Preface

These two volumes gather together the tributes of a distinguished group of colleagues and friends in honor of Professor Jean-Yves Beziau on his fiftieth birthday.

The articles in each of the two volumes (of which this is the first) fall, broadly speaking, into four categories:

1. those concerned with universal logic,
2. those concerned with hexagonal and other geometrical diagrams of opposition,
3. those concerned with paraconsistency, and
4. current work not directly connected to the work of Jean-Yves Beziau.

With these contributed papers, we want to record our gratitude for the intellectual and organizational work of Jean-Yves in uncovering a golden tradition of logical thought, and his constant encouragement to all of us to insure that tradition will continue and flourish. Many thanks, Jean-Yves. Our heartfelt thanks on this your fiftieth birthday.

With the possible exception of the last category, there are three subdivisions of universal logic as conceived by Jean-Yves Beziau. In order to understand this project, we can do no better than to recall the way in which universal logic was compactly described by Beziau in the preface to what is probably the defining collection on the subject,[1] and to expand upon it, briefly:

(i) [**Beyond particular Logical Systems**] "Universal logic is a general study of logical structures. The idea is to go beyond particular logical systems to clarify fundamental concepts of logic and to construct general proofs." (p. v)

(ii) [**Comparison of Logics**] "Comparison of logics is a central feature of universal logic." (p. v)

(iii) [**Abstraction and the central notion of Consequence**] "But the abstraction rise is not necessarily progressive, there are also some radical jumps into abstraction. In logic we find such jumps in the work of Paul Hertz on *Satzsysteme* (Part 1), and of Alfred Tarski on the notion of a *consequence operator* (Part 3). What is primary in these theories are not the notions of logical operators or logical constants (connectives and quantifiers), but a more fundamental notion: a relation of consequence defined on undetermined abstract objects that can be propositions of any science, but also data, acts, events." (p. vi)

[1] Beziau [2].

(iv) [**Beyond Syntax and Semantics**] "In universal logic, consequence is the central concept. But this consequence relation is neither syntactical (proof-theoretical), nor semantical (model-theoretical). We are beyond the dichotomy syntax/semantics (proof theory/model theory." (p. vi)

There are of course other themes that are characteristic of Universal Logic, but it seems evident to me that the first observation—(i) [**Beyond particular Logical Systems**]—indicates clearly that universal logic does not advocate a unique logical system that is the one correct, most expressive, accurate, and useful logical structure. Universal logic includes in its domain a host of logical structures in all their variety. But universal logic is not simply a catalogue of all advocated or imagined logical structures, all logical possibilities, as it would have all the utility of a telephone book that is useful for certain problems, but cognitively dumb.

It is the second observation—(ii) [**Comparison of Logics**]—which adds intellectual content to the project. Comparison is indeed central to universal logic, but not comparisons of a vapid kind. What is intended are comparisons that not only note the difference between logical structures, but explanations of why there are those differences in a way that reveal their different logical character. That is, the second observation suggests that not only are comparisons offered, but that there may be many different ways of ordering those logics, and one cannot take for granted that those orderings or comparisons are coherent when taken together. This kind of issue is nicely illustrated when we think of a paper now commonly referred to as "Beziau's translation paradox".[2] Simply put, two logical systems K (classical propositional logic), and $K/2$ are described. Two orderings or relations are proved to hold: that K is an extension of $K/2$ and also that there is a faithful translation of K into $K/2$. So there are two orderings. The first seems to indicate that K is clearly the stronger logic, yet the second result seems to say otherwise (that there is within $K/2$ a faithful translation of classical propositional logic). Each of the two orderings seems to measure the strength of one logic over another. According then to Beziau's concept of universal logic, comparisons are a central task, but it is also a task of universal logic to figure out what to do when the orderings seem to go in different directions.

Beziau has suggested that it is like the so-called Galilean "paradox", which notes that there are more square natural numbers than there are natural numbers, and also notes that those two collections are evenly matched. It is not that Galileo's solution is recommended for the Beziau example. That is not a possible way out, since Galileo thought that, in the case of infinite collections, the notion of "is larger than" just doesn't apply. The intended similarity, as we see it, is that in both cases there are two ways of explaining the notion of one collection having more members than another, and one logic being more powerful than another. The two ways give opposing verdicts, and the resolution of this situation, Beziau maintains, is a task that lies squarely within the province of universal logic.

We mentioned that the study of Hexagonal logics of opposition falls squarely within the province of universal logic, for they provide a good example of finite logical systems, with a specified particular implication relation between their sentences (taken pairwise). In fact there is a growing literature which considers consequence relations on finite geometrical arrays of different dimension. All belong comfortably within the project that is universal logic.

[2]Beziau [1].

We also mentioned that paraconsistent logics are included in the program. That should be obvious if one considers the various consequence relations to be found in that branch of logic. Also we need to mention the beautiful studies of Dov Gabbay in which he proposed the study of *restrictive access logics* as an alternative to paraconsistent logics that is an extension of classical logic.[3]

These restrictive access logics can be described by using a substructural consequence relation, where there is a modification of the Gentzen structural conditions on implication. It then becomes an interesting problem to see what features the logical operators have will have as a consequence.[4] The examples of paraconsistent and restrictive logics lie well within the province of present day logic.

In contrast, what is interesting and novel is that Beziau's observation's in (iv) [**Beyond Syntax and Semantics**] permits the extension of the program beyond the more traditional range of contemporary logical systems. As he stated it, not only can we have the notion of consequence for scientific propositions, and non-propositional, non-sentential objects including, data, acts, and events, but we do now add pictures (perhaps mathematical diagrams), and even the epistemic notion of states of belief for which consequence relations exist, and the possibility of logical operators acting on pictures as well as states of belief. We are concerned with consequence relations that are beyond the semantical or proof-theoretical.

The case for a consequence relation between pictures has recently been forcefully made by Jan Westerhoff. Here, compactly, is the claim:

"I will describe an implication relation between pictures. It is then possible to give precise definitions of conjunctions, disjunctions, negations, etc. of pictures. It will turn out that these logical operations are closely related to, or even identical with basic cognitive relations we naturally employ when thinking about pictures."[5]

This example with its particular consequence relation, and the pictures it relates, is an extension well beyond the usual restriction of logic to syntax and semantics. It illustrates the broad implications of Beziau's observations in (iv) and the fertility of the project of universal logic. It is not business as usual.

Finally we will briefly describe another case due to Peter Gärdenfors,[6] who developed a logic of propositions upon the basis of a theory about belief revision. His results can be recast in such a way that they also follow as a case where he defines propositions as special kinds of functions, and also defines a special relation among those functions that turns out to be a consequence relation. The result is fascinating: the conjunction of functions turns out to be the functional composition of functions, and Gärdenfors' special relation among the functions is a consequence relation provided that functional composition is commutative and idempotent.

More exactly, (1) let S be a set of states of belief of some person. (2) Let P be a set of functions from S to S (called propositions) which is closed under functional composition. (3) For any members $f_1, f_2, \ldots, f_n$ and g in P, let (G) be the condition

[3] D.M. Gabbay and A. Hunter [4].

[4] Private communication from D. Gabbay, 2005.

[5] Westerhoff, J. [6]. The implication relation proposed for pictures is similar to one that Corcoran [3] proposed for propositions, as noted by Westerhoff.

[6] Gärdenfors [5].

that

$$f_1, f_2, \ldots, f_n \Rightarrow g \quad \text{if and only if} \quad g f_1 f_2 \ldots f_n = f_1 f_2 \ldots f_n$$

(the concatenation of two functions here indicates their functional composition).

In particular, for any two propositions (functions) f and g, f implies g ($f \Rightarrow g$) if and only if $gf = f$. It is easy to prove that the relation (G) is a consequence condition if and only if functional composition is commutative and idempotent. The logic of these propositions has been shown by Gärdenfors to be Intuitionistic, and his consequence relation (G) is clearly epistemic. Again, it is not logic as usual, but it is just one more case of the fruitfulness of the ideas that the project of universal logic embodies.

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