

# Preface



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This volume is a tribute by several generations of proof theorists to Gerhard Gentzen, one of the greatest logicians ever to whom we owe the most profound investigation of the nature of proofs since Aristotle and Frege. The immediate stimulus for its inception was Gentzen's 100th birthday in 2009 which was celebrated with a conference in Leeds and a workshop in Coimbra at which most of the contributors to this volume spoke.

Gentzen has been described as logic's lost genius<sup>1</sup> whom Gödel sometimes called a better logician than himself.<sup>2</sup> It could be said that Gentzen and Gödel arrived, each in their own exquisite manner, at opposing extremes of a spectrum. Gödel found a very general negative result to the effect that no system embodying a correct

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<sup>1</sup>E. Menzler-Trott: *Logic's Lost Genius: The Life of Gerhard Gentzen* (AMS, Providence, 2007).

<sup>2</sup>G. Kreisel: Gödel's excursions into intuitionistic logic, in: *Gödel remembered*, (Bibliopolis, Napoli, 1987) p. 169.

amount of number theory can prove its own consistency by transferring the trick of the “Liar’s Paradox” from the context of truth to that of provability. Gentzen, on the other hand, established the positive result that elementary number theory is consistent, using at some crucial point the well-orderedness of a certain ordering called  $\varepsilon_0$  that sprang from Cantor’s normal form (for presenting ordinals). He also gave a direct proof that the latter principle is not deducible in this theory, thereby providing an entirely new proof of a mathematical incompleteness in number theory.

Gentzen can be rightly considered to be the founding father of modern proof theory. His sequent calculus and natural deduction system beautifully explain the deep symmetries of logic. They underlie modern developments in computer science such as automated theorem proving and type theory. This volume’s chapters by leading proof-theorists attest to Gentzen’s enduring legacy in mathematical logic and beyond. Their contributions range from philosophical reflections and re-evaluations of Gentzen’s original consistency proofs and results in proof theory to some of the most recent developments in this exciting area of modern mathematical logic.

**Acknowledgements** The plan for the present book evolved at the *Leeds Symposium on Proof Theory and Constructivism*<sup>3</sup> in 2009 which lasted from 4th July to 15th July. The symposium consisted of three connected events one of which was the *Gentzen Centenary Conference*. We would like to thank the Kurt Gödel Society and the Deutsche Vereinigung für Mathematische Logik und für Grundlagenforschung der Exakten Wissenschaften (DVMLG) for providing funding for the Gentzen Centenary Conference. This conference was followed by a Workshop associated with the Annual Meeting of the European Association of Computer Science Logic, *CSL 2009*, with the title *Gentzen Centenary—The Quest for Consistency*<sup>4</sup> which took place on 12th September in Coimbra, Portugal. This workshop also received support from the Kurt Gödel Society for which we are very grateful, as well as for the support of the Portuguese Science Foundation FCT, which funded the Workshop and the edition of this volume through the projects, *Dialogical Foundation of Semantics* (LOGICCC/0001/2007), within the ESF programme LogICCC, *Hilbert’s Legacy in the Philosophy of Mathematics* (PTDC/FIL-FCI/109991/2009), and *The Notion of Mathematical Proof* (PTDC/MHC-FIL/5363/2012).

The editors are especially indebted to Matthias Baaz for his enthusiastic support of both events, for stimulating the book project and for contributing towards giving the present volume its shape by providing information, encouragement, culinary highlights and counsel. Finally, let us thank the referees of the papers collected in this volume for their valuable help.

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<sup>3</sup><http://www.personal.leeds.ac.uk/~matptw/>.

<sup>4</sup><http://www.mat.uc.pt/~kahle/gentzen/>.

Gentzen's Centenary

The Quest for Consistency

Kahle, R.; Rathjen, M. (Eds.)

2015, X, 561 p. 23 illus., Softcover

ISBN: 978-3-319-10102-6