Preface

In recent years, problems of stability and control of nonlinear dynamical systems governed by partial differential equations and abstract differential equations have become an intensive field of research. We refer to works by Barbu [1], Coron [2], Curtain and Zwart [3], Fattorini [4], Krstic and Smyshlyaev [5], Lasiecka and Triggiani [6] as basic monographs in this area. The importance of this study is underpinned by the fact that systems of coupled nonlinear ordinary and partial differential equations describe the evolution of flexible structures, such as controlled flexible manipulators, spacecrafts endowed with elastic antennae, solar panels, and tethers. Illustrious examples of the control of models of flexible structures with elastic beams and plates are presented in monographs by Junkins and Kim [7], Komkov [8], Krabs [9], Lagnese [10], Lagnese et al. [11], Luo et al. [12], Meurer [13], Nabiullin [14], Oostveen [15], Sirazetdinov et al. [16]. Despite a vast literature on the control of flexible structures, there is still a gap in the qualitative theory of distributed parameter systems concerning the problems of partial stability and control.

The concept of stability with respect to a part of variables, introduced by Lyapunov [17], has been intensively studied in monographs by Rumyantsev and Oziraner [18], and Vorotnikov [19]. The significance of this study is justified by a well-known fact that a control system with integrals cannot be stabilized in the strong sense, but only the partial stabilization is possible for many physical systems. The majority of publications in the partial stability theory deal with finite-dimensional systems described by ordinary differential equations. The goal of this book is to provide a rigorous treatment of problems related to partial asymptotic stability and controllability for classes of infinite-dimensional mechanical systems with elastic elements. Our study is mostly based on Lyapunov's direct method, and we do not intend to cover here other approaches of the distributed parameter control theory, such as backstepping design, flatness, frequency-domain, or spectral methods. The interested reader can follow through the developments of these approaches in monographs [6, 13, 20–23].

In order to keep the book self-contained, some basic results from the theory of continuous semigroups of operators in Banach spaces are presented in Chap. 1. The rest of this book is organized as follows.

In Chap. 2, the problem of partial asymptotic stability with respect to a continuous functional is considered for a class of abstract multivalued systems on a metric space. A sufficient stability condition is derived by means of Lyapunov's direct method and the invariance principle. For the case of nonlinear continuous semigroups, asymptotic stability is characterized in terms of a differentiable Lyapunov functional. In finite-dimensional spaces, the problem of partial stabilization is considered for differential inclusions and autonomous systems of ordinary differential equations. A result on sufficient condition for partial stabilizability is proved, provided that the open-loop system admits a Lyapunov function with non-positive lower bounds of time-derivatives. The result proposed is applied to examples of single-axis stabilization of a satellite. Two cases are considered. In the first case, the attitude is controlled by thrust jets, while in the second case the satellite is controlled by means of a pair of flywheels. Explicit expressions of stabilizing feedback laws are given.

A mathematical model of a rotating body with elastic attachments is introduced in Chap. 3. It is shown that the equilibrium of the considered system cannot be made strongly asymptotically stable in the general case. This brings the motivation for considering the problem of partial stabilization with respect to a functional that represents "averaged" oscillations of the system. Such a problem is studied in detail in Sects. 3.1–3.4.

Section 3.5 is concentrated on the modeling of a robotic manipulator with flexible and rigid parts. The dynamics is described by a system of coupled ordinary and partial differential equations, which is transformed into an abstract differential equation in a suitable Hilbert space. A feedback control that ensures strong asymptotic stability of the equilibrium is proposed.

Chapter 4 is focused on the spillover analysis for infinite-dimensional systems with finite-dimensional controls. It is shown that a family of L^2-minimal controls, corresponding to low-frequency modes, can be used to solve approximately the steering problem for the complete system. This control design scheme is applied to the Euler–Bernoulli beam.

A mathematical model of a flexible-link manipulator is derived in Chap. 5 by exploiting the Timoshenko beam theory. The controller design is proposed for Galerkin's approximations with an arbitrary number of elastic modes.

In Chap. 6, we consider a mechanical system consisting of a rigid body with a thin elastic plate. The plate vibrations are governed by the Kirchhoff theory. We derive the equations of motion as a system of ordinary and partial differential equations and consider the angular acceleration of the body as the control. The dynamical equations are transformed to an infinite set of ordinary differential equations with respect to elastic coordinates. It is shown that such a system is neither controllable nor stabilizable in general. A theorem on partial stabilization of the equilibrium by a state feedback law is proved in Sect. 6.2. Then the approximate controllability problem is set out for the dynamics restricted to an invariant manifold. An estimate of the reachable set is proposed by using the approach of Chap. 4, and sufficient conditions for the approximate controllability are obtained as a corollary of this estimate. Simulation results are presented to illustrate the spill-over effect.

Many parts of this book exploit representations of solutions by the Fourier series with elastic coordinates or approximations by Galerkin's method. Such representations appear quite natural from the point of view of analytical mechanics, where generalized coordinates are used to characterize the dominant dynamics of a large-scale system. To justify this paradigm more rigorously, we prove the convergence of Galerkin's method in Appendix A for the Euler–Bernoulli beam model.

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Donetsk, July 2014 Alexander L. Zuyev

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