Chapter 2
Binary and Millisecond Pulsars

No, I don't understand my husband's theory of relativity, but I know my husband, and I know he can be trusted.

Elsa Einstein

2.1 The Observed Population of Binary Pulsars

The first evidence for neutron stars residing in binaries came in the early 60s when Giacconi et al. (1962) discovered the first extrasolar X-ray source, Sco X-1. Its high X-ray luminosity of $L > 10^{35}\,\mathrm{erg\,s^{-1}}$ could be naturally understood if the source is powered by a compact object (neutron star or black hole) that accretes mass from a stellar companion. Matter falling onto the surface would then result in significant release of gravitational energy which, due to the small column density of $\sim 0.3\,\mathrm{g\,cm^{-2}}$, can easily penetrate the system in the form of X-rays (Tauris and van den Heuvel 2003). This hypothesis was confirmed with the discovery of 4.9 s pulsations from the 2 d binary Cen X-3 (Schreier et al. 1972).

The first binary radio pulsar was discovered some years after by Hulse and Taylor (1975) during a sensitive survey conducted with the 300-m Arecibo radio telescope (Hulse and Taylor 1974). The "Hulse–Taylor" binary consists of two neutron stars (one of them is the pulsar) that orbit each other every 7.75 h. Among else, radio-timing observations yielded the first accurate determination of neutron-star masses and the first indirect detection of gravitational waves through the measurement of the system's orbital decay.

Today, more than 120 pulsars have been observed to orbit around planets, main-sequence stars, evolved giants, semi-degenerate stars, white dwarfs and neutron stars. Like the original Hulse–Taylor binary, their clock-like properties allow for precision measurements of their orbital dynamics that can be used to infer stellar properties, probe the physics of binary evolution and test the predictions of General Relativity and alternative theories of gravity.

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J. Antoniadis, *Multi-Wavelength Studies of Pulsars and Their Companions*,
Springer Theses, DOI 10.1007/978-3-319-09897-5_2

2.2 Timing and Orbits

As briefly mentioned before, much of the interesting science related to radio pulsars comes from the regular monitoring of their rotation.

For any astrophysical source, the time of arrival (TOA) of an emitted signal depends on its (changing) distance from the earth; for pulsars, the signal of interest is the pulse that sweeps the Earth once-per-rotation. Because the rotation is nearly constant, the rotational phase ϕ corresponding to a time of emission t can be approximated by a Taylor expansion:

$$\phi(t) = \phi_0 + \nu(t - t_0) + \frac{1}{2}\dot{\nu}(t - t_0)^2 + \cdots , \tag{2.1}$$

where ϕ_0 and t_0 are arbitrarily chosen reference phase and time. The salient property that enables precision measurements is that the difference between any two times of emission has to be an integer number, $\Delta\phi = N \in \mathbb{Z}$. The TOA differs from t by an amount that depends on propagation delays due to the motion of the Earth, the interstellar medium and the motion of the pulsar:

$$\Delta t = \Delta_{\mathrm{E}_\odot} + \Delta_{\mathrm{R}_\odot} + \Delta_{\mathrm{S}_\odot} - D/f^2 + \Delta_{\mathrm{Binary}}. \tag{2.2}$$

Here, the first three terms account for the Einstein, Roemer and and Shapiro delays of the bodies in the Solar System; the fourth term is the contribution due to the dispersion of the signal from the interstellar medium at an observing frequency f and the fifth term accounts for the binary motion of the pulsar and secular terms due to the system's motion as a hole (Lorimer and Kramer 2005). We shall now focus on the last term adopting the convention of Damour and Deruelle (1986) and Damour and Taylor (1992). A summary of the basic (Keplerian) orbital elements and naming conventions used throughout this thesis can be seen in Fig. 2.1. Obviously, Δ_{Binary} has to be a function of the orbital period, time of ascending node passage, eccentricity and projected semi-major axis ($x \equiv a \sin i/c$):

$$\left\{p^{\mathrm{K}}\right\} = \{P_{\mathrm{b}}, T_0, e_0, \omega_0, x_0\} , \tag{2.3}$$

where all subscripted values refer to an arbitrary epoch. For the detailed description of the binary motion we also need an additional set of parameters that can model any possible deviation from the classical Keplerian motion:

$$\left\{p^{\mathrm{PK}}\right\} = \left\{\dot{P}_{\mathrm{b}}, \gamma, r, s, \delta_\theta, \dot{e}, \dot{x}, \dot{\omega}, \delta_{\mathrm{r}}, A, B, D\right\} . \tag{2.4}$$

Here, $\dot{P}$, $\dot{e}$, $\dot{x}$ and $\dot{\omega}$ can be thought of as the first term of the Taylor expansions for the relevant parameters. Damour and Taylor (1992) showed that the above combination of Keplerian and *post-Kelperian* (PK) terms can correct for deviations (up-to) $O(v^5/c^5)$ weaker than the Newtonian gravitational interaction:

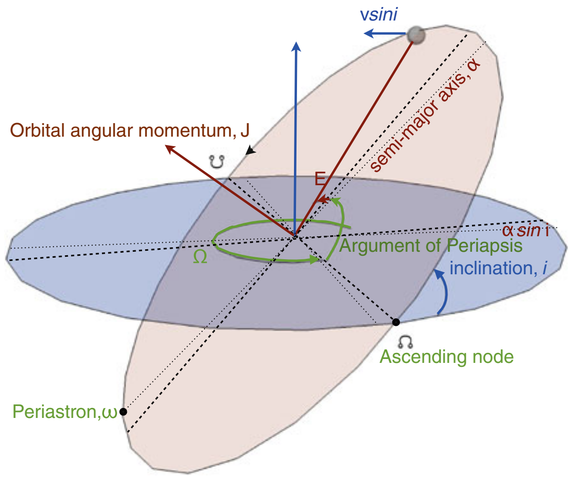

Fig. 2.1 Angles and orientation related to the pulsar orbit

$$\Delta_R = D^{-1}\left\{x \sin\omega\,[\cos u - e(1+\delta_r)] + x[1-e^2(1+\delta_\theta)^2]^{1/2}\cos\omega\sin u\right\}, \tag{2.5}$$

$$\Delta_E = D^{-1}\gamma\sin u, \tag{2.6}$$

$$\Delta_S = -D^{-1}2r\ln\left\{1 - e\cos u - s\left[\sin\omega\,(\cos u - e) + (1-e^2)^{1/2}\cos\omega\sin u\right]\right\}, \tag{2.7}$$

$$\Delta_A = D^{-1}\left\{A[\sin(\omega + A_e(u)) + e\sin\omega] + B[\cos(\omega + A_e(u)) + e\cos\omega]\right\}, \tag{2.8}$$

$$A_e(u) = 2\arctan\left[\left(\frac{1+e}{1-e}\right)^{1/2}\tan\frac{u}{2}\right], \tag{2.9}$$

$$\omega = \omega_0 + \kappa\Delta_e(u) \tag{2.10}$$

and

$$u - e\sin u = 2\pi\left[\left(\frac{t-t_0}{P_b}\right) - \frac{1}{2}\dot{P}_b\left(\frac{t-t_0}{P_b}\right)^2 + \cdots\right]. \tag{2.11}$$

Here, D^{-1} can be thought of as a Doppler term due to the secular motion of the center of mass and can be reabsorbed into re-definitions of the other parameters so that one can adopt $D = 1$ for the fitting procedure (Damour and Taylor 1991; Lorimer and Kramer 2005). Similarly, A and B are non-separable and can be neglected by redefining T_0, x, e, δ_r and δ_θ. Equations (2.5)–(2.11) resemble the Roemer, Einstein, Shapiro and aberration delays respectively; δ_θ and δ_r quantify possible relativistic orbital deformations of order $O(v^2/c^2)$ and $\{r, s\}$ parametrize the "range" and "shape" of the Shapiro delay. An important thing to note is that the former timing formula is *theory independent* and describes the orbit in a phenomenological manner. In case the system is "clean" (i.e the orbiting bodies can be approximated by non-rotating point particles), the PK parameters are (in the most general case) functions of their masses and the properties of their internal gravitational field (Will 1993).

2.2.1 Masses and Tests of General Relativity

In General Relativity, the effacement of the internal structure, the gravitational interaction between the binary's (non-rotating) components is only a function of their masses. Hence, the PK parameters become functions of (only) the masses and Keplerian parameters:

$$\dot{\omega} = 3 \left(\frac{P_{\rm b}}{2\pi} \right)^{-5/3} (T_\odot M)^{2/3} \left(1 - e^2\right)^{-1}, \tag{2.12}$$

$$\gamma = e \left(\frac{P_{\rm b}}{2\pi} \right)^{1/3} T_\odot^{2/3} M^{-4/3} m_2 \left(m_1 + 2m_2\right), \tag{2.13}$$

$$\dot{P}_{\rm b} = -\frac{192\pi}{5} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right) \left(1 - e^2\right)^{-7/2} T_\odot^{-5/3} m_1 m_2 M^{-1/3}, \tag{2.14}$$

$$r = T_\odot m_2 \tag{2.15}$$

$$s = \sin i = x \left(\frac{P_{\rm b}}{2\pi} \right)^{-2/3} T_\odot^{-1/3} M^{2/3} m_2^{-1} \tag{2.16}$$

and

$$\delta_r = \delta_\theta. \tag{2.17}$$

Here, $T_\odot = GM_\odot/c^3$ and $M = m_1 + m_2$ is the total mass of the binary. We note that the former set of equations refers to the intrinsic PK effects that can be extracted from the measured values after taking into account kinematic corrections (i.e. finding $\dot{D}/D$): if these were constant, their contribution would cancel-out directly. Unfortunately, this is normally not the case, since the accelerated motion of the system in the Galaxy results in secular variations that need to be corrected explicitly.

The measurement of *two* post-Keplerian parameters defines a set of functions on the mass-mass plane, the intersection of which yields the masses of the pulsar and the companion. If a *third* PK parameter becomes measurable and General Relativity is correct, the corresponding mass-mass curve should intersect with the previous two at the same point. For the original Hulse–Taylor binary the first PK parameters to be measured were γ and $\dot{\omega}$. Some years after, Weisberg and Taylor (1981) announced the detection of orbital decay which was a strong-field relativistic effect entering the orbital dynamics at the 2.5 ($O(v^5/c^5)$) Post-Newtonian level in GR (Will 1993). The measured value followed the prediction of the quadrupole formula (Eq. 2.13) providing the first radiative test of General Relativity.

Today, the Hulse–Taylor pulsar is outshined by the Double Pulsar for which all the former PK parameters have been measured. Owing to its short orbit (2.45 h) and proximity to the Earth, the kinematic effects can be constrained with high accuracy, yielding 5 distinct tests of General Relativity (including Shapiro delay).

Similarly, all proposed alternative theories of gravity result to unique formulas for the PK parameters. In the following Chapters we focus on two different families of alternative theories:

1. **Scalar-Tensor Gravity**: Scalar-Tensor (ST) theories of gravity are extensions of General Relativity in which gravity is mediated by a spin-2 graviton and a spin-0 scalar partner, ϕ. The motivation for these theories is multi-fold and related, among-else, to Grand Unification attempts and questions concerning Dark Matter, Dark Energy and Inflation.
   In ST-gravity the Strong Equivalence Principle is violated, leading to emission of dipole gravitational radiation that enters the orbital dynamics at the 1.5 Post-Newtonian level (Will 1993). The amount of dipolar waves emitted by the system depends on the difference of the binding energies inside the two bodies. Systems composed of two neutron stars, like the Double Pulsar and the Hulse–Taylor binary are therefore less sensitive to dipolar waves. The predictions of ST gravity can be best tested with "clean" assymetric systems composed of a pulsar and a white dwarf.
2. **Tensor-Vector-Scalar Gravity (TeVeS)**: TeVeS is a relativistic formulation of Modified-Newtonian-Dynamics (MOND) designed to explain galaxy-rotation curves without invoking Dark Matter. The gravitational interaction is mediated by spin–2, spin–1 and spin–0 particles that lead to modifications of all PK parameters. These theories can be tested with both neutron star/neutron star and neutron star/white dwarf binaries.

2.2.2 Special Cases: Circular Orbits

As we discuss further bellow, many millisecond pulsars reside in short-period, circular-orbit binaries. For these systems most PK parameters vanish and only the Shapiro delay and orbital decay can be measured. The former depends strongly on the inclination and therefore can be constrained only for systems viewed nearly edge-on; the latter is sensitive to the orbital period and can be measured only in "relativistic binaries", i.e. systems with short orbital periods.

2.2.3 Special Cases: Mass Ratios and Spectroscopy

For a handful of binary millisecond pulsars, the companion is bright enough for phase-resolved optical spectroscopy. This allows the measurement of its radial velocity which, together with the radial velocity of the pulsar measured with radio-timing, yields the mass ratio of the system. Furthermore, in case the companion is a white dwarf, comparison of its spectrum with model atmospheres yields its mass. Combined, the mass ratio and companion mass yield the mass of the pulsar. This information allows for strong-field radiative tests in relativistic binaries, even if constraints on Shapiro delay are not possible. Similarly, for the Double Pulsar where both neutron stars are visible, the Roemer delays yield a theory-independent mass ratio.

2.3 Recycled Pulsars and Their Formation

With few exceptions, the fastest spinning pulsars known, have white-dwarf or semi-degenerate companions. These systems share remarkable similarities, thought to be the relics of their evolutionary history: They spin down $\sim 10^3$–10^5 times slower than their single counterparts (Lorimer and Kramer 2005) which implies that they have relatively weak magnetic fields of order $\sim 10^8$ G. Furthermore, their orbits are almost perfectly circular and in some cases the mass of their companions is so low that they would not have formed within a Hubble-time if they were single stars.

Today, it is firmly established that these binaries emerge from systems initially formed by a massive, $M > 6\,\mathrm{M}_\odot$ star (the progenitor of the pulsar) and a lighter companion. After formation of the neutron star, the system evolves on a timescale determined by the orbital separation and companion mass. For donor star masses above $2.5\,\mathrm{M}_\odot$ and short initial periods, the evolution off the main sequence results in engulfment of the neutron star in the donor's envelope. Efficient removal of angular momentum during the common-envelope (CE) phase shrinks the orbit on a very short timescale ($\sim 10^4$ years) (Tauris and van den Heuvel 2003). If the orbital separation is larger than donor's radius during its entire lifespan, mass accretion will initiate when (and if) the donor fills its Roche-lobe. During this period the binary is observed as an X-ray binary. In X-ray binaries, the accretion episode can be long-lasting (10^8–10^{10} years) and thereby allowing the neutron star to accrete sufficient mass (and angular momentum) and spin it up to millisecond periods. Furthermore, the developed tidal torques synchronize the donor on a short timescale resulting in almost perfectly circularized orbits. For what follows, we shall only consider cases with donor star masses $\leq 2.5\,\mathrm{M}_\odot$ that ultimately lead to formation of low-mass white dwarfs or semi-degenerate stars. For a recent general review on the evolution of other systems see Tauris and van den Heuvel (2003).

2.3.1 Evolution of the Orbital Separation

During the X-ray binary phase, transfer of angular momentum changes dramatically the orbital dynamics. The orbital angular momentum is given by:

$$J_{\rm orb} = \frac{m_1 m_2}{M}\Omega a^2\sqrt{1-e^2}, \tag{2.18}$$

where $\Omega = \sqrt{GM/a^3}$ is the angular orbital velocity. We can find the evolution of the orbital separation, by differentiating the above equation:

$$\frac{\dot{a}}{a} = 2\frac{\dot{J}_{\rm orb}}{J_{\rm orb}} - 2\frac{\dot{m}_1}{m_1} - 2\frac{\dot{m}_2}{m_2} + \frac{\dot{m}_1 + \dot{m}_2}{M}. \tag{2.19}$$

Here, the total change in angular momentum can be thought of as the sum of contributions due to gravitational radiation, magnetic braking, spin-orbit couplings and mass-loss from the system (Tauris and van den Heuvel 2003):

$$\frac{\dot{J}_{\rm orb}}{J_{\rm orb}} = \frac{\dot{J}_{\rm GW}}{J_{\rm orb}} + \frac{\dot{J}_{\rm mb}}{J_{\rm orb}} + \frac{\dot{J}_{\rm ls}}{J_{\rm orb}} + \frac{\dot{J}_{\rm ml}}{J_{\rm orb}}, \tag{2.20}$$

where we have neglected changes to the eccentricity, because tidal interactions circularize the orbit on a much sorter timescale. Depending on the initial orbital separation, Roche-lobe overflow (RLO) can initiate when the star is still on the main sequence (Case-A RLO), during the RGB phase (Case-B RLO) or during helium shell-burning (Case-C RLO).

Case-A Roche-Lobe Overflow For short initial separations RLO initiates while the star is still on the main sequence. The evolution of these systems is driven by angular momentum loses due to magnetic braking (MB) and mass ejection from the system.

MB is thought to be the main mechanism responsible for the deceleration of low-mass stars with convective envelopes. In binaries, it operates at the expense of orbital angular momentum due to the tidal torques that tent to synchronize the spin. The details of the MB mechanism are uncertain but it seems that the dependence between the angular momentum loss and the stellar parameters is of the form:

$$\frac{\dot{J}_{\rm mb}}{J_{\rm orb}} \simeq -0.5\times 10^{-28} f_{\rm mb}^{-2}\frac{k^2 R_2^4}{a^5}\frac{GM^3}{m_1 m_2}\ {\rm s}^{-1} \tag{2.21}$$

where k^2 is the gyration radius of the donor and $f_{\rm mb}$ a constant of order unity (Tauris and van den Heuvel 2003).

Additionally, mass loss from the system results in an angular momentum loss rate given by:

$$\frac{\dot{J}_{\mathrm{ml}}}{J_{\mathrm{orb}}} = \frac{\alpha + \beta q^2 + \delta\gamma(1+q)^2}{1+q} \frac{\dot{m}_2}{m_2} \tag{2.22}$$

where α, β and δ are the fractions of mass lost through a direct wind, mass ejected (uniformly) from the accretor and from a circumbinary coplanar toroid with radius $r = \gamma^2 \alpha$.

Systems in this category evolve with decreasing orbital periods and eventually form binaries with a semi-degenerate companion (a.k.a "black widow" systems) or, perhaps, relativistic binaries with white dwarf companions (see next Chapters).

Case-B Roche-Lobe Overflow For larger initial neutron star-donor separations ($P_{\mathrm{b}} \geq 2\,\mathrm{d}$, Tauris and Savonije 1999), RLO initiates when the star evolves to a sub-giant and starts climbing its Hayashi track on the H-R diagram. The angular momentum loss mechanisms are generally not important and these systems evolve with increasing orbital period and descent to binaries with helium-core white dwarf companions.

For stars on the RGB, the growth of the helium core is directly related to the luminosity which is generated entirely by hydrogen-shell burning. Additionally, during this phase the temperature remains nearly constant and therefore the luminosity is also proportional to the stellar radius ($L = 4\pi\sigma T^4 R^2$) which is equal to the Roche lobe radius. Consequently, the mass of the core is correlated with the orbital period and therefore the *final* mass of the white dwarf is also a function of the *final* orbital period. This theoretical mass-orbital period relation has been studied extensively in the literature (Pylyser and Savonije 1989; Tauris and Savonije 1999, e.g) and seems to follow fairly well the observational data (Fig. 2.2).

An additional relation that can be verified observationally is a positive correlation between the orbital period and the eccentricity arising from tidal perturbations due to the convective envelope of the donor that prohibit perfect circularization (Phinney 1992).

Case C Roche-Lobe Overflow In this case the initial separation is wide and mass transfer initiates when the star fuses its helium layer to carbon. These systems descent to binary pulsars with typical $\sim$0.4 $\mathrm{M}_{\odot}$ white dwarf companions.

2.4 Low-Mass He-Core White Dwarf Companions

Low-mass white dwarf companions accompanying millisecond pulsars are, in principle, very simple objects: they consist of a degenerate helium core, surrounded by a residual hydrogen envelope of size inversely proportional to the core mass (see Chap. 5). Unlike regular white dwarfs, the main source of energy is not the latent heat of the core but residual hydrogen burning in the envelope. This allows them

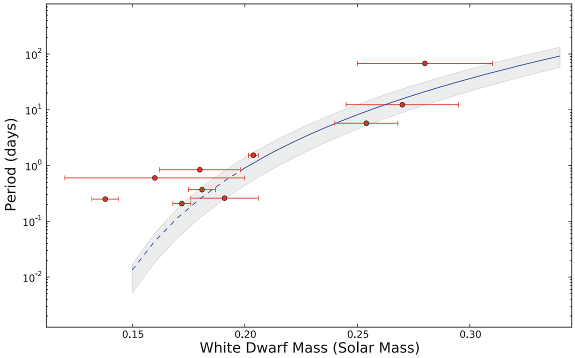

Fig. 2.2 White dwarf mass–orbital period relation based on the detailed binary evolution calculations of Tauris and Savonije (1999). The *shaded area* reflects the underlying modeling uncertainties due to the metallicity of the progenitor, which affects the cessation of mass-transfer through regulation of the magnetic-braking and size of the convective zone. The *dotted line* is an extrapolation of the relation to values below the bifurcation period where, in principle, the correlation does not hold. The *points* depict all currently known systems with determined masses. The plotted periods are inferred from the current orbital-periods and cooling ages of the white-dwarf companions, assuming that the orbit after the Roche-lobe decoupling phase was only affected by gravitational-wave damping

to stay hot and therefore be observable for timescales of several Gyr (van Kerkwijk et al. 2005).

The first low-mass white dwarf detected in the optical was the companion of PSR B0655+66 (Kulkarni 1986). It was immediately recognized that its cooling age can be used as an independent clock that can probe the magnetic field decay of the pulsar. Furthermore as mentioned above, sufficiently hot white dwarfs could be used as tools to infer pulsar masses independently of any post-Keplerian parameters.

In practice, both these uses are somewhat complicated by several issues: First, the calibration of white dwarf cooling ages requires detailed modelling of diffusive and convective processes in their interiors as well as treatment of non-gray effects on their atmospheres (van Kerkwijk et al. 2005). Furthermore, gravitational settling of CNO nuclei towards the core results in runaway hydrogen burning that consumes a large fraction of the envelope and ceases the stable nuclear fusion. It is now established that hydrogen flashes are significant only for white dwarfs more massive than a certain threshold. The latter however depends on a large number of parameters and remains poorly constrained. In the next Chapter we discuss this in more detail.

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http://www.springer.com/978-3-319-09896-8

Multi-Wavelength Studies of Pulsars and Their Companions
Antoniadis, J.
2015, XXI, 83 p. 23 illus., 18 illus. in color., Hardcover
ISBN: 978-3-319-09896-8