Model of Hybrid Active Power Filter in the Frequency Domain

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Abstract A model of the hybrid active power filter in the frequency domain has been presented in the paper. Control algorithm for detecting current harmonics uses method with Park transformation in this case. The derivation of the frequency model for this method and its application to the analyzed hybrid active power filter have been shown. The presented model allows the assessment of the impact of system parameters on the filtration characteristics of the analyzed hybrid filter.

1 Introduction

Application of an active power filter [1–4] is one of methods for reducing harmonics in three-phase power lines. Regardless of the type and configuration of the system, it is necessary to determine currents or voltages harmonics based on their measurements. These can be selected harmonics, fundamental frequency components or all of the higher harmonics. There are many methods of determining harmonics [2–5], from which the most popular are methods in the frequency domain based on the Fourier transform i.e. DFT, FFT, and their variations. Another popular group of methods are time-domain methods, which can include the analyzed method with Park transformation or a method based on p-q instantaneous power theory [1].

In contrast to the frequency methods, time-domain methods are characterized by their lower computational complexity. This allows implementation of the control algorithm in a real system without the need to use controllers with high computational power.

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In some cases, information about control method in the frequency domain is needed. For example in hybrid active power filters where passive harmonic filters are used and the filtration characteristics of whole hybrid filter should be determined. This helps in the selection of system parameters and assessing their impact on the frequency characteristics [6–9].

The paper shows the derivation of the frequency model for harmonic detection method using Park transformation [10] and an example for use of the derived model for the analysis of hybrid power filter control algorithm in the configuration with a single harmonic passive filter [6, 11].

2 Park Transformation

The transformation from the three-phase 1-2-3 system to the d-q-0 system, also known as Park transformation, was presented for the first time by R. H. Park in 1929 [10]. This conversion is very often used in the theory of electrical machines and allows to treat quantities related to the rotor as constant in time. It is based on the transformation of the natural 1-2-3 orthogonal system to the d-q-0 system rotating with angular frequency ω_0. Park transformation for currents can be written as:

$$\begin{bmatrix} i_{\mathrm{d}} \\ i_{\mathrm{q}} \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\omega_0 t) & \cos(\omega_0 t - 2\pi/3) & \cos(\omega_0 t + 2\pi/3) \\ \sin(\omega_0 t) & \sin(\omega_0 t - 2\pi/3) & \sin(\omega_0 t + 2\pi/3) \\ \sqrt{2}/2 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}. \tag{1}$$

This form is rarely used, and the most commonly used is form with intermediate orthogonal transformation to the α-β system (Clarke transformation) which, for three-wire network is:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}, \tag{2}$$

$$\begin{bmatrix} i_{\mathrm{d}} \\ i_{\mathrm{q}} \end{bmatrix} = \begin{bmatrix} \cos(\omega_0 t) & \sin(\omega_0 t) \\ -\sin(\omega_0 t) & \cos(\omega_0 t) \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}. \tag{3}$$

The transformation of the 1-2-3 natural system to d-q system, rotating with angular frequency ω_0 system causes that variables of the synchronous angular frequency ω_0 are constant over time. This allows the detection of the selected harmonic.

To filter the fundamental frequency components of the symmetrical three-phase currents, DC components from d-q currents should be removed as it has been shown in the block diagram in Fig. 1. Result of the shown transformations are higher harmonics of currents ($i_{1,2,3(\mathrm{hh})}$). In a similar way one can achieve detection of selected harmonic components and their positive and negative components in the case of an unsymmetrical system.

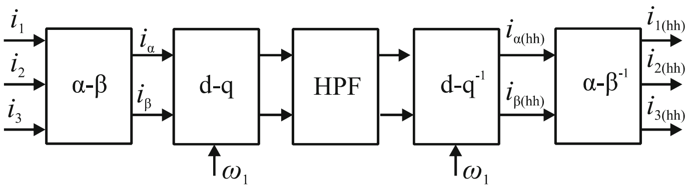


Fig. 1 Block diagram of the algorithm of detecting the higher harmonic

3 Model in Frequency Domain

The transformation of algorithm shown in Fig. 1 to the frequency domain has been performed in the α-β system (Fig. 2). High-pass filter (HPF) at the diagram in Fig. 1 (the same for both components) in this case is represented by the impulse response function $k(t)$, while the searched transfer function is designated as $G(\mathrm{j}\omega)$.

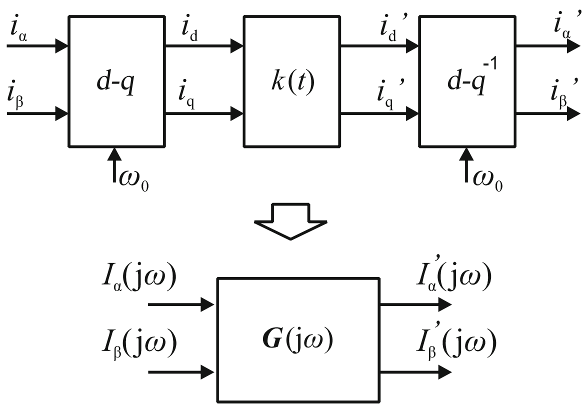
The relationship between input and output values in d-q rotating coordinates can be written as:

$$\begin{bmatrix} i'_\mathrm{d} \\ i'_\mathrm{q} \end{bmatrix} = \int\limits_{-\infty}^{\infty} \left(k(t-\tau) \begin{bmatrix} i_\mathrm{d} \\ i_\mathrm{q} \end{bmatrix} \right) \mathrm{d}\tau, \tag{4}$$

which after applying the Fourier transform to both sides of the equation, and next using theorem of convolution in time domain, gives:

$$\mathcal{F}\left\{ \begin{bmatrix} i_\mathrm{d}{}' \\ i_\mathrm{q}{}' \end{bmatrix} \right\} = K(\mathrm{j}\omega)\,\mathcal{F}\left\{ \begin{bmatrix} i_\mathrm{d} \\ i_\mathrm{q} \end{bmatrix} \right\}, \tag{5}$$

Fig. 2 Model in frequency domain



where: $K(\mathrm{j}\omega) = N(\mathrm{j}\omega)/D(\mathrm{j}\omega)$—frequency transfer function of the applied signal filters (with the same parameters). $N(\mathrm{j}\omega)$ and $D(\mathrm{j}\omega)$ are polynomials in this case, the degree of the numerator is less than or equal to the degree of the denominator.

After applying the transformation to the α-β system, Eq. (5) takes the form:

$$\begin{aligned} D(\mathrm{j}\omega)\mathcal{F}\left\{\begin{bmatrix}\cos(\omega_0 t) & \sin(\omega_0 t)\\ -\sin(\omega_0 t) & \cos(\omega_0 t)\end{bmatrix}\begin{bmatrix}i_\alpha{}'\\ i_\beta{}'\end{bmatrix}\right\} = \\ = N(\mathrm{j}\omega)\mathcal{F}\left\{\begin{bmatrix}\cos(\omega_0 t) & \sin(\omega_0 t)\\ -\sin(\omega_0 t) & \cos(\omega_0 t)\end{bmatrix}\begin{bmatrix}i_\alpha\\ i_\beta\end{bmatrix}\right\}. \end{aligned} \tag{6}$$

Then using the relationship [6, 8, 9]:

$$\begin{aligned} &\frac{\mathrm{d}^n}{\mathrm{d}t^n}\left(\begin{bmatrix}\cos(\omega_0 t) & \sin(\omega_0 t)\\ -\sin(\omega_0 t) & \cos(\omega_0 t)\end{bmatrix}\begin{bmatrix}i'_\alpha\\ i'_\beta\end{bmatrix}\right) \\ &= \begin{bmatrix}\cos(\omega_0 t) & \sin(\omega_0 t)\\ -\sin(\omega_0 t) & \cos(\omega_0 t)\end{bmatrix}\left(\begin{bmatrix}\frac{\mathrm{d}}{\mathrm{d}t} & \omega_0\\ -\omega_0 & \frac{\mathrm{d}}{\mathrm{d}t}\end{bmatrix}^n\begin{bmatrix}i'_\alpha\\ i'_\beta\end{bmatrix}\right), \end{aligned} \tag{7}$$

and the derivative transform theorem, while bearing in mind that $N(\mathrm{j}\omega)$ and $D(\mathrm{j}\omega)$ are polynomials, Eq. (6) can be converted to the form:

$$\boldsymbol{D}(\boldsymbol{\lambda})\Bigg|_{\lambda=\begin{bmatrix}\mathrm{j}\omega & \omega_0\\ -\omega_0 & \mathrm{j}\omega\end{bmatrix}}\mathcal{F}\left\{\begin{bmatrix}i_\alpha{}'\\ i_\beta{}'\end{bmatrix}\right\} = \boldsymbol{N}(\boldsymbol{\lambda})\Bigg|_{\lambda=\begin{bmatrix}\mathrm{j}\omega & \omega_0\\ -\omega_0 & \mathrm{j}\omega\end{bmatrix}}\mathcal{F}\left\{\begin{bmatrix}i_\alpha\\ i_\beta\end{bmatrix}\right\}. \tag{8}$$

The searched transfer function can be so written as:

$$\mathbf{G}(\mathrm{j}\omega) = \mathbf{N}(\boldsymbol{\lambda})^{-1}\mathbf{D}(\boldsymbol{\lambda}), \boldsymbol{\lambda} = \begin{bmatrix}\mathrm{j}\omega & \omega_0\\ -\omega_0 & \mathrm{j}\omega\end{bmatrix}. \tag{9}$$

In this form it is not possible to obtain independent characteristics for α and β components. However, in case of the filters described in the form of transfer function of a rational function, the transfer function $G(\mathrm{j}\omega)$ can be expressed as [8, 9]:

$$\mathbf{G}(\mathrm{j}\omega) = \begin{bmatrix}a(\mathrm{j}\omega) & b(\mathrm{j}\omega)\\ -b(\mathrm{j}\omega) & a(\mathrm{j}\omega)\end{bmatrix}, \tag{10}$$

which after transformation into symmetrical components:

$$\begin{bmatrix}I'^{+}(\mathrm{j}\omega)\\ I'^{-}(\mathrm{j}\omega)\end{bmatrix} = \begin{bmatrix}a(\mathrm{j}\omega) - \mathrm{j}b(\mathrm{j}\omega) & 0\\ 0 & a(\mathrm{j}\omega) + \mathrm{j}b(\mathrm{j}\omega)\end{bmatrix}\begin{bmatrix}I^{+}(\mathrm{j}\omega)\\ I^{-}(\mathrm{j}\omega)\end{bmatrix}, \tag{11}$$

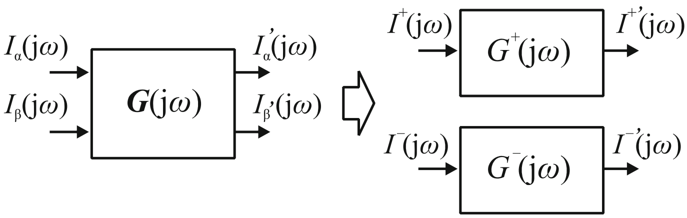


Fig. 3 Transforming model to the symmetrical components

gives independent transfer functions for positive and negative sequences:

$$\begin{aligned}\mathbf{G}^{+,-}(j\omega) &= \begin{bmatrix} G^+(j\omega) & 0 \\ 0 & G^-(j\omega) \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix}\mathbf{G}(j\omega)\begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}.\end{aligned} \tag{12}$$

In this way, two separate transfer functions describing the analyzed system in the frequency domain (Fig. 3) have been obtained.

For example, using high-pass first order signal filters with ω_c cutoff angular frequency described by transfer function:

$$K_{\mathrm{HPF}}(j\omega) = \frac{j\omega}{j\omega + \omega_c}, \tag{13}$$

we obtain the following transfer functions:

$$G^+(j\omega) = \frac{j\omega - j\omega_0}{j\omega - j\omega_0 + \omega_c} = K_{\mathrm{HPF}}(j\omega - j\omega_0), \tag{14}$$

$$G^-(j\omega) = \frac{j\omega + j\omega_0}{j\omega + j\omega_0 + \omega_c} = K_{\mathrm{HPF}}(j\omega + j\omega_0). \tag{15}$$

As an effect of components filtration which are the result of Park transformation (and inverse transform), we obtain a shift in the frequency of the used signal filters characteristics. In case of positive components, the shift occurs in the positive direction by the value of ω_0 angular frequency. For negative components, shift takes place in the negative direction. Sample frequency responses of resultant transfer functions $G_1^+(j\omega)$ for first and second order low pass filters have been shown in Fig. 4.

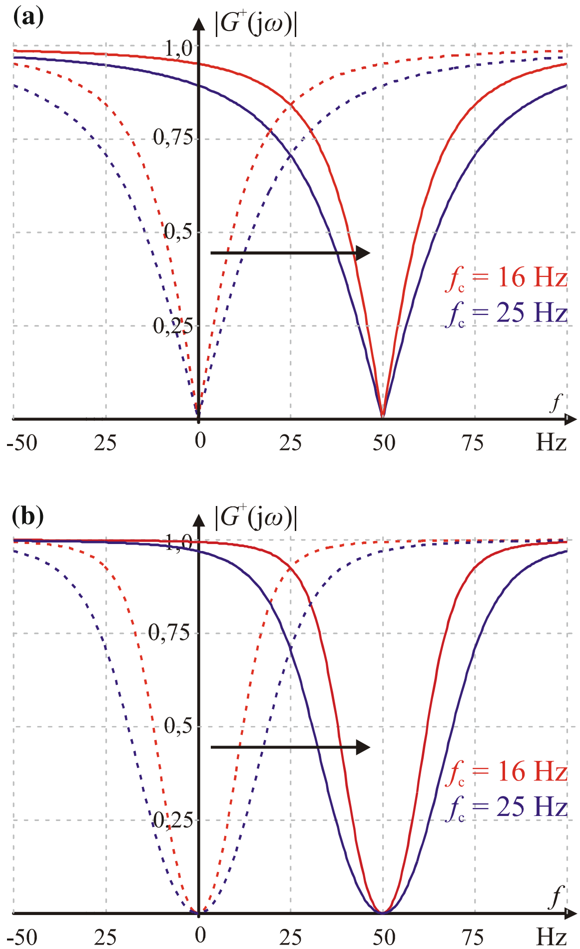


Fig. 4 Resultant transfer functions frequency responses $G_1^+(\mathrm{j}\omega)$ for filters: **a** first order **b** Butterworth second order

4 Application of the Frequency Model for Hybrid Power Filter Analysis

Hybrid power filters combine active power filter with traditional passive filters. This connection allows to take advantages from both solutions, usually by improving the properties of the filter in relation to the passive filters and by reducing the required power rate of the active part [1, 4, 6, 8]. Depending on the connection method and applied systems, hybrid filters are used in order to reduce currents and voltages harmonics and to compensate reactive power. One of the possible configurations of hybrid system is a series connection of parallel passive filter with a shunt active power filter (Fig. 5). The configuration with passive part reduced to the 7th harmonic filter has been used in this case [6, 8]. Diagram of the control algorithm for

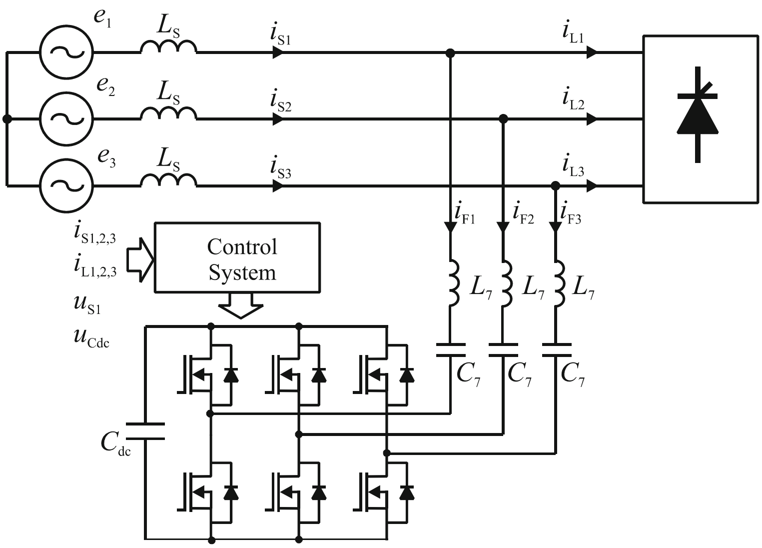


Fig. 5 Hybrid power filter with single tuned passive filter

this solution is shown in Fig. 6. In order to determine high harmonics of source currents the discussed method with d-q synchronous reference frame has been used. The upper branch of the shown control algorithm is responsible for determining higher harmonics of the source currents, as in the case of usual control of hybrid filter [1, 9]. Lower branch is responsible for determining voltage at the output of the filter which is necessary to reduce the selected harmonic current (5th and 11th in this case). In order to reduce other harmonics (e.g. 13th, 17th) additional similar branches should be connected to the lower branch of the algorithm. In the paper a simplified version of the algorithm without the part responsible for regulating the u_{DC} has been presented. The obtained voltages $v_{1,2,3(\mathrm{ref})}$ give rise for determining control signals of the of hybrid filter transistors.

Based on the schema of the analyzed system (Fig. 5), and the diagram of its control (Fig. 6) it is possible to create a frequency model according to the previous derivations. This model has been shown in Fig. 7.

However, the relationship between source currents and load currents can be described by the equation:

$$\begin{aligned}\begin{bmatrix} I_{\mathrm{S}\alpha}(\mathrm{j}\omega) \\ I_{\mathrm{S}\beta}(\mathrm{j}\omega) \end{bmatrix} &= \left[Z_{\mathrm{F}}(\mathrm{j}\omega)\mathbf{1}_2 + Z_{\mathrm{S}}(\mathrm{j}\omega)\mathbf{1}_2 + K\mathbf{G}_1(\mathrm{j}\omega)\right]^{-1} \\ &\quad \times \left[Z_{\mathrm{F}}(\mathrm{j}\omega)\mathbf{1}_2 - \sum_{\mathrm{h}=5,11,13} \mathbf{G}_h(\mathrm{j}\omega)\mathbf{Z}_{\mathrm{F}h}\right] \begin{bmatrix} I_{\mathrm{L}\alpha}(\mathrm{j}\omega) \\ I_{\mathrm{L}\beta}(\mathrm{j}\omega) \end{bmatrix},\end{aligned} \tag{16}$$

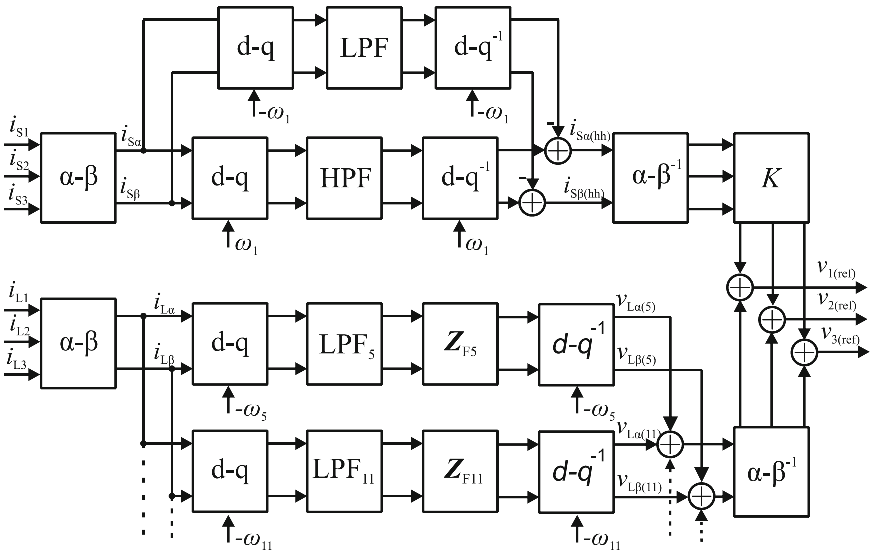


Fig. 6 Diagram of control algorithm of the analyzed hybrid power filter

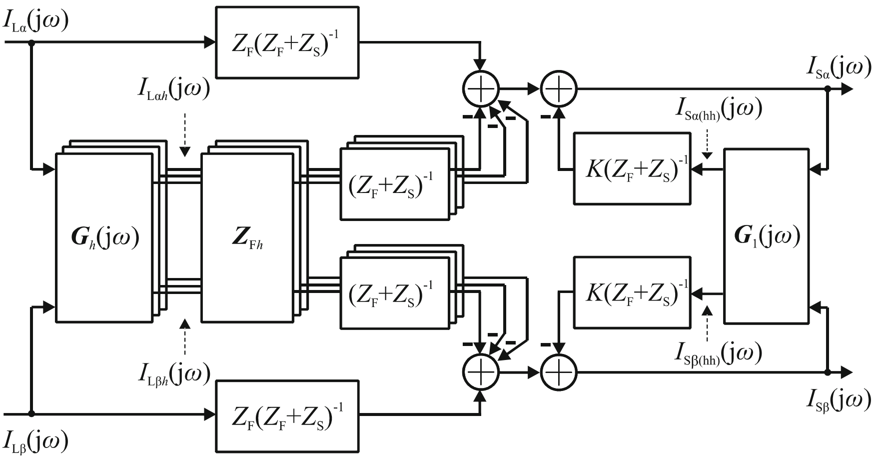


Fig. 7 Frequency model of the analyzed hybrid power filter

where: $\mathbf{1}_2$ - 2x2 identity matrix, $Z_F(j\omega)$—passive filter impedance, $Z_S(j\omega)$—source impedance, K—hybrid filter gain.

The remaining quantities in Eq. (16) are associated with the Park transformation and the used signal filters:

$$\mathbf{G}_1(\mathrm{j}\omega) = \mathbf{G}'(\mathrm{j}\omega) - \mathbf{G}''(\mathrm{j}\omega). \tag{17}$$

$\mathbf{G}'(\mathrm{j}\omega)$—matrix connected with Park transformation and high pass filter (HPF):

$$\mathbf{G}'(\mathrm{j}\omega) = \mathbf{D}_{\mathrm{HPF}}^{-1}(\boldsymbol{\lambda}')\mathbf{N}_{\mathrm{HPF}}(\boldsymbol{\lambda}'), \boldsymbol{\lambda}' = \begin{bmatrix} \mathrm{j}\omega & \omega_1 \\ -\omega_1 & \mathrm{j}\omega \end{bmatrix}, \tag{18}$$

$$K_{\mathrm{LPF}}(\mathrm{j}\omega) = \frac{N_{\mathrm{HPF}}(\mathrm{j}\omega)}{D_{\mathrm{HPF}}(\mathrm{j}\omega)}. \tag{19}$$

$\mathbf{G}''(\mathrm{j}\omega)$—matrix corresponding with Park transformation and low pass filter (LPF) (used for computation negative components of source currents):

$$\mathbf{G}''(\mathrm{j}\omega) = \mathbf{D}_{\mathrm{LPF}}^{-1}(\boldsymbol{\lambda}'')\mathbf{N}_{\mathrm{LPF}}(\boldsymbol{\lambda}''), \boldsymbol{\lambda}'' = \begin{bmatrix} \mathrm{j}\omega & -\omega_1 \\ \omega_1 & \mathrm{j}\omega \end{bmatrix} \tag{20}$$

$$K_{\mathrm{LPF}}(\mathrm{j}\omega) = \frac{N_{\mathrm{LPF}}(\mathrm{j}\omega)}{D_{\mathrm{LPF}}(\mathrm{j}\omega)}. \tag{21}$$

$\mathbf{G}_h(\mathrm{j}\omega)$—matrixes related to Park transformation and low pass filters (LPF_h):

$$\mathbf{G}_h(\mathrm{j}\omega) = \mathbf{D}_{\mathrm{LPF}_h}^{-1}(\boldsymbol{\lambda}_h)\mathbf{N}_{\mathrm{LPF}_h}(\boldsymbol{\lambda}_h), \boldsymbol{\lambda}_h = \begin{bmatrix} \mathrm{j}\omega & \omega_h \\ -\omega_h & \mathrm{j}\omega \end{bmatrix}, \tag{22}$$

$$K_{\mathrm{LPF}_h}(\mathrm{j}\omega) = \frac{N_{\mathrm{LPF}_h}(\mathrm{j}\omega)}{D_{\mathrm{LPF}_h}(\mathrm{j}\omega)}. \tag{23}$$

$\mathbf{Z}_{\mathrm{F}h}$—matrix related to passive filter impedance for h-harmonic:

$$\mathbf{Z}_{\mathrm{F}h} = \begin{bmatrix} \mathrm{Re}\{Z_{\mathrm{F}}(\mathrm{j}\omega_h)\} & -\mathrm{Im}\{Z_{\mathrm{F}}(\mathrm{j}\omega_h)\} \\ \mathrm{Im}\{Z_{\mathrm{F}}(\mathrm{j}\omega_h)\} & \mathrm{Re}\{Z_{\mathrm{F}}(\mathrm{j}\omega_h)\} \end{bmatrix} \tag{24}$$

Similarly as in the derivation (10)–(13), it is possible to transform relation (16) to symmetrical components which allows to determine independent frequency responses for positive and negative components. An example of frequency responses has been shown in Fig. 8. It was assumed that all signal filters are first order with cut-off frequency of 25 Hz.

Figure 9 shows the effect of a different cut-off frequency of signal filters on frequency response. First-order filters have been assumed in this case. Characteristics are much more selective for the filtered harmonics for signal filters with lower cut-off frequency. It allows to obtain better filtering properties of the hybrid system. Unfortunately, reducing the cut-off frequency of filters causes deterioration of the

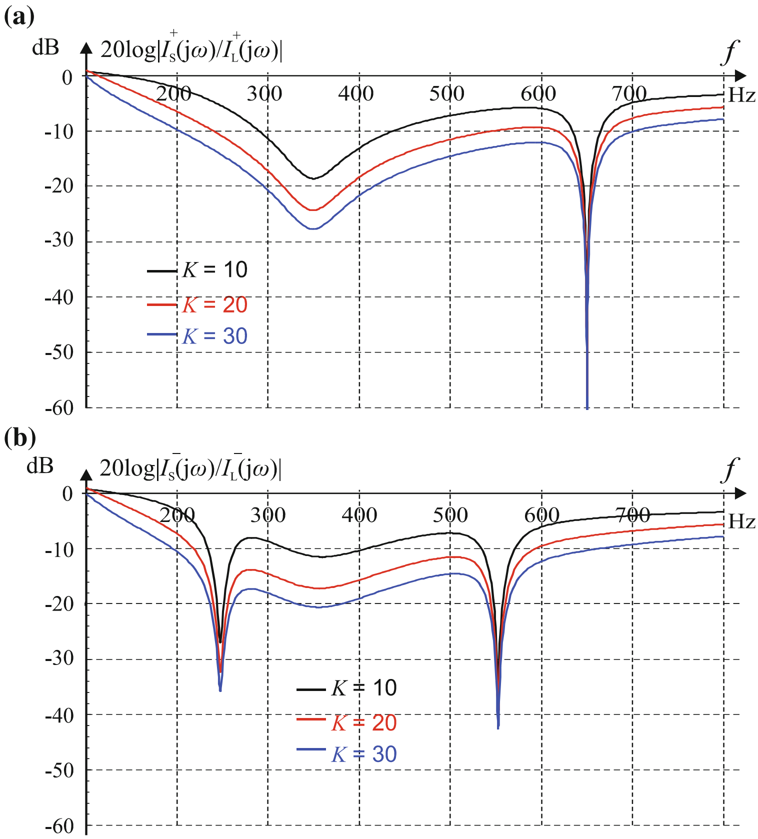


Fig. 8 Frequency responses of the analyzed hybrid filter for different value of *K* gain

system dynamic properties by extending the time transients [3, 6, 7]. Therefore it is necessary to compromise and to use in real applications signal filters with cut-off frequency in the range (10–20) Hz.

5 Summary

The paper presents a frequency model derivation of method for determining harmonics in three-phase systems. The analyzed method uses the Park transformation and is implemented in the time domain, however, the obtained model in frequency domain allows to analyze the frequency characteristics of the system in which it is used.

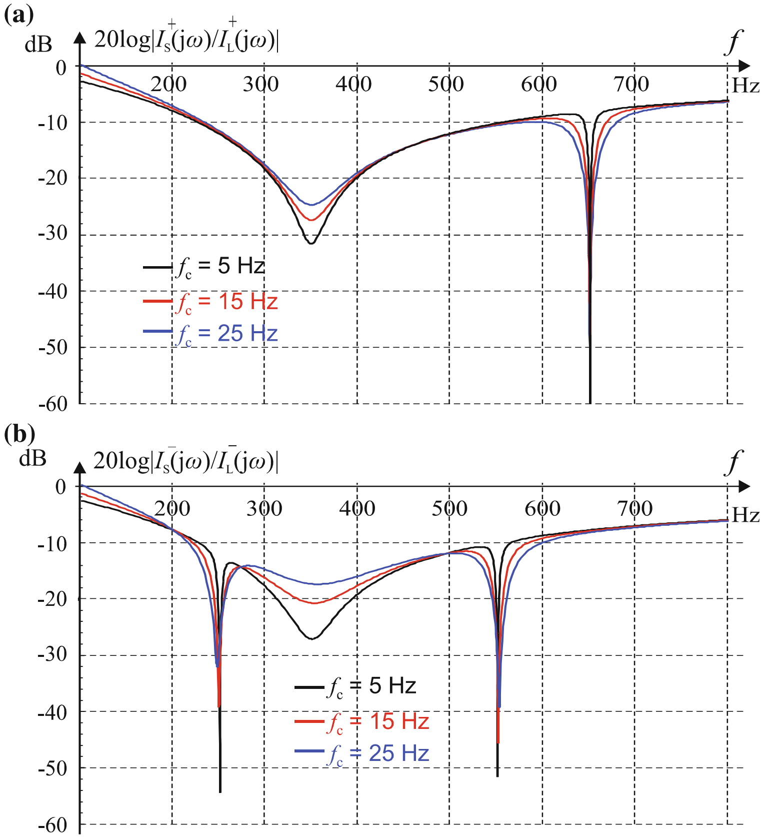


Fig. 9 Frequency responses of the analyzed hybrid filter for different cut-off frequency of signal filters (K = 20)

Hybrid active power filter in configuration with single tuned passive harmonic filter has been used as an example. Based on the derived model, it was possible to obtain a harmonic filtration characteristics of the analyzed hybrid filter, separately for positive and negative components. This allows for assessment of the system parameters impact on properties of the analyzed filter.

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