero compatibility estimates between the selected design alternatives.*

A discrete space of the system quality (a poset) is based on the following vector (Fig. 2.5): $N(S) = (w(S); e(S))$, where $w(S)$ is the minimum of pairwise compatibility between DAs, which correspond to different system components (i.e., $\forall P_{j_1}$ and P_{j_2}, $1 \leq j_1 \neq j_2 \leq m$) in S, $e(S) = (\eta_1, \dots, \eta_\iota, \dots, \eta_l)$, where η_ι is the number of DAs of the ιth quality in S $(\sum_{\iota=1}^{l} \eta_\iota = m)$. Here, composite solutions (composite DAs) are searched for, which are nondominated by $N(S)$ (i.e., Pareto-efficient solutions) (Fig. 2.5). Thus, the basic version of HMMD corresponds to the following problem (two objectives, one constraint):

$$\max \ e(S), \quad \max \ w(S), \quad s.t. \ w(S) \geq 1.$$

"Maximization" of $e(S)$ is based on the corresponding poset.

This problem is NP-hard (because a more simple its subproblem is NP hard [562]). Generally, the following layers of system excellence can be considered

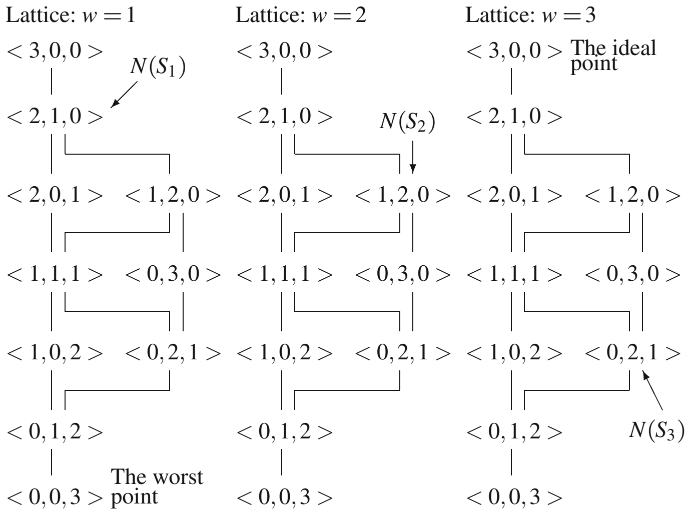
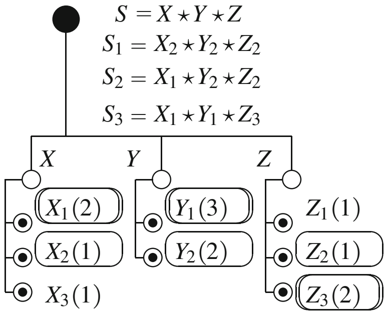


Fig. 2.5 Poset of quality (3 system parts, 3 levels of element quality)

Fig. 2.6 Example of composition



(e.g., [628]): (i) ideal point; (ii) Pareto-efficient points; (iii) a neighborhood of Pareto-efficient DAs (e.g., a composite decision of this set can be transformed into a Pareto-efficient point on the basis of an improvement action(s)). Clearly, the compatibility component of vector $N(S)$ can be considered on the basis of a poset-like scale too (as $e(S)$). In this case, the discrete space of system excellence will be an analogical lattice [631, 636].

Figures 2.6 and 2.7 illustrate HMMD (by a numerical example for three part system $S = X \star Y \star Z$). Priorities of DAs are shown in Fig. 2.6 in parentheses and are depicted in Fig. 2.7. Table 2.5 contains compatibility estimates (they are pointed out in Fig. 2.7 too). In the example, composite decisions are (Pareto-efficient solutions) (Figs. 2.5, 2.6, 2.7 and 2.8): $S_1 = X_2 \star Y_2 \star Z_2$, $N(S_1) = (1; 2, 1, 0)$; $S_2 = X_1 \star Y_2 \star Z_2$, $N(S_2) = (2; 1, 2, 0)$; $S_3 = X_1 \star Y_1 \star Z_3$, $N(S_3) = (3; 0, 2, 1)$.

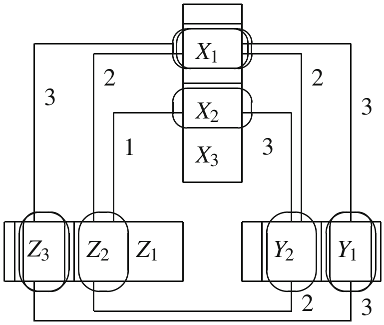


Fig. 2.7 Concentric presentation

Table 2.5 Compatibility

	Y_1	Y_2	Z_1	Z_2	Z_3
X_1	3	2	0	2	3
X_2	0	3	0	1	0
X_3	0	0	0	0	1
Y_1			0	0	3
Y_2			0	2	0

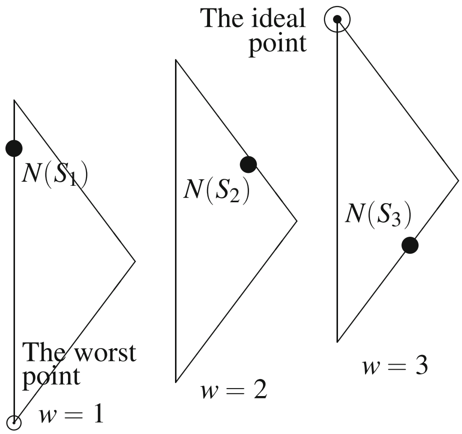


Fig. 2.8 Illustration for space of quality

2.3 Design Examples for GSM Network

In recent two decades, the significance of GSM network has been increased (e.g., [200, 391, 435, 713, 752, 873, 1026]). Thus, there exists a need of the design and maintenance of this kind of communication systems. Here, a numerical example for design of GSM network (a modification of an example from [682]) is used to illustrate and to compare several MA-based methods: basic MA, method of closeness to ideal point, Pareto-based MA, multiple choice problem, and HMMD.

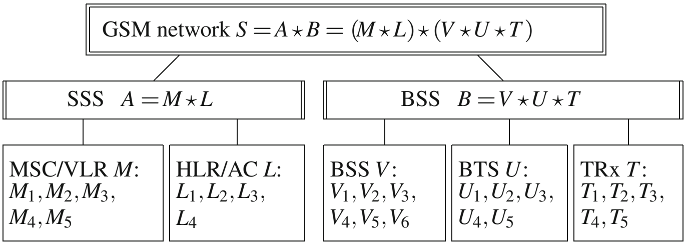


Fig. 2.9 General simplified structure of GSM network

2.3.1 Initial Example

A simplified tree-like model of GSM network is the following (Fig. 2.9):

0. GSM network $S = A \star B$.
1. Switching SubSystem SSS $(A = M \star L)$.

   1.1. Mobile Switching Center/Visitors Location Register MSC/VLR M : M_1 (Motorola), M_2 (Alcatel), M_3 (Huawei), M_4 (Siemens), and M_5 (Ericsson).
   1.2. Home Location Register/Authentification Center HLR/AC L : L_1 (Motorola), L_2 (Ericsson), L_3 (Alcatel), and L_4 (Huawei).

2. Base Station SubSystem BSS $(B = V \star U \star T)$.

   2.1. Base Station Controller BSC V : V_1 (Motorola), V_2 (Ericsson), V_3 (Alcatel), V_4 (Huawei), V_5 (Nokia), and V_6 (Siemens).
   2.2. Base Transceiver Station BTS U : U_1 (Motorola), U_2 (Ericsson), U_3 (Alcatel), U_4 (Huawei), and U_5 (Nokia).
   2.3. Transceivers TRx T : T_1 (Alcatel), T_2 (Ericsson), T_3 (Motorola), T_4 (Huawei), and T_5 (Siemens).

Note, an initial set of possible composite decisions contained 3,000 combinations $(5 \times 4 \times 6 \times 5 \times 5)$.

The following criteria for system components are considered (weights of criteria are pointed out in parentheses):

1. M: maximal number of data pathes (1,000 pathes) $(C_{m1}, 0.2)$; maximal capacity VLR (100,000 subscribers) $(C_{m2}, 0.2)$; price index (100,000/price (USD)) $(C_{m3}, 0.2)$; power consumption (1/power consumption (kWt)) $(C_{m4}, 0.2)$; and number of communication and signaling interfaces $(C_{m5}, 0.2)$.
2. L: maximal number of subscribers (100,000 subscribers) $(C_{l1}, 0.25)$; volume of service provided $(C_{l2}, 0.25)$; reliability (scale $[1, \dots, 10]$) $(C_{l3}, 0.25)$; and integratability (scale $[1, \dots, 10]$) $(C_{l4}, 0.25)$.
3. V: price index (100,000/cost (USD)) $(C_{v1}, 0.25)$; maximal number of BTS $(C_{v2}, 0.25)$; handover quality $(C_{v3}, 0.25)$; and throughput $(C_{v4}, 0.25)$.

Table 2.6 Estimates for M

DAs	C_{m1}	C_{m2}	C_{m3}	C_{m4}	C_{m5}	Priority r
M_1	3.7	8.6	6	5.1	4	2
M_2	4.0	11	8	7	5	3
M_3	4.1	10	9	7	4	3
M_4	3.2	7	5	6	3	1
M_5	3.5	8.7	6.2	5	4	2

Table 2.7 Estimates for V, L

DAs	C_{v1}	C_{v2}	C_{v3}	C_{v4}	Priority r
V_1	6	4	3	4	1
V_2	7	5	7	7	2
V_3	9	7	10	7	3
V_4	7	5	8	6	2
V_5	6	3	4	4	1
V_6	10	6	9	7	3
DAs	C_{l1}	C_{l2}	C_{l3}	C_{l4}	Priority r
L_1	9	7	7	8	1
L_2	10	4	9	8	1
L_3	12	8	10	10	2
L_4	9	5	8	8	1

4. U: maximal number of TRx $(C_{u1}, 0.25)$; capacity $(C_{u2}, 0.25)$; price index (100,000/cost (USD)) $(C_{u3}, 0.25)$; and reliability (scale $[1, \ldots, 10]$) $(C_{u4}, 0.25)$.
5. T: maximum power-carrying capacity $(C_{t1}, 0.3)$; throughput $(C_{t2}, 0.2)$; price index (100,000/cost(USD)) $(C_{t3}, 0.25)$; and reliability (scale $[1, \ldots, 10]$) $(C_{t4}, 0.25)$.

Tables 2.6, 2.7 and 2.8, contain estimates of DAs upon criteria above (data from catalogues, expert judgment) and their resultant priorities (the priorities are based on multicriteria ranking by an Electre-like technique [674, 910]). Compatibility estimates are contained in Table 2.9 (expert judgment).

2.3.2 Morphological Analysis

In the case of basic MA, binary compatibility estimates are used. To decrease the dimension of the considered numerical example, the following version of MA is examined. Let us consider more strong requirements to compatibility (Table 2.10): (i) new compatibility estimate equals 1 if the old estimate was equal 3, (ii) new compatibility estimate equals 1 if the old estimate was equal 0 or 1 or 2. Clearly, here we can get some negative results, for example: (a) admissible solutions are absent, (b)

Table 2.8 Estimates for U, T

DAs	C_{u1}	C_{u2}	C_{u3}	C_{u4}	Priority r
U_1	2	7	5	8	1
U_2	4	10	6	10	3
U_3	3	9	6	10	2
U_4	3	6	3	7	1
U_5	3	10	6	9	2
DAs	C_{t1}	C_{t2}	C_{t3}	C_{t4}	Priority r
T_1	9	7	10	7	3
T_2	6	4	3	4	1
T_3	7	5	7	7	2
T_4	7	5	8	6	2
T_5	6	3	4	4	1

Table 2.9 Compatibility

	U_1	U_2	U_3	U_4	U_5	T_1	T_2	T_3	T_4	T_5
V_1	2	2	2	2	3	3	2	2	2	2
V_2	3	3	3	2	0	0	3	0	3	2
V_3	3	3	3	2	0	0	3	0	3	2
V_4	3	2	0	2	3	0	2	0	2	2
V_5	3	0	0	2	0	2	2	0	2	2
V_6	0	3	2	3	2	3	0	2	2	0
U_1						2	0	0	2	3
U_2						0	2	0	3	0
U_3						0	2	0	3	0
U_4						0	3	3	0	0
U_5						3	0	2	2	0
	L_1	L_2	L_3	L_4						
M_1	3	2	0	3						
M_2	2	3	2	0						
M_3	0	2	3	2						
M_4	2	3	3	3						
M_5	3	3	0	3						

some sufficiently good solutions (e.g., solutions with one/two compatibility estimate at the only admissible/good levels as 1 or 2) will be lost.

As a result, the following admissible DAs can be analyzed:

(1) nine DAs for A: $A_1 = M_1 \star L_1$, $A_2 = M_1 \star L_4$, $A_3 = M_2 \star L_2$, $A_4 = M_3 \star L_3$, $A_5 = M_4 \star L_2$, $A_6 = M_4 \star L_3$, $A_7 = M_5 \star L_1$, $A_8 = M_5 \star L_2$, and $A_9 = M_5 \star L_4$;

Table 2.10 Compatibility

	U_1	U_2	U_3	U_4	U_5	T_1	T_2	T_3	T_4	T_5
V_1	0	0	0	0	1	1	0	0	0	0
V_2	1	1	1	0	0	0	1	0	1	0
V_3	1	1	1	0	0	0	1	0	1	0
V_4	1	0	0	0	1	0	0	0	0	0
V_5	1	0	0	0	0	0	0	0	0	0
V_6	0	1	0	1	0	1	0	0	0	0
U_1						0	0	0	0	1
U_2						0	0	0	1	0
U_3						0	0	0	1	0
U_4						0	1	1	0	0
U_5						1	0	0	0	0
	L_1	L_2	L_3	L_4						
M_1	1	0	0	1						
M_2	0	1	0	0						
M_3	0	0	1	0						
M_4	0	1	1	1						
M_5	1	1	0	1						

(2) five DAs for B: $B_1 = V_1 \star U_5 \star T_1$, $B_2 = V_2 \star U_2 \star T_4$, $B_3 = V_2 \star U_3 \star T_4$, $B_4 = V_3 \star U_2 \star T_4$, and $B_5 = V_3 \star U_3 \star T_4$;

and the resultant composite DAs are: $S_1 = A_1 \star B_1$, $S_2 = A_2 \star B_1$, $S_3 = A_3 \star B_1$, $S_4 = A_4 \star B_1$, $S_5 = A_5 \star B_1$, $S_6 = A_6 \star B_1$, $S_7 = A_7 \star B_1$, $S_8 = A_8 \star B_1$, $S_9 = A_9 \star B_1$; $S_{10} = A_1 \star B_2$, $S_{11} = A_2 \star B_2$, $S_{12} = A_3 \star B_2$, $S_{13} = A_4 \star B_2$, $S_{14} = A_5 \star B_2$, $S_{15} = A_6 \star B_2$, $S_{16} = A_7 \star B_2$, $S_{17} = A_8 \star B_2$, $S_{18} = A_9 \star B_2$; $S_{19} = A_1 \star B_3$, $S_{20} = A_2 \star B_3$, $S_{21} = A_3 \star B_3$, $S_{22} = A_4 \star B_3$, $S_{23} = A_5 \star B_3$, $S_{24} = A_6 \star B_3$, $S_{25} = A_7 \star B_3$, $S_{26} = A_8 \star B_3$, $S_{27} = A_9 \star B_3$; $S_{28} = A_1 \star B_4$, $S_{29} = A_2 \star B_4$, $S_{30} = A_3 \star B_4$, $S_{31} = A_4 \star B_4$, $S_{32} = A_5 \star B_4$, $S_{33} = A_6 \star B_4$, $S_{34} = A_7 \star B_4$, $S_{35} = A_8 \star B_4$, $S_{36} = A_9 \star B_4$; $S_{37} = A_1 \star B_5$, $S_{38} = A_2 \star B_5$, $S_{39} = A_3 \star B_5$, $S_{40} = A_4 \star B_5$, $S_{41} = A_5 \star B_5$, $S_{42} = A_6 \star B_5$, $S_{43} = A_7 \star B_5$, $S_{44} = A_8 \star B_5$, and $S_{45} = A_9 \star B_5$.

Finally, the next step has to consist in selection of the best solution.

2.3.3 Method of Closeness to Ideal Point

Here, the initial set of admissible solutions corresponds to the solution set, which was obtained in previous case (i.e., basic MA). Evidently, this approach depends on the kind of the proximity between the ideal point (S^I) and examined solutions.

First of all, let us consider estimate vector for each admissible solution (basic estimates are contained in Tables 2.6, 2.7 and 2.8):

$$\overline{z} = (z_M \bigcup z_L \bigcup z_V \bigcup z_U \bigcup z_T)$$
$$= (z_{m1}, z_{m2}, z_{m3}, z_{m4}, z_{m5}, z_{l1}, z_{l2}, z_{l3}, z_{l4}, z_{v1}, z_{v2}, z_{v3}, z_{v4}, z_{u1}, z_{u2}, z_{u3}, z_{u4}, z_{t1}, z_{t2}, z_{t3}, z_{t4}).$$

On the other hand, it may be reasonable to consider a simplified version of the estimate vector as follows: $\widehat{z} = (r_M, r_L, r_V, r_U, r_T)$, where r_M, r_L, r_V, r_U, r_T are the priorities of DAs, which are obtained for local DAs (for M, for L, for V, for U, and for T; Tables 2.6, 2.7 and 2.8). To simplify the considered example, the second case of the estimate vector is used. Thus, the resultant vector estimates (i.e., $\{\widehat{z}\}$) for examined 45 admissible solutions are contained in Table 2.11.

Evidently, it is reasonable to consider the estimate vector for the ideal solution as follows: $\widehat{z_I} = (1, 1, 1, 1, 1)$. Now, let us use a simplified proximity function between ideal solution I and design alternative as follows (i.e., metric like l^2):

$$\rho(I, DA) = \sqrt{\sum_{k \in \{M,L,V,U,T\}} (z_k(I) - z_k(DA))^2}.$$

Table 2.11 Estimates of admissible solutions

DAs	$\widehat{z}$	Proximity to ideal point	Membership of Pareto-set
S_1	(2, 1, 1, 2, 3)	2.4495	No
S_2	(2, 1, 1, 2, 3)	2.4495	No
S_3	(3, 1, 1, 2, 3)	3.0	No
S_4	(3, 2, 1, 2, 3)	3.1623	No
S_5	(1, 1, 1, 2, 3)	2.2361	Yes
S_6	(1, 2, 1, 2, 3)	2.4495	No
S_7	(2, 1, 1, 2, 3)	2.4495	No
S_8	(2, 1, 1, 2, 3)	2.4495	No
S_9	(2, 1, 1, 2, 3)	2.4495	No
S_{10}	(2, 1, 2, 3, 2)	2.6458	No
S_{11}	(2, 1, 2, 3, 2)	2.6458	No
S_{12}	(3, 1, 2, 3, 2)	3.1623	No
S_{13}	(3, 2, 2, 3, 2)	3.3166	No
S_{14}	(1, 1, 2, 3, 2)	2.4495	No
S_{15}	(1, 2, 2, 3, 2)	2.6458	No

(continued)

Table 2.11 (continued)

DAs	$\widehat{z}$	Proximity to ideal point	Membership of Pareto-set
S_{16}	(2, 1, 2, 3, 2)	2.6458	No
S_{17}	(2, 1, 2, 3, 2)	2.6458	No
S_{18}	(2, 1, 2, 3, 2)	2.6458	No
S_{19}	(2, 1, 2, 2, 2)	2.0	No
S_{20}	(2, 1, 2, 2, 2)	2.0	No
S_{21}	(3, 1, 2, 2, 2)	2.6458	No
S_{22}	(3, 2, 2, 2, 2)	2.8284	No
S_{23}	(1, 1, 2, 2, 2)	1.7321	Yes
S_{24}	(1, 2, 2, 2, 2)	2.0	No
S_{25}	(2, 1, 2, 2, 2)	2.0	No
S_{26}	(2, 1, 2, 2, 2)	2.0	No
S_{27}	(2, 1, 2, 2, 2)	2.0	No
S_{28}	(2, 1, 3, 3, 2)	3.1623	No
S_{29}	(2, 1, 3, 3, 2)	3.1623	No
S_{30}	(3, 1, 3, 3, 2)	3.6056	No
S_{31}	(3, 2, 3, 3, 2)	3.7417	No
S_{32}	(1, 1, 3, 3, 2)	3.0	No
S_{33}	(1, 2, 3, 3, 2)	3.1623	No
S_{34}	(2, 1, 3, 3, 2)	3.1623	No
S_{35}	(2, 1, 3, 3, 2)	3.1623	No
S_{36}	(2, 1, 3, 3, 2)	3.1623	No
S_{37}	(2, 1, 3, 2, 2)	2.6458	No
S_{38}	(2, 1, 3, 2, 2)	2.6458	No
S_{39}	(3, 1, 3, 2, 2)	3.1623	No
S_{40}	(3, 2, 3, 2, 2)	3.3166	No
S_{41}	(1, 1, 3, 2, 2)	2.4495	No
S_{42}	(1, 2, 3, 2, 2)	2.6458	No
S_{43}	(2, 1, 3, 2, 2)	2.658	No
S_{44}	(2, 1, 3, 2, 2)	2.6458	No
S_{45}	(2, 1, 3, 2, 2)	2.6458	No

The resultant proximity is presented in Table 2.11. Finally, the best composite DA (by the minimal proximity) is: $S^I_0 = S_{23} = A_5 \star B_3 = M_3 \star L_1 \star V_1 \star U_2 \star T_3$ ($\rho = 1.7321$). Several composite DAs are very close to the best one, for example:

$S^I_1 = S_{19} = A_1 \star B_3 = M_1 \star L_1 \star V_2 \star U_3 \star T_4$ ($\rho = 2.0$), $S^I_2 = S_{20} = A_2 \star B_3 = M_1 \star L_4 \star V_2 \star U_3 \star T_4$ ($\rho = 2.0$), $S^I_3 = S_{24} = A_6 \star B_3 = M_4 \star L_3 \star V_2 \star U_3 \star T_4$ ($\rho = 2.0$), $S^I_4 = S_{25} = A_7 \star B_3 = M_5 \star L_1 \star V_2 \star U_3 \star T_4$ ($\rho = 2.0$), $S^I_5 = S_{26} = A_8 \star B_3 = M_5 \star L_2 \star V_3 \star U_2 \star T_4$ ($\rho = 2.0$), and $S^I_6 = S_{27} = A_9 \star B_3 = M_5 \star L_4 \star V_2 \star U_3 \star T_4$ ($\rho = 2.0$).

It may be reasonable to point out several prospective directions for the improvement of this method:

(1) consideration of special types of proximity between solutions and the ideal point (e.g., ordinal proximity, vector-like proximity [628], etc.);
(2) usage of special interactive procedures (expert judgment) for the assessment of the proximity;
(3) consideration of a set of ideal points (the set can be generated by domain expert(s)); and
(4) design of special support visualization tools, which will aid domain expert(s) in his/her (their) activity (i.e., generation of the ideal point and assessment of proximity).

In addition, let us list the basic approaches to generation of the ideal point(s):

1. consideration of design alternative with the estimate vector, in which each component equals the best value of the design alternatives estimates (by the corresponding criterion, i.e., minimum or maximum);
2. consideration of design alternative with the estimate vector, in which each component equals the best value of the corresponding criterion scale (i.e., minimum or maximum);
3. expert judgment based generation of the best design alternative(s);
4. projection of expert judgment based design alternatives into convex shell of the set of Pareto-efficient points; etc.

2.3.4 Pareto-Based Morphological Analysis

Here, the initial set of admissible solutions corresponds to the previous design case (basic MA). Two approaches can be used for mulricriteria assessment of admissible solutions:

1. Basic method: selection of Pareto-efficient solutions over the set of admissible composite solutions on the basis of of usage of the initial set of criteria for assessment of each admissible composite DAs;
2. Two-stage method:
   (i) assessment of initial components by the corresponding criteria and ranking of the alternative components the get an ordinal priority for each components,
   (ii) selection of Pareto-efficient solutions over the set of admissible composite solutions on the basis of of usage of the vector estimates, which integrate priorities of solution components above. The results of the Pareto-based MA are presented in Table 2.11, i.e., the resultant (Pareto-efficient) DAs are: (i) $S^P_1 = S_5 = A_5 \star B_1 = M_4 \star L_2 \star V_1 \star U_5 \star T_1$ and (ii) $S^P_2 = S_{23} = A_5 \star B_3 = M_4 \star L_2 \star V_2 \star U_3 \star T_4$.

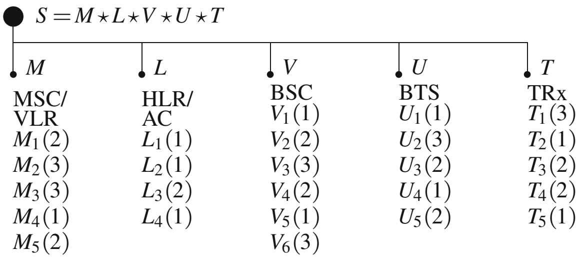


Fig. 2.10 Structure of designed GSM network

It is important to note, the estimate vector for each DA can contain estimates of compatibility as well.

2.3.5 Multiple Choice Problem

Multiple choice problem with 5 groups of elements (i.e., for M, L, V, U, T) is studied (Fig. 2.10, priorities of DAs are shown in parentheses). Here, it is reasonable to examine multicriteria multiple choice problem. In the example, a simplified problem solving approach is considered (Table 2.12):

(i) a simple greedy algorithm based on element priorities is used;
(ii) for each element (i.e., i, j) 'profit' is computed as follows: $c_{i,j} = 4 - r_{i,j}$;
(iii) for each element (i.e., i, j) a required resource is computed as follows: $a_{i,j} = 11 - z_{i,j}$ where $z_{i,j}$ equals: (a) for M: the estimate upon criterion C_{m3} (Table 2.6), (b) for L: 1.0, (c) for V: the estimate upon criterion C_{v1} (Table 2.7), (d) for U: the estimate upon criterion C_{u3} (Table 2.8), and (e) for T: the estimate upon criterion C_{mt3} (Table 2.8).

Thus, the following simplified one-objective problem is considered:

$$\max \sum_{i=1}^{5} \sum_{j=1}^{q_i} c_{ij} x_{ij} \quad s.t. \sum_{i=1}^{5} \sum_{j=1}^{q_i} a_{ij} x_{ij} \leq b, \quad \sum_{j=1}^{q_i} x_{ij} = 1 \ \forall i = \overline{1,5}, \quad x_{ij} \in \{0, 1\},$$

where $q_1 = 5$, $q_2 = 4$, $q_3 = 6$, $q_4 = 5$, $q_5 = 5$. After the usage of the greedy algorithm, the following composite DAs are obtained (Table 2.12):

(1) resource constraint $b = 14$: $S_1^C = M_4 \star L_1 \star V_6 \star U_3 \star T_1$,
(2) resource constraint $b = 15$: $S_2^C = M_4 \star L_1 \star V_6 \star U_1 \star T_1$.

Table 2.12 Example for multiple choice problem

No. (i, j)	DAs	Priority r	Resource requirement $a_{i,j}$	$c_{i,j}/a_{i,j}$	Selection (constraint: ≤ 14)	Selection (constraint: ≤ 15)
(1, 1)	M_1	2	5.0	0.4	No	No
(1, 2)	M_2	3	3.0	0.33	No	No
(1, 3)	M_3	3	2.0	0.5	No	No
(1, 4)	M_4	1	6.0	0.5	Yes	Yes
(1, 5)	M_5	2	4.8	0.38	No	No
(2, 1)	L_1	1	1.0	3.0	Yes	Yes
(2, 2)	L_2	1	1.0	3.0	No	No
(2, 3)	L_3	2	1.0	2.0	No	No
(2, 4)	L_4	1	1.0	3.0	No	No
(3, 1)	V_1	1	5.0	0.6	No	No
(3, 2)	V_2	2	4.0	0.5	No	No
(3, 3)	V_3	3	2.0	0.5	No	No
(3, 4)	V_4	2	4.0	0.5	No	No
(3, 5)	V_5	1	5.0	0.6	No	No
(3, 6)	V_6	3	1.0	1.0	Yes	Yes
(4, 1)	U_1	1	6.0	0.5	No	Yes
(4, 2)	U_2	3	5.0	0.2	No	No
(4, 3)	U_3	2	5.0	0.4	Yes	No
(4, 4)	U_4	3	8.0	0.39	No	No
(4, 5)	U_5	2	5.0	0.4	No	No
(5, 1)	T_1	3	1.0	1.0	Yes	Yes
(5, 2)	T_2	1	8.0	0.39	No	No
(5, 3)	T_3	2	4.0	0.5	No	No
(5, 4)	T_4	2	3.0	0.66	No	No
(5, 5)	T_5	1	7.0	0.42	No	No

2.3.6 Hierarchical Morphological Design

A preliminary example for HMMD was presented in [682] (Fig. 2.11, priorities of DAs are shown in parentheses). For system part A, the following Pareto-efficient composite DAs are obtained: (1) $A_1 = M_4 \star L_2$, $N(A_1) = (3; 2, 0, 0)$; (2) $A_2 = M_4 \star L_4$, $N(A_2) = (3; 2, 0, 0)$. For system part B, the following Pareto-efficient composite DAs are obtained: (1) $B_1 = V_5 \star U_1 \star T_5$, $N(B_1) = (2; 3, 0, 0)$; (2) $B_2 = V_5 \star U_4 \star T_2$, $N(B_2) = (2; 3, 0, 0)$; (3) $B_3 = V_1 \star U_5 \star T_1$, $N(B_3) = (3; 1, 1, 1)$, and (4) $B_4 = V_2 \star U_3 \star T_4$, $N(B_4) = (3; 0, 3, 0)$. Figure 2.12 illustrates system quality for B. Now, it is possible to combine the resultant composite DAs as follows (Fig. 2.11):

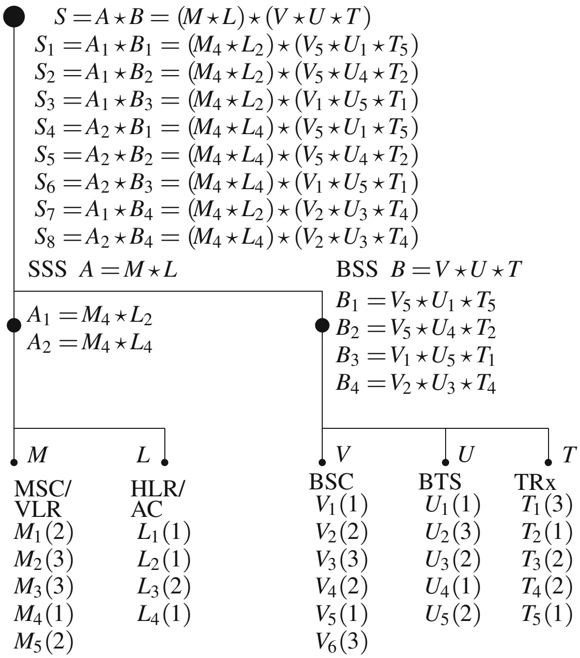


Fig. 2.11 Designed GSM network

(1) $S_1^H = A_1 \star B_1 = (M_4 \star L_2) \star (V_5 \star U_1 \star T_5)$;

(2) $S_2^H = A_1 \star B_2 = (M_4 \star L_2) \star (V_5 \star U_4 \star T_2)$;

(3) $S_3^H = A_1 \star B_3 = (M_4 \star L_2) \star (V_1 \star U_5 \star T_1)$;

(4) $S_4^H = A_2 \star B_1 = (M_4 \star L_4) \star (V_5 \star U_1 \star T_5)$;

(5) $S_5^H = A_2 \star B_2 = (M_4 \star L_4) \star (V_5 \star U_4 \star T_2)$;

(6) $S_6^H = A_2 \star B_3 = (M_4 \star L_4) \star (V_1 \star U_5 \star T_1)$;

(7) $S_7^H = A_1 \star B_3 = (M_4 \star L_2) \star (V_2 \star U_3 \star T_4)$; and (8) $S_8^H = A_2 \star B_3 = (M_4 \star L_4) \star (V_2 \star U_3 \star T_4)$.

Finally, it is reasonable to integrate quality vectors for components A and B to obtain the following quality vectors: $N(S_1^H) = (2; 5, 0, 0)$, $N(S_2^H) = (2; 5, 0, 0)$, $N(S_3^H) = (3; 3, 1, 1)$, $N(S_4^H) = (2; 5, 0, 0)$, $N(S_5^H) = (3; 3, 1, 1)$, and $N(S_6^H) = (3; 3, 1, 1)$. $N(S_7^H) = (3; 2, 3, 0)$, and $N(S_8^H) = (3; 2, 3, 0)$. Further, the obtained eight resultant composite decisions can be analyzed to select the best decision (e.g., additional multicriteria analysis, expert judgment).

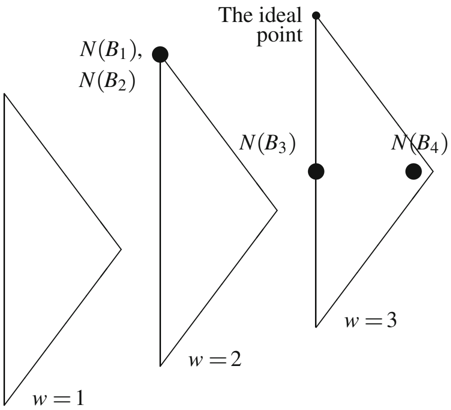


Fig. 2.12 Space of system quality for B

2.3.7 Comparison of Methods and Discussion

Note, 45 resultant solutions were obtained by basic MA. Table 2.13 integrates resultant composite solutions for four methods: (1) closeness to ideal point method (the best solution and six close solutions), (2) Pareto-based morphological analysis (two solutions), (3) multiple choice problem (two solutions), (4) HMMD (eight solutions).

Now, let us consider a comparison of solution sets above via the following notes:

1. In the case of the first three methods (i.e., MA, closeness to ideal point method, and Pareto-based morphological analysis), compatibility estimates in examples are considered at levels 0 (incompatible) and 1 (compatible). Generally, this situation corresponds of a simplified case.
2. In the case of MA, a sufficiently large and rich set of admissible solutions was obtained: 45. Note, this solution set covers solutions sets for other methods (i.e., closeness to ideal-point method, Pareto-based morphological analysis, HMMD). At the same time, the problem is: *to analyze this large solution set*.
3. In the case of closeness to ideal point method, only solution S^I_0 belongs to the set of Pareto-efficient solutions. Considered solutions $\{S^I_1, S^I_2, S^I_3, S^I_4, S^I_5, S^I_6\}$, which are close to the above-mentioned solution, are not sufficiently good by elements. At the same time, some good solutions are lost, for example: S^H_3, S^H_5, S^H_6, S^H_8.
4. In the case of Pareto-based morphological analysis, many good solutions are lost, for example: S^H_5, S^H_6, S^H_8, etc.
5. In the case of multiple choice problem, compatibility estimates are not examined. As a result, all obtained solutions are inadmissible. It can be reasonable to extend this kind of optimization models by additional logical constraints, which will formalize the compatibility requirements. But it may lead to complicated models.
6. In the case of HMMD, the set of solutions is sufficiently rich and not very large at the same time (eight solutions).

Table 2.13 Integration of composite solutions

	Method	Resultant composite DAs	Quality vector (HMMD)
1.	Closeness to ideal point	$S_0^I = M_4 \star L_2 \star V_2 \star U_3 \star T_4$	(3; 2, 3, 0)
		$S_1^I = M_1 \star L_1 \star V_2 \star U_3 \star T_4$	(3; 1, 3, 1)
		$S_2^I = M_1 \star L_4 \star V_2 \star U_3 \star T_4$	(3; 1, 4, 0)
		$S_3^I = M_4 \star L_3 \star V_2 \star U_3 \star T_4$	(3; 1, 4, 0)
		$S_4^I = M_5 \star L_1 \star V_2 \star U_3 \star T_4$	(3; 1, 4, 0)
		$S_5^I = M_5 \star L_2 \star V_3 \star U_2 \star T_4$	(3; 1, 2, 2)
		$S_6^I = M_5 \star L_4 \star V_2 \star U_3 \star T_4$	(3; 1, 4, 0)
2.	Pareto-based MA	$S_1^P = M_4 \star L_2 \star V_1 \star U_5 \star T_1$	(3; 3, 1, 1)
		$S_2^P = M_4 \star L_2 \star V_2 \star U_3 \star T_4$	(3; 2, 3, 0)
3.	Multiple choice problem	$S_1^C = M_4 \star L_1 \star V_6 \star U_3 \star T_1$	(0; 2, 1, 2)
		$S_2^C = M_4 \star L_1 \star V_6 \star U_1 \star T_1$	(0; 3, 0, 2)
4.	HMMD	$S_1^H = M_4 \star L_2 \star V_5 \star U_1 \star T_5$	(2; 5, 0, 0)
		$S_2^H = M_4 \star L_2 \star V_5 \star U_4 \star T_2$	(2; 5, 0, 0)
		$S_3^H = M_4 \star L_2 \star V_1 \star U_5 \star T_1$	(3; 3, 1, 1)
		$S_4^H = M_4 \star L_4 \star V_5 \star U_1 \star T_5$	(2; 5, 0, 0)
		$S_5^H = M_4 \star L_4 \star V_5 \star U_4 \star T_2$	(3; 3, 1, 0)
		$S_6^H = M_4 \star L_4 \star V_1 \star U_5 \star T_1$	(3; 3, 1, 1)
		$S_7^H = M_4 \star L_2 \star V_2 \star U_3 \star T_4$	(3; 2, 3, 0)
		$S_8^H = M_4 \star L_4 \star V_2 \star U_3 \star T_4$	(3; 2, 3, 0)

Table 2.14 contains an additional qualitative author's comparison of used methods. Here, computational complexity is depended on enumerative computing and analysis of all admissible combinatorial solutions (i.e., admissible combinations). In the case of HMMD, the usage of hierarchical system structure decreases complexity of the computing process. In the case of Pareto-based MA, an analysis of

Table 2.14 Qualitative comparison of used methods

	Method	Computational complexity	Taking into account compatibility	Usefulness for selection of the best solutions	Usefulness for expert(s)
1.	MA	High	Yes, binary	Hard	Hard
2.	Closeness to Ideal-point(s)	High	Yes, binary	Easy	Good
3.	Pareto-based MA	High	Yes, binary	Medium, analysis of pareto-efficient solutions	Good
4.	Multiple choice Problem	Low/medium	None	Easy	Medium
5.	HMMD	Low/medium	Yes, ordinal	Easy	Good

Pareto-efficient solutions will required additional enumerative computing. Finally, column "Usefulness for expert(s)" (Table 2.14) corresponds to the following: (i) possibility to include the domain(s) expert(s) or/and decision maker(s) into the solving process (i.e., to include cognitive man-machine procedures into the design framework), (ii) understandability of the used design method to domain(s) expert(s) and/or decision maker(s).

Generally, the selection of the certain kind of morphological methods for a designed system has to be based on the following: (a) a type of the examined system class (structure, complexity of component interaction, etc.); (b) structure and complexity of the examined representative of the system class; (c) existence of an experienced design team; (d) possibility to implement some assessment procedures (for assessment of DAs and/or compatibility); (e) possibility to use computational recourses (e.g., computing environment, power software, computing personnel), and (f) possibility to use qualified domain(s) experts and/or decision makers.

2.4 Towards Other Approaches

Generally, hierarchical design approaches are often based on a hierarchical model of the designed system and 'Bottom-Up' design framework (Fig. 2.13). The list of some hierarchical design approaches, which are close to MA-based approaches and based on the framework above, is the following: (1) hierarchical design frameworks (e.g., [582, 957]); (2) structural synthesis of technical systems based on MA, cluster analysis, and parametric optimization [875]; (3) HTN planning (e.g., [317]); (4) hierarchical decision making in design and manufacturing (e.g., [73, 74, 92, 449, 593]); and (5) linguistic geometry approach (e.g., [990]).

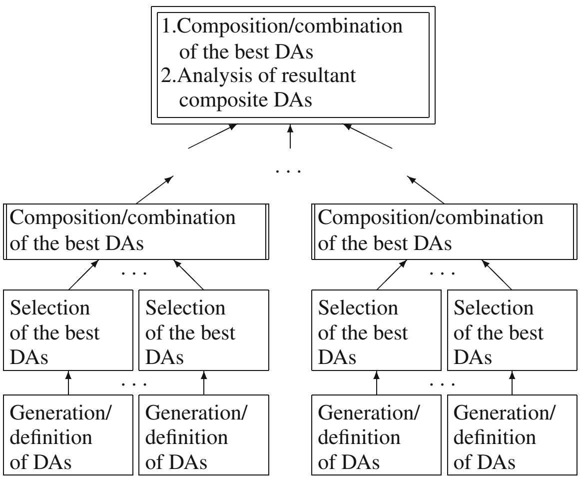


Fig. 2.13 'Bottom-Up' design scheme

Here, it is reasonable to point out some nonlinear programming models, which are targeted to modular system design as well. First, modular design of series and series-parallel information processing from the viewpoint of reliable software design while taking into account a total budget (i.e., multi-version software design) was investigated in [41, 42, 96]. The authors suggested several generalizations of knapsack problem with non-linear objective function. Thus, the following kind of the optimization model for reliable modular software design can be examined (a basic case) [96]:

$$\max \prod_{i=1}^{m} (1 - \prod_{j=1}^{q_i} (1 - p_{ij} x_{ij}))$$

$$s.t. \sum_{i=1}^{m} \sum_{j=1}^{q_i} d_{ij} x_{ij} \leq b, \quad \sum_{j=1}^{q_i} x_{ij} \geq 1 \quad \forall i = \overline{1, m}, \quad x_{ij} \in \{0, 1\},$$

where p_{ij} is a reliability estimate of software module version (i, j) (i.e., version j for module i), d_{ij} is a cost of software module version (i, j). Figure 2.14 illustrates the design problems above. Evidently, the obtained models are complicated ones and heuristics or enumerative techniques are used for the solving process [41, 42, 96].

In [1104], the problems above are considered regarding the usage of multi-objective genetic algorithms. Second, design problems in chemical engineering systems require often examination of integer and continuous variables at the same time and, as a result, nonlinear mixed-integer optimization models are formulated and used (e.g., [343, 413]).

Further, it is reasonable to point out constraint-based approaches (e.g., [341, 734, 993]) including composite constraint satisfaction problems and AI-based solving methods (e.g., [914, 987]).

Table 2.15 contains some other research directions, which are close to morphological design (models or/and applications).

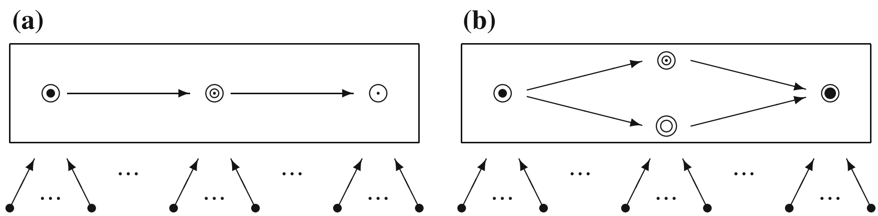


Fig. 2.14 Modular design of series or series-parallel system. **a** Series scheme. **b** Series-parallel scheme

Table 2.15 Research directions closed to morphological design

	Research directions/models	Some sources
1.	Problems of representatives	[437, 562]
2.	Design structure matrix	[150, 595]
3.	Morphological tables	[475, 969]
4.	Clustering in multipartite graph	[187, 1047]
5.	Maximal clique in multipartite graph	[254]
6.	Method engineering, method service	[144, 264]
7.	OLAP systems	[1024, 1093]
8.	Coresets problems	[330, 444, 1125]
9.	Mining of association rules	[10, 984]

2.5 Summary

In the chapter, several MA-based system design approaches were described. It can be very useful and prospective to extend studies of the MA-based approaches, for example:

1. design of interactive MA-based methods (e.g., participation of experts),
2. integration of MA-based methods and special expert based systems;
3. integration of MA-based methods and TOPSIS methods; and
4. special research projects as generation of benchmarks for evaluation of MA-based methods.

Springer

http://www.springer.com/978-3-319-09875-3

Modular System Design and Evaluation
Levin, M.S.
2015, XXI, 473 p. 405 illus., Hardcover
ISBN: 978-3-319-09875-3