

Chapter 2

Free Space Propagation

Objectives

- Define free space propagation in time and space
- Derive free space pathloss formula
- Show that free space pathloss formula exhibits an equation of straight line
- Define ERP and RSL
- Problems
- Group exercise

2.1 Free-Space Propagation in Time and Space

Electromagnetic waves differ in energy according to their wavelength. Their ability to propagate is also different in different propagation environments. In free space (vacuum) they are characterized by their ability to propagate without obstruction and without atmospheric effects. The path loss under these conditions is said to be free space path loss.

For example, we consider an isotropic RF (Radio Frequency) source, which radiates electromagnetic energy uniformly in all directions, as shown in Fig. 2.1a in three-dimensional space. The radiating source is located at the center, which begins its emission at a given time. Maxwell's theory of Electromagnetic Radiation implies that the energy radiates uniformly in all directions, at the speed of light (3×10^8 m/s or $3.3 \mu\text{s/km}$) [1]. This may be viewed as a sphere, expanding in time and space.

Since it is difficult to represent time and space in four dimensions, we can represent this time-space relationship by means of a cross-sectional view of the energy sphere in two dimensions in space and one dimension in time [2]. This is shown in Fig. 2.1b, where time is represented in the vertical axis.

There is no signal outside the cone, since the velocity of electromagnetic wave is constant. For example, we assume that $d_1 = 1$ km, $d_2 = 2$ km and $d_3 = 3$ km, the RF signal that originates at time $t=0$, will arrive in those locations exactly after 3.3, 6.6 and 9.9 μs respectively. This implies that the propagated signal exists within

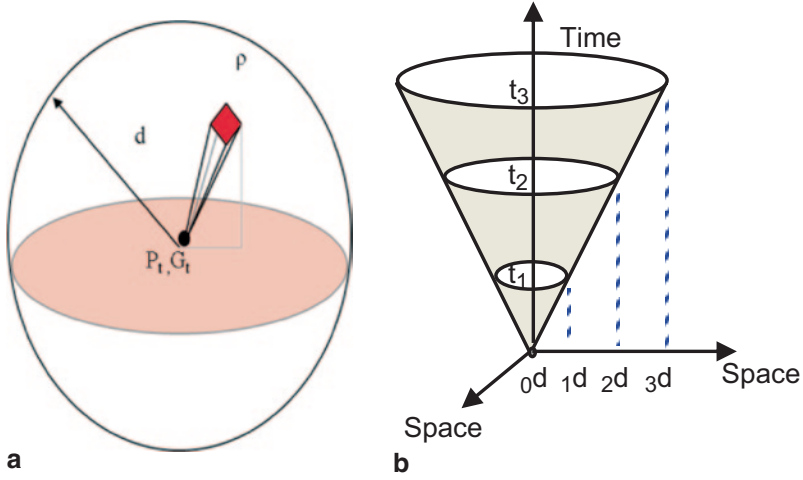


Fig. 2.1 Illustration of free Space propagation in time and space. (a) An isotropic RF (Radio Frequency) source, which radiates electromagnetic energy uniformly in all directions. (b) Representation of time-space relationship by means of a cross-sectional view of the energy sphere in two dimensions in space and one dimension in time

a space-time coordinate (d_i, t_i) where d_i is the location of the signal and t_i is the corresponding instant of time. The propagation delay is given by

$$t_p \approx 3.3 \mu s/km \quad (2.1)$$

This propagation delay is an important parameter in cellular communication systems. It determines the maximum cell size and inter symbol interference in digital cellular radios.

For example the signal that arrives at d_3 , has a propagation delay of $9.9 \mu s$. This delay determines the radius of the sphere, which has uniform signal strength throughout the surface of the sphere. In cellular communication this sphere is known as an ideal cell, as shown in Fig. 2.1a. The total energy within the cell is constant irrespective of time and space.

2.2 Derivation of Free Space Pathloss Formula

Consider the free space propagation model as shown in Fig. 2.1a to derive the well-known free space pathloss formula [3]. Assuming that the total transmit power at the source as P_t , whose gain in a particular direction is G_t , and then the radiated power density at a given distance d will be given by

$$\rho = \frac{P_t G_t}{4\pi d^2} \text{ watts / m}^2 \quad (2.2)$$

If a receive antenna is located at a distance d , whose gain is G_r and the effective area is A [2], where

$$A = G_r \frac{\lambda^2}{4\pi} \quad (2.3)$$

The received power P_r at the terminal of the receive antenna will be given by

$$P_r = \rho A = P_t G_t G_r \left(\frac{\lambda}{4\pi d} \right)^2 \quad (2.4)$$

In the above analysis, it was assumed that the transmission began at t_o and that it was received at a distance d at t . The time difference $t - t_o$ is the propagation delay (t_p), which is given by,

$$t_p = t - t_o \quad (2.5)$$

Thus, by knowing the start time, the propagation delay and hence the distance can be determined.

Now, referring to Eq. 2.4 we find that the received signal attenuates as square of the distance. The pathloss formula is given by the ratio of the received power to the transmit power, i.e.,

$$L_p = \left(\frac{P_r}{P_t G_t G_r} \right) \quad (2.6)$$

Combining Eqs. 2.4 and 2.6, we get,

$$L_p = \left(\frac{\lambda}{4\pi d} \right)^2 \quad (2.7)$$

In decibel, the free space path loss formula (L_p), can be obtained as

$$L_p(dB) = 10 \log \left[\left(\frac{4\pi d}{\lambda} \right)^2 \right] \quad (2.8)$$

or

$$L_p(dB) = 32.5 + 20 \log(f) + 20 \log(d) \quad (2.9)$$

Where $\lambda = c/f$, $c = 3 \times 10^8$ m/s, the frequency (f) is measured in MHz and the distance (d) is measured in km. Equation (2.9) is the familiar free space path loss formula.

2.3 Free Space Path Loss Formula Exhibits Equation of Straight Line [4]

Consider the free space path loss formula again as given below:

$$L_p(\text{dB}) = 32.5 + 20 \log(f) + 20 \log(d)$$

For a given frequency, $20 \log(f)$ is constant. Therefore, we can express the above equation as

$$L_p(\text{dB}) = L_o(\text{dB}) + \gamma 10 \log(d) \quad (2.10)$$

Notice that, Eq. (2.10) is similar to the equation of straight line of the form:

$$y = c + mx \quad (2.11)$$

Where,

$y = L_p$ in dB

$c = L_o(\text{dB}) = 32.5 + 20 \log(f)$ is the intercept in dB

$m = \gamma = 2$ is the slope

$x = 10 \log(d)$ is the distance in logarithmic scale

Figure 2.2 Shows the path loss characteristics for a given frequency.

From the above analysis, we see that free space propagation exhibits an equation of a straight line having a pathloss slope of 2 ($\gamma = 2$). The intercept L_o depends on the frequency. Later in this book we shall see that all propagation models can be approximated as an equation of a straight line having different pathloss slopes, depending on the propagation environment.

Problem 2.1

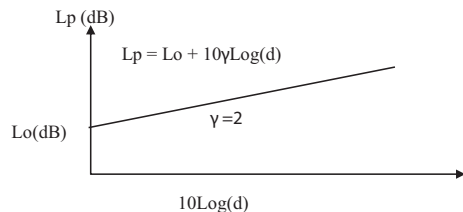
Given:

- Frequency $f = 1 \text{ GHz}$ (10^9 Hz)
- Distance $d = 10 \text{ km}$
- Free space Path loss slope $\gamma = 2$

Find:

- a. The intercept L_o in dB
- b. The free space pathloss L_p in dB

Fig. 2.2 Free space path loss characteristics in a log-log scale. The path loss slope $\gamma = 2$



Solution:

- a. Intercept: $L_o(\text{dB}) = 32.5 + 20\text{Log}(1000 \text{ MHz})$
 $= 32.5 + 60$
 $= 92.5 \text{ dB}$
- b. Path loss $L_p(\text{dB}) = L_o(\text{dB}) + \gamma 10\text{Log}(d)$
 $= 98.5 + 10 \times 2 \times 10\text{Log}(10 \text{ km})$
 $= 98.5 + 20 \times 1$
 $= 118.5 \text{ dB}$

Problem 2.2

Given:

- Frequency $f = 2 \text{ GHz}$
- Distance $d = 10 \text{ km}$
- Free space Path loss slope $\gamma = 2$

Find:

- a. The intercept L_o in dB
 b. The free space path loss L_p in dB

Solution:

- a. Intercept: $L_o(\text{dB}) = 32.5 + 20\text{Log}(2000 \text{ MHz})$
 $= 32.5 + 66$
 $= 98.5 \text{ dB}$
- c. Path loss $L_p(\text{dB}) = L_o(\text{dB}) + \gamma 10\text{Log}(d)$
 $= 98.5 + 10 \times 2 \times 10\text{Log}(10 \text{ km})$
 $= 98.5 + 20 \times 1$
 $= 118.5 \text{ dB}$

From the above two problems, we see that, for a given distance, the intercept and pathloss increase by 6 dB when the frequency doubles.

2.4 ERP and RSL

The Effective Radiated Power (ERP) and the Received Signal Level (RSL) are two design parameters used in cellular communications. ERP is the power radiated from the tip of the antenna and RSL is the power received at the receiver. The receiver is located at a distance d from the transmitter. We examine this by means of Fig. 2.3:

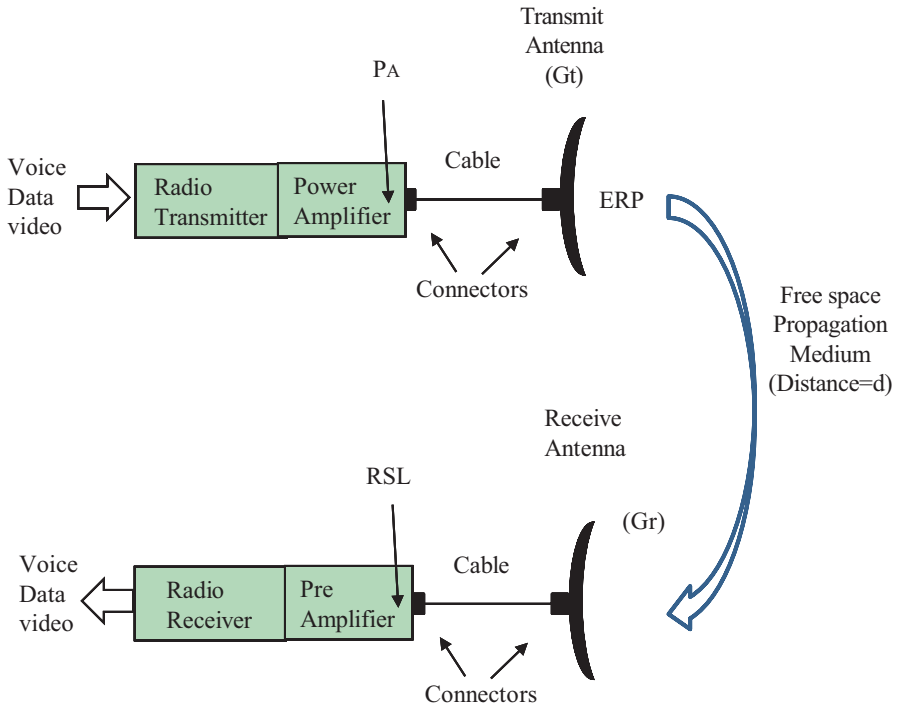


Fig. 2.3 A free space radio link. ERP is the power radiated from the tip of the antenna and RSL is the power received at the receiver. The transmitter and the receiver are separated by a distance d

In Fig. 2.3, let the effective radiated power be defined as ERP and the received signal level be defined as RSL. Then, we can write:

$$ERP = P_A - 2L_{\text{connector}} - L_{\text{cable}} + G_t \quad (2.12)$$

$$RSL = ERP - L_p + G_r - L_{\text{cable}} - 2L_{\text{connector}} \quad (2.13)$$

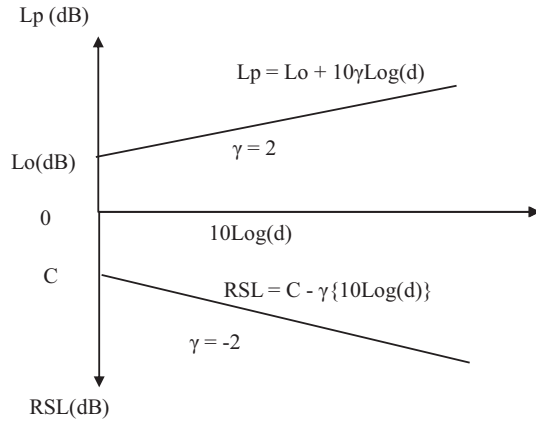
Where,

- P_A is the output power from the power amplifier
- $L_{\text{connector}}$ is the connector loss. The factor 2 is due to two connectors
- L_{cable} is the cable loss
- G_t is the transmit antenna gain
- G_r is the receive antenna gain
- L_p is the pathloss

The effective radiated power ERP is constant since all the parameters in Eq. 2.12 are constants. Now, Substituting for L_p , in Eq. 2.13 we get,

$$RSL = ERP - 32.5 - 20\text{Log}(f) - 20\text{Log}(d) + G_r - L_{\text{cable}} - 2L_{\text{connector}}$$

Fig. 2.4 Illustration of path-loss slope and RSL



Since ERP, frequency, G_r , Cable loss and Connector losses are constant parameters, the above equation reduces to,

$$\begin{aligned} \text{RSL} &= C - 20\text{Log}(d) \\ &= C - \gamma 10\text{Log}(d) \end{aligned} \quad (2.14)$$

Where,

$$C = \text{ERP} - 32.5 - 20\text{Log}(f) + G_r - L_{\text{cable}} - 2L_{\text{connector}} \quad (2.15)$$

Notice that the received signal level RSL also exhibits an equation of a straight line having an intercept C and a slope $\gamma = -2$. This is plotted in Fig. 2.4 along with the path loss characteristic.

Problem 2.3

Consider a radio link as shown in Fig. 2.3 with the following design parameters:

Frequency	$f = 1 \text{ GHz}$
Propagation medium	Free space
Distance	$d = 10 \text{ km}$
Power from the amplifier	$P_A = 10 \text{ watts}$
Transmit cable and connector losses	3 dB
Transmit antenna gain	$G_t = 10 \text{ dB}$
Receiver antenna gain	$G_r = 10 \text{ dB}$
Receive cable and connector losses	3 dB

Find:

- a. ERP in dB
- b. Path loss in dB
- c. Received Signal Level RSL in dBm

Solution:

$$\begin{aligned}
 \text{a. ERP (dB)} &= 10\text{Log}(P_A) - \text{Cable \& Connector losses} + G_t \\
 &= 10 \text{ dB} - 3 \text{ dB} + 10 \text{ dB} \\
 &= 17 \text{ dBW}
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } L_p &= 32.5 + 20\text{Log}(1000 \text{ MHz}) + 20\text{Log}(10 \text{ km}) \\
 &= 32.5 + 60 + 20 \\
 &= 112.5 \text{ dB}
 \end{aligned}$$

$$\begin{aligned}
 \text{c. RSL} &= \text{ERP} - L_p + G_r - \text{Cable \& Connector Losses} \\
 &= 17 \text{ dB} - 112.5 \text{ dB} + 10 \text{ dB} - 3 \text{ dB} \\
 &= 27 \text{ dB} - 115.5 \text{ dB} \\
 &= -88.5 \text{ dB} \\
 &= -88.5 + 30 \\
 &= -58.5 \text{ dBm}
 \end{aligned}$$

[Note: 1 W = 0 dBW. Also, 1 W = 1000 mW = 30 dBm. Therefore, 0 dBW = 30 dBm]

2.5 Conclusions

- We have derived the free-space path loss formula and have shown that it is proportional to the square of the distance.
- Free space pathloss is also proportional to the square of the frequency.
- It is shown that free space pathloss exhibits an equation of a straight line, having a pathloss slope of 2.

References

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