Preface

The mathematical theory of counterpoint developed by Guerino Mazzola around 1984 was originally aimed at simulating the composition rules described in Johann Joseph Fux's *Gradus ad Parnassum* [29]. It soon came out that the algebraic apparatus used in this model could also serve to define entirely new systems of rules for composition. Those are generated by new choices of consonances and dissonances, which in turn lead to new restrictions governing the succession of intervals.

Discovering new systems of counterpoint in the usual twelve-note chromatic gamut was a first demonstration of the model's potential for variation. In this book we propose to pursue this exploration of exotic counterpoint worlds, along two axes. First, a *longitudinal* one where we vary the division of the octave and show how macro- and microtonal consonances are embedded into a sequence, up to the pitch continuum. In a second, more *cross-sectional* perspective, we will see how counterpoint structures can be mapped across different systems of the same or different temperaments. This work originated in two PhD dissertations, which were merged to summarize these recent developments in the field of mathematical counterpoint theory: *Extensiones microtonales de contrapunto* was submitted to the Universidad Nacional Autonóma de México in 2011 by Octavio Alberto Agustín Aquino, and *Counterpoint Worlds and Morphisms* to the Universität Zürich in 2010 by Julien Junod. This book is also the first comprehensive exposition of this counterpoint model, bringing together many aspects of the subject previously published in different places, in particular *Geometrie der Töne* [55] and the *Topos of Music* [57]. Guerino Mazzola, the advisor for both theses, joined Octavio and Julien in this project with his recent reformulation of the theory in terms of gestures.

The counterpoint described by the model is the first-species case: two voices playing simultaneously. While handling only the most basic case, we believe that it already contains all the fundamental principles of counterpoint and that higher species only provide a variation of the basic rules. Simplicity has the pedagogical advantage of showing the essential structure of the model without hiding it behind complex configurations.

The explicit nature of counterpoint rules has always attracted computer scientists, and it is very tempting to use them as constraints in algorithmic composition. The

first example of a computer-generated composition, Lejaren Hiller's *Illiac Suite*, dates back to the 1950s.[1] It is not our aim to simulate faithfully vocal polyphonies of the Renaissance. The counterpoint model raises a far-deeper-reaching question: counterpoint has a centuries-long history and, despite its limitations, has proved to be very successful in Western music history. Now, our model tells us that this system is only one instance among many others of a more general structure. So what is the musical value of the other systems? Is the traditional Fuxian system really the best one? Do other worlds as effective for composition exist? And how would such an exotic counterpoint sound? It is our desire that this theory opens up new territories for musical composition, not by starting everything from scratch, but by exporting proven principles.

Chapter 1 provides some justifications for our choice to diverge from standard counterpoint theory. Chapter 2 briefly describes how the model is constructed for any equally tempered scale with an even number of tones, and the third chapter specializes the results to the twelve-tone scale, and a series of further peculiarities that arise in this setup. Chapter 4 introduces the notion of counterpoint world, plus the required graph theory and notation setting the stage for the fifth chapter, which shows under which conditions and how the contrapuntal structure can be morphed from one world to another.

The sixth chapter is essentially independent from the rest, and constitutes a user's guide to the Bolly rubettes, which allow musicians to perform experiments composing traditional and exotic counterpoint and morphing it between worlds.

The following two chapters study with more detail the combinatorial aspects of the model and how consonances can be progressively embedded between tunings.

The last pair of chapters introduce the reader to categorical gesture theory and its consequences for counterpoint, an exciting and novel development that, together with the possibility of infinite sets of consonances and dissonances, announce the notion of continuous counterpoint.

A good command of algebra is necessary to understand the construction of the model, which uses cyclic groups and tangent spaces to describe interval classes. Graph theory plays a role in describing how counterpoint worlds can be morphed into one another and in the proofs of some theorems. While the reader may need some mathematical training to follow the theoretical developments, a layman should be able to apply it for composing music, i.e., using software tools to compose and play with the rules in different systems.

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[1] Premiered on August 9th, 1956, the score of these *experiments* was published in 1957 [39]. The algorithmic process was first described in [40] and more thoroughly explained in [41], which also contains a reproduction of the original score in its appendix on page 181.