

Contents

1	Discrete Stochastic Processes, Numerical Methods for Markov Chains and Polynomial Time Algorithms.	1
1.1	The Basic Definitions and Some Preliminary Results.	2
1.1.1	Discrete Markov Processes and Determining the State-Time Probabilities of the System	2
1.1.2	Definition of the Limit Matrix and the Classification of the States in Markov Chains.	6
1.1.3	A Decomposition Algorithm for Determining the Limit Matrix	8
1.1.4	The z -Transform and the Asymptotic Behavior of State-Time Probabilities	11
1.2	An Algorithm for Determining the Limit Matrix Based on the z -Transform and Classical Numerical Methods	15
1.2.1	The Main Approach and the General Scheme of the Algorithm.	15
1.2.2	The Calculation of the Coefficients of the Characteristic Polynomial	19
1.2.3	Determining the z -Transform Function.	21
1.2.4	An Algorithm for Calculating the Limit Matrix.	22
1.3	An Algorithm for Determining the Differential Matrices in Markov Chains	28
1.3.1	Determining the Differential Matrices Based on the z -Transform	28
1.3.2	Linear Recurrent Equations and its Main Properties.	29
1.3.3	The Main Results and an Algorithm	30
1.3.4	Comments on the Computational Complexity of the Algorithm	32

1.4	An Algorithm for Determining the Limit and the Differential Matrices	33
1.4.1	Some Auxiliary Results Concerning the Representation of the z -Transform	33
1.4.2	Expansion of the z -Transform with Respect to Nonzero Characteristic Values	34
1.4.3	The Main Conclusion and the Description of the Algorithm	39
1.4.4	Numerical Examples	41
1.5	A Dynamic Programming Approach for Discrete Markov Processes and Combinatorial Algorithms for Determining the Limit Matrix	51
1.5.1	Calculation of the State-Time Probability of the System with the Restriction on the Number of Transitions	52
1.5.2	Determining the Limiting State Probabilities in Markov Chains Based on Dynamic Programming	60
1.5.3	An Algorithm for the Calculation of the Limit Matrix in Markov Chains with Running Time $O(n^3)$	66
1.5.4	An Algorithm for Determining the Limit Probabilities Based on the Ergodicity Condition	72
1.5.5	Determining the First Hitting Limiting Probability of the State in a Markov Chain	77
1.6	Determining the State-Time Probabilities of the System for Non-stationary Markov Processes	79
1.7	Markov Processes with Rewards and Transition Costs	84
1.7.1	Definition and Calculation of the Expected Total Cost	84
1.7.2	An Asymptotic Behavior Analysis of the Expected Total Cost Based on the z -Transform	87
1.7.3	Determining the Expected Total Cost for Non-stationary Markov Processes	91
1.7.4	Definition and Calculation of the Variance of the Total Cost	92
1.8	Markov Processes with Discounted Costs	93
1.9	Semi-Markov Processes with Transition Costs	96
1.10	Determining the Expected Total Cost for Markov Processes with Stopping States	99

2	Stochastic Optimal Control Problems and Markov Decision Processes with Infinite Time Horizon	103
2.1	Problem Formulation and the Main Concept of Optimal Control Models with Infinite Time Horizon	104
2.2	An Optimal Stationary Control with an Average Cost Criterion and Algorithms for Solving Stochastic Control Problems on Networks	108
2.2.1	Problem Formulation	108
2.2.2	A Linear Programming Approach for Determining Optimal Stationary Strategies on Perfect Networks	111
2.2.3	Remark on the Application of the Unichain Linear Programming Model for an Arbitrary Network	117
2.2.4	Determining the Solutions for an Arbitrary Unichain Control Problem and for the Deterministic Case	118
2.2.5	Dual Linear Programming for the Unichain Control Problem and an Algorithm for Determining the Optimal Strategies	120
2.2.6	The Potential Transformation and Optimality Conditions for Multichain Control Problems	122
2.2.7	Linear Programming for Multichain Control Problems and an Algorithm for Determining Optimal Stationary Strategies	133
2.2.8	Primal and Dual Linear Programming Models for the Multichain Problem	139
2.2.9	An Algorithm for Solving the Multichain Control Problem Using a Dual Unichain Model	139
2.2.10	An Approach for Solving the Multichain Control Problem Using a Reduction Procedure to a Unichain Problem	146
2.3	A Linear Programming Approach for Markov Decision Problems with an Average Cost Optimization Criterion	150
2.3.1	Problem Formulation	150
2.3.2	Reduction of Markov Decision Problems to Stochastic Control Problems	152
2.3.3	A Linear Programming Approach for the Average Markov Decision Problem and an Algorithm for Determining the Optimal Strategies	157
2.3.4	A Dual Linear Programming Model for an Average Markov Decision Problem	163

2.3.5	Optimality Conditions for Multichain Decision Problems and a Linear Programming Approach	164
2.3.6	Primal and Dual Linear Programming Models for a Multichain Markov Decision Problem	166
2.4	Iterative Algorithms for Markov Decision Processes and Control Problems with an Average Cost Criterion	166
2.5	A Discounted Stochastic Control Problem and Algorithms for Determining the Optimal Strategies on Networks.	171
2.5.1	Problem Formulation	171
2.5.2	A Linear Programming Approach for a Discounted Control Problem on Networks.	173
2.5.3	Dual Linear Programming Models for a Discounted Control Problem.	180
2.6	A Linear Programming Approach for a Discounted Markov Decision Problem	181
2.6.1	A Dual Linear Programming Model for the Discounted Markov Decision Problem	186
2.7	An Iterative Algorithm for Discounted Markov Decision Processes and Stochastic Control Problems	187
2.8	Determining the Optimal Expected Total Cost for Markov Decision Problems with a Stopping State	190
2.8.1	Problem Formulation and a Linear Programming Approach	190
2.8.2	Optimality Conditions for the Control Problem on Network with a Stopping State.	194
2.8.3	A Dynamic Programming Algorithm for Solving Deterministic Non-stationary Control Problems on Networks.	196
2.9	Discrete Decision Problems with Varying Time of State's Transitions and Special Solution Algorithms.	199
2.9.1	Problem Formulation	199
2.9.2	A Linear Programming Approach for the Problem with Arbitrary Transition Costs	201
2.9.3	Reduction of the Problem to the Case with Unit Time of States' Transitions	207
2.10	Determining the Optimal Strategies for Semi-Markov Decision Problems	209

3	A Game-Theoretical Approach to Markov Decision Processes, Stochastic Positional Games and Multicriteria Control Models.	213
3.1	Stochastic Positional Games with Average Payoff	
	Functions of the Players.	214
3.1.1	Problem Formulation.	214
3.1.2	Determining Nash Equilibria for Stochastic Positional Games with Average Payoff Functions.	216
3.1.3	Determining Nash Equilibria for Average Stochastic Positional Games Using the Potential Transformation.	223
3.1.4	Necessary and Sufficient Conditions for the Existence of Nash Equilibria in Average Stochastic Positional Games.	225
3.1.5	Nash Equilibria Conditions for Cyclic Games.	226
3.1.6	Average Stochastic Positional Games on Networks.	230
3.1.7	Saddle Point Conditions for Average Stochastic Antagonistic Positional Games and Determining the Optimal Strategies of the Players.	234
3.1.8	Saddle Point Conditions for Average Stochastic Antagonistic Positional Games on Networks.	239
3.1.9	Applications of Average Stochastic Positional Games for Studying Shapley Stochastic Games.	240
3.2	Stochastic Positional Games with Discounted Payoff	
	Functions.	242
3.2.1	Problem Formulation.	243
3.2.2	Determining Nash Equilibria for Stochastic Positional Games with Discounted Payoff Functions.	244
3.2.3	Discounted Stochastic Positional Games with Different Discount Factors for the Players.	248
3.2.4	Determining Nash Equilibria for Discounted Stochastic Positional Games Using a Potential Transformation.	249
3.2.5	Nash Equilibria Conditions for Discounted Games on Networks.	252
3.3	Nash Equilibria Conditions for Stochastic Positional Games with Stopping States and Determining Optimal Strategies of the Players in Dynamic c -Games.	254

3.3.1	Problem Formulation and the Main Results	254
3.3.2	Stochastic Positional Games with a Stopping State on Networks and Determining Nash Equilibria	257
3.3.3	Determining Optimal Stationary Strategies of the Players in Dynamic c -Games.	258
3.3.4	On Determining Nash Equilibria for Stationary Dynamic c -Games and Multicriteria Problems with Restrictions on the Number of Moves	273
3.4	Determining Pareto Solutions for Multicriteria Markov Decision Problems	274
3.5	Deterministic Antagonistic Positional Games on Networks and Algorithms for Finding Optimal Strategies of the Players.	275
3.5.1	Zero-Sum Games on Networks and Polynomial Time Algorithms for Max-Min Path Problems	276
3.5.2	An Algorithm for Solving the Problem on Acyclic Networks.	278
3.5.3	The Main Results for the Problem on an Arbitrary Network.	280
3.5.4	A Polynomial Time Algorithm for Determining the Optimal Strategies of the Players in an Antagonistic Dynamic c -Game.	282
3.5.5	Pseudo-polynomial Time Algorithms for Solving Antagonistic Dynamic c -Games	287
3.6	A Polynomial Time Algorithm for Solving Acyclic l -Games on Networks	293
3.6.1	Problem Formulation.	294
3.6.2	The Main Properties of Optimal Strategies in Acyclic l -Games	294
3.6.3	A Polynomial Time Algorithm for Finding the Value and the Optimal Strategies in the Acyclic l -Game.	296
3.7	Algorithms for Finding the Optimal Strategies of the Players in a Cyclic Game.	298
3.7.1	Problem Formulation and the Main Properties	298
3.7.2	Some Preliminary Results	300
3.7.3	The Reduction of Cyclic Games to Ergodic Ones	301
3.7.4	A Polynomial Time Algorithm for Solving Ergodic Zero-Value Cyclic Games	301
3.7.5	A Polynomial Time Algorithm for Solving Cyclic Games Based on a Reduction to Acyclic l -Games.	304

3.7.6	An Approach for Solving Cyclic Games Based on the Dichotomy Method and Solving a Dynamic c -Game	306
3.8	On Determining Pareto Optima for Cyclic Games with m Players	307
3.9	Multi-objective Control Based on the Concept of Noncooperative Games: Nash Equilibria	308
3.9.1	Stationary and Non-stationary Multi-objective Control Models	311
3.9.2	Multi-objective Control Problems with Infinite Time Horizon	311
3.10	Hierarchical Control and Stackelberg's Optimization Principle	312
3.10.1	Multi-objective Control Based on the Concept of Cooperative Games: Pareto Optima	314
3.11	Alternate Players' Control Condition and Nash Equilibria for Dynamic Games in Positional Form.	315
3.12	Determining a Stackelberg Solution for Hierarchical Control Problems	319
3.12.1	A Stackelberg Solution for Static Games	319
3.12.2	Hierarchical Control on Networks and Determining Stackelberg Stationary Strategies.	321
3.12.3	An Algorithm for Determining Stackelberg Stationary Strategies on Acyclic Networks	324
3.12.4	Algorithms for Solving Hierarchical Control Problems	330
3.13	Extensions and Generalizations of the Dynamic Decision Problem Based on Concept of Multi-objective Games	331
3.13.1	Problem Formulation	331
3.13.2	Main Results	333
3.13.3	Discrete and Matrix Multi-objective Games	337
3.13.4	Some Comments on Multi-objective Games	338
3.13.5	Determining a Pareto-Stackelberg Solution for Multi-objective Games	339
4	Dynamic Programming Algorithms for Finite Horizon Control Problems and Markov Decision Processes	341
4.1	Problem Formulation	341
4.2	Algorithms for Solving Stochastic Control Problems Using Time-Expanded Networks	345
4.2.1	Construction of the Time-Expanded Network with a Fixed Number of Transitions	345
4.2.2	An Example of Constructing the Time-Expanded Network.	347

4.2.3	Algorithms for Determining the State-Time Probabilities and the Optimal Control for the Problem with a Fixed Number of Transitions	351
4.2.4	Algorithms for Determining the State-Time Probabilities and the Optimal Control for the Problem with the Number of Transitions from a Given Interval	357
4.2.5	Determining the State-Time Probabilities and the Optimal Control Using a Modified Time-Expanded Network	361
4.3	Algorithms for Determining the Expected Total Cost and the Optimal Control Using a Time-Expanded Network.	365
4.3.1	Determining the Expected Total Cost and the Optimal Control in the Problems with a Fixed Number of Transitions	365
4.3.2	Determining the Expected Total Cost and the Optimal Control with a Given Number of Transitions and a Fixed Final State	367
4.3.3	Determining the Optimal Control and the Expected Total Cost for the Control Problem with a Restriction on the Number of Transitions	371
4.4	Discrete Control Problems with Varying Time of States' Transitions of the Dynamical System.	376
4.4.1	Deterministic Control Problem with Varying Time of States' Transitions.	377
4.4.2	The Time-Expanded Network Method for a Deterministic Control Problem with Varying Time of States' Transitions.	378
4.4.3	The Stochastic Discrete Control Problem with a Varying Time of States' Transitions	382
4.5	Dynamic Programming Algorithms for Finite Horizon Markov Decision Problems	383
4.5.1	Problem Formulations	383
4.5.2	Construction of the Time-Expanded Network for a Finite Horizon Markov Decision Process	385
4.5.3	Backward Dynamic Programming Algorithms for Finite Horizon Markov Decision Problems	386

Errata to: Optimization of Stochastic Discrete Systems and Control on Complex Networks	E1
Conclusion	389
References.	391
Index	397

Optimization of Stochastic Discrete Systems and
Control on Complex Networks

Computational Networks

Lozovanu, D.; Pickl, S.

2015, XIX, 400 p. 54 illus., Hardcover

ISBN: 978-3-319-11832-1